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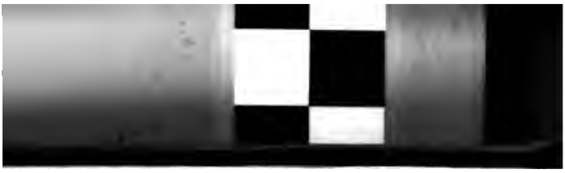

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THE FIFTH EDITION.

Philoſophia mater omnium bonarum artium eſt. CICERO. I. Tuſc.

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Prince EDWARD.

SIR,

AS Heaven has inſpired your ROYAL HIGHNESS with ſuch love of ingenious and uſeful arts, that you not only ſtudy their theory, but have often condeſcended to honour the profeſſors of mechanical and experimental philoſophy with your preſence

ſence and particular favour; I am thereby encouraged to lay myſelf and the following work at your Royal Highness's feet; and at the ſame time beg leave to expreſs that veneration with which I am,

SIR,

Your Royal Highness's

Moſt obliged,

And moſt obedient,

Humble Servant,

James Ferguson.

THE

PREFACE.

EVER ſince the days of the LORD CHANCELLOR BACON, *natural philoſophy hath been more and more cultivated in England.* THAT *great genius firſt ſet out with taking a general ſurvey of all the natural ſciences, dividing them into diſtinct branches, which he enumerated with great exactneſs. He inquired ſcrupulouſly into the degree of knowledge already attained to in each, and drew up a liſt of what ſtill remained to be diſcovered: this was the ſcope of his firſt undertaking. Afterward he carried his views* *much farther; and ſhewed the neceſſity of an experimental philoſophy, a thing never before thought of. As he was a profeſſed enemy to ſyſtems, he conſidered philoſophy, no otherwiſe than as that part of knowledge which contributes to make men better and happier: he ſeems to limit it to the knowledge of things uſeful, recommending above all the ſtudy of nature, and ſhewing that no progreſs can be made therein, but by collecting facts, and comparing experiments, of which he points out a great number proper to be made.*

But notwithſtanding the true path to ſcience was thus exactly marked out, the old notions of the ſchools ſo ſtrongly poſſeſſed people's minds at that time, as not to be eradicated by any new opinions, how rationally ſoever advanced, until the illuſtrious Mr. BOYLE, *the firſt who purſued* LORD BACON'S *plan, began to put experiments in practice with an aſſiduity equal to his great talents. Next, the* ROYAL SOCIETY *being eſtabliſhed, the true philoſophy began to be the reigning taſte of the age, and continues ſo to this day.*

The immortal SIR ISAAC NEWTON *inſiſted even in his early years, that it was high time to baniſh vague conjectures and hypotheſes from natural philoſophy, and to bring that ſcience under an entire ſubjection to experiments and geometry. He frequently called it* the experimental philoſophy, *ſo as to expreſs ſignificantly the difference between it and the numberleſs ſyſtems which had ariſen merely out of the conceits of inventive brains: the one ſubſiſting no longer than the ſpirit of novelty laſts; the other never failing whilſt the nature of things remain unchanged.*

The method of teaching and laying the foundation of phyſics, by public courſes of experiments, was firſt undertaken in this kingdom, I believe, by Dr. JOHN KEILL, *and ſince improved and enlarged by Mr.* HAUKSBEE, *Dr.* DESAGULIERS, *Mr.* WHISTON, *Mr.* COTES, *Mr.* WHITESIDE, *Dr.* BRADLEY, *our late Regius and Savilian profeſſor of Aſtronomy, and the Reverend Dr.* BLISS *his ſucceſſor.*——

Nor has the ſame been neglected by Dr. JAMES, *and Dr.* DAVID GREGORY, *Sir* ROBERT STEWART, *and after him Mr.* MACLAURIN.——*Dr.* HELSHAM *in Ireland, Meſſieurs* s'GRAVESANDE *and* MUSCHENBROEK, *and the Abbé* NOLLET *in France, have alſo acquired juſt applauſe thereby.*

The ſubſtance of my own attempt in this way of inſtrumental inſtruction, the following ſheets (excluſive of the aſtronomical part) will ſhew: the ſatisfaction they have generally given, read as lectures to different audiences, affords me ſome hope that they may be favourably received in the ſame form by the Public.

I ought to obſerve, that though the five laſt lectures connot be properly ſaid to concern experimental philoſophy, I conſidered, however, that they were not of ſo different a claſs, but that they might, without much impropriety, be ſubjoined to the preceding ones.

My apparatus (part of which is deſcribed here, and the reſt in a [*] *former work) is rather ſimple than magnificent; which is owing to a particular point I had in view at firſt ſetting out, namely, to avoid all ſuperfluity, and to render every thing as plain and intelligible as I thought the ſubject would admit of.*

---

[*] Aſtronomy explained upon SIR ISAAC NEWTON's principles, and made eaſy to thoſe who have not ſtudied mathematics.

DIRECTIONS to the BOOKBINDER.

The plates muſt open to the left-hand, fronting the right-hand pages in the following order.

PLATE I	fronting page 5
II	19
III	25
IV	27
V	49
VI	59
VII	69
VIII	83
IX	97
X	101
XI	115
XII	125
XIII	151
XIV	179
XV	201
XVI	209
XVII	215
XVIII	221
XIX	237
XX	311
XXI	323
XXII	331
XXIII	353

THE

CONTENTS.

LECTURE I.

Of matter and its properties. Page 1

LECTURE II.

Of central forces. 19

LECTURE III.

Of the mechanical powers. 47

LECTURE IV.

Of mills, cranes, wheel-carriages, and the engine for driving piles. 71

LECTURE V.

Of hydroſtatics, and hydraulic machines in general. 101

LECTURE VI.

Of pneumatics. 168

LEC-

CONTENTS.

LECTURE VII.

Of optics. Page 201

LECTURE VIII. and IX.

The deſcription and uſe of the globes and armillary ſphere. 247, 296

LECTURE X. and XI.

The principles and art of dialing. 316, 350

LECTURE XII.

Shewing how to calculate the mean time of any new or full moon, or eclipſe from the creation of the world to the year of Chriſt 5800.

The SUPPLEMENT.

LECTURES ON SELECT SUBJECTS.

LECT. I.

Of Matter and its Properties.

AS the deſign of the firſt part of this courſe is to explain and demonſtrate thoſe laws by which the material univerſe is governed, regulated, and continued; and by which the various appearances in nature are accounted for; it is requiſite to begin with explaining the properties of matter.

By the word *matter* is here meant every thing that has length, breadth, and thickneſs, and reſiſts the touch. Matter, what.

The inherent properties of matter are ſolidity, inactivity, mobility, and diviſibility. Its properties.

The *ſolidity* of matter ariſes from its having length, breadth, thickneſs; and hence it is that all bodies are comprehended under ſome ſhape or other, and that each particular body hinders all others from occupying the ſame part of ſpace which it poſſeſſeth. Thus, if a piece of wood or metal be ſqueezed ever ſo hard between two plates, they cannot be brought into contact. And even water or air has this property; for if a ſmall quantity of it be fixed between any other Solidity.

bodies, they cannot be brought to touch one another.

Inactivity. A ſecond property of matter is *inactivity*, or *paſſiveneſs*; by which it always endeavours to continue in the ſtate that it is in, whether of reſt or motion. And therefore, if one body contains twice or thrice as much matter as another body does, it will have twice or thrice as much inactivity; that is, it will require twice or thrice as much force to give it an equal degree of motion, or to ſtop it after it hath been put into ſuch a motion.

That matter can never put itſelf into motion is allowed by all men. For they ſee that a ſtone, lying on the plane ſurface of the earth, never removes itſelf from that place, nor does any one imagine it ever can. But moſt people are apt to believe that all matter has a propenſity to fall from a ſtate of motion into a ſtate of reſt; becauſe they ſee that if a ſtone or a cannon-ball be put into ever ſo violent a motion, it ſoon ſtops; not conſidering that this ſtoppage is cauſed, 1. By the gravity or weight of the body, which ſinks it to the ground in ſpite of the impulſe; and, 2. By the reſiſtance of the air through which it moves, and by which its velocity is retarded every moment till it falls.

A bowl moves but a ſhort way upon a bowling-green; becauſe the roughneſs and unevenneſs of the graſſy ſurface ſoon creates friction enough to ſtop it. But if the green were perfectly level, and covered with poliſhed glaſs, and the bowl were perfectly hard, round, and ſmooth, it would go a great way farther; as it would have nothing but the air to reſiſt it; if then the air were taken away, the bowl would go on without any friction, and conſequently without any diminution of the velocity it had at ſetting out: and therefore, if the green were extended quite around the earth, the bowl would go on, round and round the earth, for ever.

If the bowl were carried ſeveral miles above the earth, and there projected in a horizontal direction, with ſuch a velocity as would make it move more than a ſemidiameter of the earth, in the time it would take to fall to the earth by gravity; in that caſe, and if there were no reſiſting medium in the way, the bowl would not fall to the earth at all; but would continue to circulate round it, keeping always in the ſame tract, and returning to the ſame point from which it was projected, with the ſame velocity as at firſt. In this manner the moon goes round the earth, although ſhe be as unactive and dead as any ſtone upon it.

Mobility. The third property of matter is *mobility*; for we find that all matter is capable of being moved, if a ſufficient degree of force be applied to overcome its inactivity or reſiſtance.

Diviſibility. The fourth property of matter is *diviſibility*, of which there can be no end. For, ſince matter can never be annihilated by cutting or breaking, we can never imagine it to be cut into ſuch ſmall particles, but that if one of them be laid on a table, the uppermoſt ſide of it will be further from the table than the undermoſt ſide. Moreover, it would be abſurd to ſay that the greateſt mountain on earth has more halves, quarters, or tenth parts, than the ſmalleſt particle of matter has.

We have many ſurpriſing inſtances of the ſmallneſs to which matter can be divided by art: of which the two following are very remarkable.

1. If a pound of ſilver be melted with a ſingle grain of gold, the gold will be equally diffuſed through the whole ſilver; ſo that taking one grain from any part of the maſs (in which there can be no more than the 5760th part of a grain of gold) and diſſolving it in *aqua fortis*, the gold will fall to the bottom.

2. The gold beaters can extend a grain of gold into a leaf containing 50 ſquare inches; and this leaf may be divided into 500000 viſible parts. For an inch in length can be divided into 100 parts, every one of which will be viſible to the bare eye: conſequently a ſquare inch can be divided into 10000 parts, and 50 ſquare inches into 500000. And if one of theſe parts be viewed with a microſcope that magnifies the diameter of an object only 10 times, it will magnify the area 100 times; and then the 100th part of a 500000th part of a grain (that is, the 50 millionth part) will be viſible. Such leaves are commonly uſed in gilding; and they are ſo very thin, that if 124500 of them were laid upon one another, and preſſed together, they would not exceed one inch in thickneſs.

Yet all this is nothing in compariſon of the lengths that nature goes in the diviſion of matter. For Mr. *Leewenhoek* tells us, that there are more animals in the milt of a ſingle cod-fiſh, than there are men upon the whole earth: and that, by comparing theſe animals in a microſcope with grains of common ſand, it appeared that one ſingle grain is bigger than four millions of them. Now each animal muſt have a heart, arteries, veins, muſcles, and nerves, otherwiſe they could neither live nor move. How inconceivably ſmall then muſt the particles of their blood be, to circulate through the ſmalleſt rami-

fications

fications and joinings of their arteries and veins? It has been found by calculation, that a particle of their blood muſt be as much ſmaller than a globe of the tenth part of an inch in diameter, as that globe is ſmaller than the whole earth; and yet, if theſe particles be compared with the particles of light, they will be found to exceed them as much in bulk as mountains do ſingle grains of ſand. For, the force of any body ſtriking againſt an obſtacle is directly in proportion to its quantity of matter multiplied into its velocity:- and ſince the velocity of the particles of light is demonſtrated to be at leaſt a million times greater than the velocity of a cannon-ball, it is plain, that if a million of theſe particles were as big as a ſingle grain of ſand, we durſt no more open our eyes to the light, than we durſt expoſe them to ſand ſhot point-blank from a cannon.

Plate I. Fig. 1.

The infinite diviſibility of matter proved.

That matter is infinitely diviſible, in a mathematical ſenſe, is eaſy to be demonſtrated. For, let *AB* be the length of a particle to be divided; and let it be touched at oppoſite ends by the parallel lines *CD* and *EF*, which, ſuppoſe to be infinitely extended beyond *D* and *F*. Set off the equal diviſions *BG*, *GH*, *HI*, &c. on the line *EF*, towards the right hand from *B*; and take a point, as at *R*, any where toward the left hand from A, in the line *CD*: Then, from this point, draw the right lines *RG*, *RH*, *RI*, &c. each of which will cut off a part from the particle *AB*. But after any finite number of ſuch lines are drawn, there will ſtill remain a part, as *AP*, at the top of the particle, which can never be cut off: becauſe the lines *DR* and *EF* being parallel, no line can ever be drawn from the point *R* to any point of the line *EF* that will coincide with the line *RD*. Therefore the particle *AB* contains more than any finite number of parts.

Attraction.

A fifth property of matter is *attraction*, which ſeems rather to be infuſed than inherent. Of this there are four kinds, viz. *coheſion*, *gravitation*, *magnetiſm*, and *electricity*.

Coheſion.

The *attraction of coheſion* is that by which the ſmall parts of matter are made to ſtick and cohere together. Of this we have ſeveral inſtances, ſome of which follow.

1. If a ſmall glaſs tube, open at both ends, be dipt in water, the water will riſe up in the tube to a conſiderable height above its level in the baſon: which muſt be owing to the attraction of a ring of particles of the glaſs all round in the tube, immediately above thoſe to which the water at any inſtant riſes. And when it has riſen ſo high, that the weight of the column balances the attraction of the tube, it riſes no higher. This can be no ways owing to the preſſure of the air upon the water in the baſon; for, as the tube is open at top, it is full of air above the water, which will preſs as much upon the water in the tube as the neighbouring air does upon any column of an equal diameter in the baſon. Beſides, if the ſame experiment be made in an exhauſted receiver of the air-pump, there will be found no difference.

2. A piece of loaf-ſugar will draw up a fluid, and a ſpunge will draw in water: and on the ſame principle ſap aſcends in trees.

3. If two drops of quickſilver be placed near each other, they will run together and become one large drop.

4. If two pieces of lead be ſcraped clean, and preſſed together with a twiſt, they will attract

each other ſo ſtrongly, as to require a force much greater than their own weight to ſeparate them. And this cannot be owing to the preſſure of the air, for the ſame thing will hold in an exhauſted receiver.

5. If two poliſhed plates of marble or braſs be put together, with a little oil between them to fill up the pores in their ſurfaces, and prevent the lodgement of any air; they will cohere ſo ſtrongly, even if ſuſpended in an exhauſted receiver, that the weight of the lower plate will not be able to ſeparate it from the upper one. In putting theſe plates together, the one ſhould be rubbed upon the other, as a joiner does two pieces of wood when he glues them.

6. If two pieces of cork, equal in weight, be put near each other in a baſon of water, they will move equally faſt toward each other with an accelerated motion, until they meet: and then, if either of them be moved, it will draw the other after it. If two corks of unequal weights be placed near each other, they will approach with accelerated velocities inverſely proportionate to their weights: that is, the lighter cork will move as much faſter than the heavier, as the heavier exceeds the lighter in weight. This ſhews that the attraction of each cork is in direct proportion to its weight or quantity of matter.

This kind of attraction reaches but to a very ſmall diſtance; for, if two drops of quickſilver be rolled in duſt, they will not run together, becauſe the particles of duſt keep them out of the ſphere of each other's attraction.

Repulſion.

Where the ſphere of attraction ends, a *repulſive force* begins; thus, water repels moſt bodies till they are wet; and hence it is, that a ſmall needle, if dry, ſwims upon water; and flies walk upon it without wetting their feet.

The repelling force of the particles of a fluid is but ſmall; and therefore, if a fluid be divided, it eaſily unites again. But if glaſs, or any other hard ſubſtance, be broke into ſmall parts, they cannot be made to ſtick together again without being firſt wetted: the repulſion being too great to admit of a re-union.

The repelling force between water and oil is ſo great, that we find it almoſt impoſſible to mix them ſo, as not to ſeparate again. If a ball of light wood be dipt in oil, and then put into water, the water will recede ſo as to form a channel of ſome depth all around the ball.

The repulſive force of the particles of air is ſo great, that they can never be brought ſo near togather by condenſation as to make them ſtick or cohere. Hence it is, that when the weight of the incumbent atmoſphere is taken off from any ſmall quantity of air, that quantity will diffuſe itſelf ſo as to occupy (in compariſon) an infinitely greater portion of ſpace than it did before.

Gravitation.

Attraction of gravitation is that power by which diſtant bodies tend towards one another. Of this we have daily inſtances in the falling of bodies to the earth. By this power in the earth it is, that bodies, on whatever ſide, fall in lines perpendicular to its ſurface; and conſequently, on oppoſite ſides, they fall in oppoſite directions; all towards the center, where the force of gravity is as it were accumulated: and by this power it is, that bodies on the earth's ſurface are kept to it on all ſides, ſo that they cannot fall from it. And as it acts upon all bodies in proportion to their reſpective quantities of matter, without any regard to their bulks or figures, it accordingly conſtitutes their weight. Hence,

If two bodies which contain equal quantities of matter, were placed at ever ſo great a diſtance from one another, and then left at liberty in free ſpace; if there were no other bodies in the univerſe to affect them, they would fall equally ſwift towards one another by the power of gravity, with velocities accelerated as they approached each other; and would meet in a point which was half-way between them at firſt. Or, if two bodies, containing unequal quantities of matter, were placed at any diſtance, and left in the ſame manner at liberty, they would fall towards one another with velocities which would be in an inverſe proportion to their reſpective quantities of matter; and moving faſter and faſter in their mutual approach, would at laſt meet in a point as much nearer to the place from which the heavier body began to fall, than to the place from which the lighter body began to fall, as the quantity of matter in the former exceeded that in the latter.

All bodies that we know of have gravity or weight. For, that there is no ſuch thing as poſitive levity, even in ſmoke, vapours, and fumes, is demonſtrable by experiments on the air-pump; which ſhews, that although the ſmoke of a candle aſcends to the top of a tall receiver when full of air, yet, upon the air's being exhauſted out of the receiver, the ſmoke falls down to the bottom of it. So, if a piece of wood be immerſed in a jar of water, the wood will riſe to the top of the water, becauſe it has a leſs degree of weight than its bulk of water has: but if the jar be emptied of water, the wood falls to the bottom.

Gravity demonſtrated to be as the quantity of matter in bodies.

As every particle of matter has its proper gravity, the effect of the whole muſt be in proportion to the number of the attracting particles; that is, as the quantity of matter in the whole body. This is demonſtrable by experiments on pendulums; for, if they are of equal lengths, whatever their weights be, they vibrate in equal times. Now it is plain, that if one be double or triple the weight of another, it muſt require a double or triple power of gravity to make it move with the ſame celerity: juſt as it would require a double or triple force to project a bullet of twenty or thirty pounds weight with the ſame degree of ſwiftneſs that a bullet of ten pounds would require. Hence it is evident, that the power or force of gravity is always proportional to the quantity of matter in bodies, whatever their bulks or figures are.

It decreaſes as the ſquare of the diſtance increaſes.

Gravity alſo, like all other virtues or emanations which proceed or iſſue from a center, decreaſes as the diſtance multiplied by itſelf increaſes: that is, a body at twice the diſtance of another, attracts with only a fourth part of the force; at thrice the diſtance, with a ninth part; at four times the diſtance, with a ſixteenth part; and ſo on. This too is confirmed by comparing the diſtance which the moon falls in a minute, from a right line touching her orbit, with the diſtance through which heavy bodies near the earth fall in that time. And alſo by comparing the forces which retain Jupiter's moons in their orbits, with their reſpective diſtances from Jupiter. Theſe forces will be explained in the next lecture.

The velocity which bodies near the earth acquire in deſcending freely by the force of gravity, is proportional to the times of their deſcent.

For, as the power of gravity does not conſiſt in a ſingle impulſe, but is always operating in a conſtant and uniform manner, it muſt produce equal effects in equal times; and conſequently in a double or triple time, a double or triple effect. And ſo, by acting uniformly on the body, muſt accelerate its motion proportionably to the time of its deſcent.

To be a little more particular on this ſubject, let us ſuppoſe that a body begins to move with a celerity conſtantly and gradually increaſing, in ſuch a manner, as would carry it through a mile in a minute; at the end of this ſpace it will have acquired ſuch a degree of celerity, as is ſufficient to carry it two miles the next minute, though it ſhould then receive no new impulſe from the cauſe by which its motion had been accelerated; but if the ſame accelerating cauſe continues, it will carry the body a mile farther; on which account, it will have run through four miles at the end of two minutes; and then it will have acquired ſuch a degree of celerity as is ſufficient to carry it through a double ſpace in as much more time, or eight miles in two minutes, even though the accelerating force ſhould act upon it no more. But this force ſtill continuing to operate in an uniform manner, will again, in an equal time, produce an equal effect; and ſo, by carrying it a mile further, cauſe it to move through five miles the third minute; for, the celerity already acquired, and the celerity ſtill acquiring, will have each its complete effect. Hence we learn, that if the body ſhould move one mile the firſt minute, it would move three miles the ſecond, five the third, ſeven the fourth, nine the fifth, and ſo on in proportion.

And thus it appears, that the ſpaces deſcribed in ſucceſſive equal parts of time, by an uniformly accelerated motion, are always as the odd numbers 1, 3, 5, 7, 9, &c. and conſequently, the whole ſpaces are as the ſquares of the times, or of the laſt acquired velocities. For, the continued addition of the odd numbers yields the ſquares of all numbers from unity upwards. Thus, 1 is the firſt odd number, and the ſquare of 1 is 1; 3 is the ſecond odd number, and this added to 1 makes 4, the ſquare of 2; 5 is the third odd number, which added to 4 makes 9, the ſquare of 3; and ſo on for ever. Since, therefore, the times and velocities proceed evenly and conſtantly as 1, 2, 3, 4, &c. but the ſpaces deſcribed in each equal times are as 1, 3, 5, 7, &c. it is evident that the ſpace deſcribed

In 1 minute will be - - - $1=$ſquare of 1
In 2 minutes - - $1+3=4=$ſquare of 2
In 3 minutes - $1+3+5=9=$ſquare of 3
In 4 minutes $1+3+5+7=16=$ſquare of 4, &c.

N. B. The character + ſignifies *more*, and = *equal*.

The deſcending velocity will give a power of equal aſcent.

As heavy bodies are uniformly accelerated by the power of gravity in their deſcent, it is plain that they muſt be uniformly retarded by the ſame power in their aſcent. Therefore, the velocity which a body acquires by falling, is ſufficient to carry it up again to the ſame height from whence it fell: allowance being made for the reſiſtance of the air, or other medium in which the body is moved. Thus, the body *D* in rolling down the inclined plane *AB* will acquire ſuch a velocity by the time it arrives at *B*, as

Fig. 2.

B, as will carry it up the inclined plane *BC*, almoſt to *C*; and would carry it quite up to *C*, if the body and plane were perfectly ſmooth, and the air gave no reſiſtance.—So, if a pendulum were put into motion, in a ſpace quite free of air, and all other reſiſtance, and had no friction on the point of ſuſpenſion, it would move for ever: for the velocity it had acquired in falling through the deſcending part of the arc, would be ſtill ſufficient to carry it equally high in the aſcending part thereof.

The center of gravity,

The *center of gravity* is that point of a body in which the whole force of its gravity or weight is united. Therefore, whatever ſupports that point bears the weight of the whole body: and whilſt it is ſupported, the body cannot fall; becauſe all its parts are in a perfect equilibrium about that point.

and line of direction.

An imaginary line drawn from the center of gravity of any body towards the center of the earth, is called the *line of direction*. In this line all heavy bodies deſcend, if not obſtructed.

Since the whole weight of a body is united in its center of gravity, as that center aſcends or deſcends, we muſt look upon the whole body to do ſo too. But as it is contrary to the nature of heavy bodies to aſcend of their own accord, or not to deſcend when they are permitted; we may be ſure that, unleſs the center of gravity be ſupported, the whole body will tumble or fall. Hence it is, that bodies ſtand upon their baſes when the line of direction falls within the baſe; for in this caſe the body cannot be made to fall without firſt raiſing the center of gravity higher than it was before. Thus, the inclining body *ABCD*, whoſe center of gravity is *E*, ſtands firmly on its baſe *CDIK*, becauſe the line Fig. 3.

of direction *E F* falls within the baſe. But if a weight, as *A B G H*, be laid upon the top of the body, the center of gravity of the whole body and weight together is raiſed up to *I*; and then, as the line of direction *I D* falls without the baſe at *D*, the center of gravity *I* is not ſupported; and the whole body and weight tumble down together.

Hence appears the abſurdity of people's riſing haſtily in a coach or boat when it is likely to overſet: for, by that means they raiſe the center of gravity ſo far as to endanger throwing it quite out of the baſe; and if they do, they overſet the vehicle effectually. Whereas, had they clapt down to the bottom, they would have brought the line of direction, and conſequently the center of gravity, farther within the baſe, and by that means might have ſaved themſelves.

The broader the baſe is, and the nearer the line of direction is to the middle or center of it, the more firmly does the body ſtand. On the contrary, the narrower the baſe, and the nearer the line of direction is to the ſide of it, the more eaſily may the body be overthrown, a leſs change of poſition being ſufficient to remove the line of direction out of the baſe in the latter caſe than in the former. And hence it is, that a ſphere is ſo eaſily rolled upon a horizontal plane; and that it is ſo difficult, if not impoſſible, to make things which are ſharp-pointed to ſtand upright on the point.—From what hath been ſaid, it plainly appears that if the plane be inclined on which the heavy body is placed, the body will ſlide down upon the plane whilſt the line of direction falls within the baſe; but it will tumble or roll down when that line falls without the baſe. Thus, the body *A* will only ſlide down the inclined plane *C D*, whilſt the body *B* rolls down upon it. Fig. 4.

When the line of direction falls within the baſe of our feet, we ſtand; and moſt firmly when it is in the middle: but when it is out of that baſe, we immediately fall. And it is not only pleaſing, but even ſurpriſing, to reflect upon the various and unthought-of methods and poſtures which we uſe to retain this poſition, or to recover it when it is loſt. For this purpoſe we bend our body forward when we riſe from a chair, or when we go up ſtairs: and for this purpoſe a man leans forward when he carries a burden on his back, and backward when he carries it on his breaſt; and to the right or left ſide as he carries it on the oppoſite ſide. A thouſand more inſtances might be added.

The quantity of matter in all bodies is in exact proportion to their weights, bulk for bulk. Therefore, heavy bodies are as much more denſe or compact than light bodies of the ſame bulk, as they exceed them in weight.

All bodies porous.

All bodies are full of pores, or ſpaces void of matter: and in gold, which is the heavieſt of all known bodies, there is perhaps a greater quantity of ſpace than of matter. For the particles of heat and magnetiſm find an eaſy paſſage through the pores of gold; and even water itſelf has been forced through them. Beſides, if we conſider how eaſily the rays of light paſs through ſo ſolid a body as glaſs, in all manner of directions, we ſhall find reaſon to believe that bodies are much more porous than is generally imagined.

The expanſion of metals.

All bodies are ſome way or other affected by heat; and all metallic bodies are expanded in length, breadth, and thickneſs thereby.—The proportion of the expanſion of ſeveral metals, according to the beſt experiments I have been able to make with my pyrometer, is nearly thus: Iron and ſteel as 3, copper 4 and an half; braſs 5, tin 6, lead 7. An iron rod 3 feet long is about one 70th part of an inch longer in ſummer than in winter.

The pyrometer. The pyrometer here mentioned being (for aught I know) of a new conſtruction, a deſcription of it may perhaps be agreeable to the reader.

Fig. 5. *AA* is a flat piece of mahogany, in which are fixed four braſs ſtuds *B*,*C*,*D*,*L*; and two pins, one at *F* and the other at *H*. On the pin *F* turns the crooked index *E I*, and upon the pin *H* the ſtraight index *G K*, againſt which a piece of watch-ſpring *R* bears gently, and ſo preſſes it towards the beginning of the ſcale *MN*, over which the point of that index moves. This ſcale is divided into inches and tenth parts of an inch: the firſt inch is marked 1000, the ſecond 2000, and ſo on. A bar of metal *O* is laid into notches in the top of the ſtuds *C* and *D*; one end of the bar bearing againſt the adjuſting ſcrew *P*, and the other end againſt the crooked index *E I*, at a 20th part of its length from its center of motion *F*.—Now it is plain, that however much the bar *O* lengthens, it will move that part of the index *EI*, againſt which it bears, juſt as far: but the crooked end of the ſame index, near *H*, being 20 times as far from the center of motion *F* as the point is againſt which the bar bears, it will move 20 times as far as the bar lengthens. And as this crooked end bears againſt the index *G K* at only a 20th part of the whole length *GS* from its center of

motion *H*, the point S will move through 20 times the ſpace that the point of bearing near *H* does. Hence, as 20 multiplied by 20 produces 400, it is evident that if the bar lengthens but a 400th part of an inch, the point *S* will move a whole inch on the ſcale; and as every inch is divided into 10 equal parts, if the bar lengthens but the 10th part of the 400th part of an inch, which is only the 4000th part of an inch, the point *S* will move the tenth part of an inch, which is very perceptible.

To find how much a bar lengthens by heat, firſt lay it cold into the notches of the ſtuds, and turn the adjuſting ſcrew *P* until the ſpring *R* brings the point *S* of the index *G K* to the beginning of the diviſions of the ſcale at *M:* then, without altering the ſcrew any farther, take off the bar and rub it with a dry woollen cloth till it feels warm; and then, laying it on where it was, obſerve how far it puſhes the point *S* upon the ſcale by means of the crooked index *E I*; and the point *S* will ſhew exactly how much the bar has lengthened by the heat of rubbing. As the bar cools, the ſpring *R* bearing againſt the index *K G*, will cauſe its point *S* to move gradually back towards *M* in the ſcale: and when the bar is quite cold, the index will reſt at *M*, where it was before the bar was made warm by rubbing. The indexes have ſmall rollers under them at *I* and *K*; which, by turning round on the ſmooth wood as the indexes move, make their motions the eaſier, by taking off a great part of the friction, which would otherwiſe be on the pins *F* and *H*, and of the points of the indexes themſelves on the wood.

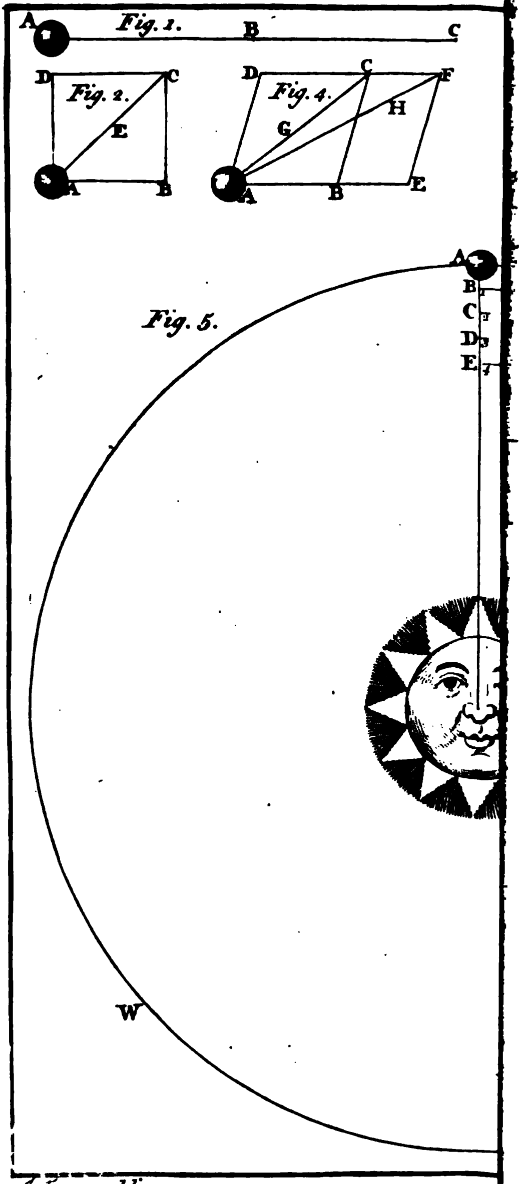
Beſides the univerſal properties above mentioned, there are bodies which have properties peculiar Magnetiſm.

peculiar to themſelves: ſuch as the loadſtone, in which the moſt remarkable are theſe: 1. It attracts iron and ſteel only. 2. It conſtantly turns one of its ſides to the north and another to the ſouth, when ſuſpended by a thread that does not twiſt. 3. It communicates all its properties to a piece of ſteel when rubbed upon it, without loſing any itſelf.

According to Dr. *Helſham*'s experiments, the attraction of the loadſtone decreaſes as the ſquare of the diſtance increaſes. Thus, if a loadſtone be ſuſpended at one end of a balance, and counterpoiſed by weights at the other end, and a flat piece of iron be placed beneath it, at the diſtance of four tenths of an inch, the ſtone will immediately deſcend and adhere to the iron. But if the ſtone be again removed to the ſame diſtance, and as many grains be put into the ſcale at the other end as will exactly counterbalance the attraction, then, if the iron be brought twice as near the ſtone as before, that is, only two tenth parts of an inch from it, there muſt be four times as many grains put into the ſcale as before, in order to be a juſt counterbalance to the attractive force, or to hinder the ſtone from deſcending and adhering to the iron. So, if four grains will do in the former caſe, there muſt be ſixteen in the latter. But from ſome later experiments, made with the greateſt accuracy, it is found that the force of magnetiſm decreaſes in a ratio between the reciprocal of the ſquare and the reciprocal of the cube of the diſtance; approaching to the one or the other, as the magnitudes of the attracting bodies are varied.

Electricity. Several bodies, particularly amber, glaſs, jet, ſealing-wax, agate, and almoſt all precious ſtones, have a peculiar property of attracting and

PLATE II.



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and repelling light bodies when heated by rubbing. This is called *electrical attraction*, in which the chief things to be obſerved are, 1. If a glaſs tube about an inch and a half diameter, and two or three feet long, be heated by rubbing, it will alternately attract and repel all light bodies when held near them. 2. It does not attract by being heated without rubbing. 3. Any light body being once repelled by the tube, will never be attracted again till it has touched ſome other body. 4. If the tube be rubbed by a moiſt hand, or any thing that is wet, it totally deſtroys the electricity. 5. Any body, except air, being interpoſed, ſtops the electricity. 6. The tube attracts ſtronger when rubbed over with bees-wax, and then with a dry woollen-cloth. 7. When it is well rubbed, if a finger be brought near it, at about the diſtance of half an inch, the effluvia will ſnap againſt the finger, and make a little crackling noiſe; and if this be performed in a dark place, there will appear a little flaſh of light.

LECT. II.

Of central Forces.

All bodies equally indifferent to motion or reſt. Plate II. Fig. 1.

WE have already mentioned it as a neceſſary conſequence ariſing from the deadneſs or inactivity of matter, that all bodies endeavour to continue in the ſtate they are in, whether of reſt or motion. If the body *A* were placed in any part of free ſpace, and nothing either drew or impelled it any way, it would for ever remain in that part of ſpace, becauſe it could have no tendency of itſelf to remove any way from thence. If it receives a ſingle impulſe any way, as ſuppoſe from *A* towards *B*, it will go on in that direction; for, of itſelf, it could never ſwerve from a right line, nor ſtop its courſe.—When it has gone through the ſpace *AB*, and met with no reſiſtance, its velocity will be the ſame at *B* as it was at *A*: and this velocity, in as much more time, will carry it through as much more ſpace, from *B* to *C*; and ſo on for ever. Therefore, when we ſee a body in motion, we conclude that ſome other ſubſtance muſt have given it that motion; and when we ſee a body fall from motion to reſt, we conclude that ſome other body or cauſe ſtopt it.

All motion naturally rectilineal.

As all motion is naturally rectilineal, it appears, that a bullet projected by the hand, or ſhot from a cannon, would for ever continue to move in the ſame direction it received at firſt, if no other power diverted its courſe. Therefore when we ſee a body move in a curve of any kind whatever, we conclude it muſt be acted upon by two powers at leaſt; one putting it in motion, and another drawing it off from the rectilineal courſe it would otherwiſe have continued to move in: and whenever that power, which bent the motion of the body from a ſtraight line into a curve, ceaſes to act, the body will again move on in a ſtraight line touching that point of the curve in which it was when the action of that power ceaſed. For example, a pebble moved round in a ſling ever ſo long a time, will fly off the moment it is ſet at liberty, by ſlipping one end of the ſling cord: and will go on in a line touching the circle it deſcribed before: which line would actually be a ſtraight one, if the earth's attraction did not affect the pebble, and bring it down to the ground. This ſhews that the natural tendency of the pebble, when put into motion, is to continue moving in a ſtraight line, although by the force that moves the ſling it be made to revolve in a circle.

The effects of combined forces.

The change of motion produced is in proportion to the force impreſſed: for the effects of natural cauſes are always proportionate to the force or power of thoſe cauſes.

By theſe laws it is eaſy to prove that a body will deſcribe the diagonal of a ſquare or parallelogram, by two forces conjoined, in the ſame time that it would deſcribe either of the ſides, by one force ſingly. Thus, ſuppoſe the body *A* to repreſent a ſhip at ſea; and that it is driven by the wind, in the right line *AB*, with ſuch a force as would carry it uniformly from *A* to *B* in a minute: then ſuppoſe a ſtream or current of water running in the direction *AD*, with ſuch a force as would carry the ſhip through an equal ſpace from *A* to *D* in a minute. By theſe two forces, acting together at right angles to each other, the ſhip will deſcribe the line *AEC* in a minute: which line (becauſe the forces are equal and perpendicular to each other) will be the diagonal of an exact ſquare. To confirm this law by an experiment, let there be a wooden ſquare *ABCD* ſo contrived, as to have the part *BEFC* made to draw out or puſh into the ſquare at pleaſure. To this part let the pully *H* be joined, ſo as to turn freely on an axis, which will be at *H* when the piece is puſhed in, and at *b* when it is drawn out. To this part let the ends of a ſtraight wire *k* be fixed, ſo as to move along with it, under the pulley; and let the ball *G* be made to ſlide eaſily on the wire. A thread *m* is fixed to this ball, and goes over the pulley to *I*; by this thread the ball may be drawn up on the wire, parallel to the ſide *AD*, when the

Fig. 2.

Fig. 3.

to revolve about *S* in the circle *ATW*; in the ſame manner, and by the ſame law, that a pebble is moved round in a ſling. And if, when the body is in any part of its orbit (as ſuppoſe at *K*) a ſmaller body as *L*, within the ſphere of attraction of the body *K*, be projected in the right line *LM*, with a force duly adjuſted, and perpendicular to the line of attraction *LK*; then, the ſmall body *L* will revolve about the large body *K* in the orbit *NO*, and accompany it in its whole courſe round the yet larger body *S*. But then, the body *K* will no longer move in the circle *ATW*; for that circle will now be deſcribed by the common center of gravity between *K* and *L*. Nay, even the great body *S* will not keep in the center; for it will be the common center of gravity between all the three bodies *S*, *K*, and *L*, that will remain immoveable there. So, if we ſuppoſe *S* and *K* connected by a wire *P* that has no weight, and *K* and *L* connected by a wire *q* that has no weight, the common center of gravity of all theſe three bodies will be a point in the wire *P* near *S*; which point being ſupported, the bodies will be all in *equilibrio* as they move round it. Though indeed, ſtrictly ſpeaking, the common center of gravity of all the three bodies will not be in the wire *P* but when theſe bodies are all in the right line. Here, *S* may repreſent the ſun, *K* the earth, and *L* the moon.

In order to form an idea of the curves deſcribed by two bodies revolving about their common center of gravity, whilſt they themſelves with a third body are in motion round the common center of gravity of all the three; let us firſt ſuppoſe *E* to be the ſun, and *e* the earth going round him without any moon; and their

See Plate III.

their moving forces regulated as above. In this caſe, whilſt the earth goes round the ſun in the dotted circle *RTUWX*, &c. the ſun will go round the circle *ABD*, whoſe center *C* is the common center of gravity between the ſun and earth: the right line βd repreſenting the mutual attraction between them, by which they are as firmly connected as if they were fixed at the two ends of an iron bar ſtrong enough to hold them. So, when the earth is at *e*, the ſun will be at *E*; when the earth is at *T*, the ſun will be at *F*; and when the earth is at *g*, the ſun will be at *G*, &c.

The curves deſcribed by bodies revolving about their common center of gravity.

Now, let us take in the moon *q* (at the top of the figure) and ſuppoſe the earth to have no progreſſive motion about the ſun; in which caſe, whilſt the moon revolves about the earth in her orbit 𝔄𝔅ℭ𝔇, the earth will revolve in the circle *S* 13, whoſe center *R* is the common center of gravity of the earth and moon; they being connected by the mutual attraction between them in the ſame manner as the earth and ſun are.

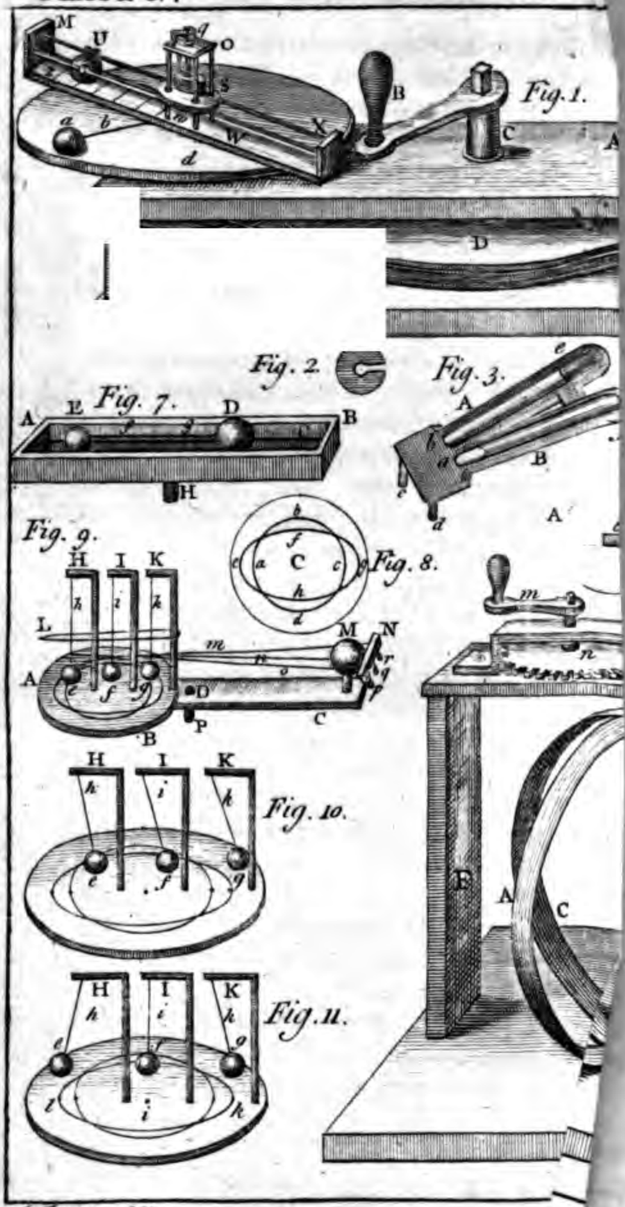
But the truth is, that whilſt the moon revolves about the earth, the earth is in motion about the ſun: and now, the moon will cauſe the earth to deſcribe an irregular curve, and not a true circle, round the ſun; it being the common center of gravity of the earth and moon that will then deſcribe the ſame circle which the earth would have moved in, if it had not been attended by a moon. For, ſuppoſing the moon to deſcribe a quarter of her progreſſive orbit about the earth in the time that the earth moves from *e* to *f*; it is plain, that when the earth comes to *f*, the moon will be found at *r*; in which time, their common center of gravity will have deſcribed the dotted arc *R* 1 *T*, the earth the curve *R* 5 *f*, and the moon the curve *q* 14 *r*. In the time that the moon deſcribes another quarter of her orbit, the center of gravity of the earth and moon will deſcribe the dotted arc *T* 2 *U*, the earth the curve *f* 6 *g*, the moon the curve *r* 15 *s*, and ſo on—And thus, whilſt the moon goes once round the earth in her progreſſive orbit, their common center of gravity deſcribes the regular portion of a circle *R* 1 *T* 2 *U* 3 *V* 4 *W*, the earth the irregular curve *R* 5 *f* 6 *g* 7 *h* 8 *i*, and the moon the yet more irregular curve *q* 14 *r* 15 *s* 16 *t* 17 *u*; and then, the ſame kind of tracks over again.

The center of gravity of the earth and moon is 6000 miles from the earth's center towards the moon; therefore the circle *S* 13 which the earth deſcribes round that center of gravity (in every courſe of the moon round her orbit) is 12000 miles in diameter. Conſequently the earth is 12,000 miles nearer the ſun at the time of full moon than at the time of new. [See the earth at *f* and at *h*.]

To avoid confuſion in ſo ſmall a figure, we have ſuppoſed the moon to go only twice and a half round the earth, in the time that the earth goes once round the ſun: it being impoſſible to take in all the revolutions which ſhe makes in a year, and to give a true figure of her path, unleſs we ſhould make the ſemidiameter of the earth's orbit at leaſt 95 inches; and then, the proportional ſemidiameter of the moon's orbit would be only a quarter of an inch.—For a true figure of the moon's path, I refer the reader to my treatiſe of aſtronomy.

If the moon made any complete number of revolutions about the earth in the time that the earth

PLATE IV.



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one revolution about the ſun, the
e ſun and moon would return into
at the end of every year; and ſo be
ver again: but they return not into
n leſs than 19 years nearly; in which
earth makes nearly 19 revolutions
n, and the moon 235 about the earth.
inet *A* be attracted towards the ſun,
force as would make it fall from *A*
ie time that the projectile impulſe
carried it from *A* to *F*, it will de-
: *AG* by the combined action of theſe
he ſame time that the former would
it to fall from *A* to *B*, or the latter
it from *A* to *F*. But, if the projec-
l been twice as great, that is, ſuch as
carried the planet from *A* to *H*, in
ie that now, by the ſuppoſition, it
y from *A* to *F*; the ſun's attraction
ve been four times as ſtrong as for-
iave kept the planet in the circle
is, it muſt have been ſuch as would
the planet to fall from *A* to *E*,
times the diſtance of *A* from *B*, in
: the projectile force ſingly would
it from *A* to *H*, which is only twice
of *A* from *F**. Thus, a double pro-
will balance a quadruple power of
e ſame circle; as appears plain by
ind ſhall ſoon be confirmed by an

Plate II. Fig. 5.

A double projectile force balances a quadruple power of gravity.

ng-table is a machine contrived for
riments of this nature. *A A* is a
of wood, *B* a winch or handle

Plate IV. Fig. 1.

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cs *AG*, *AI* muſt be ſuppoſed to be very
AE, which is equal to *HI*, will be more
o *AB*, which is equal to *FG*.

fixed

The whirling table deſcribed.

fixed on the axis *C* of the wheel *D*, round which is the catgut ſtring *F*; which alſo goes round the ſmall wheels *G* and *K*, croſſing between them and the great wheel *D*. On the upper end of the axis of the wheel *G*, above the frame, is fixed the round board *d*, to which the bearer *M S X* may be faſtened occaſionally, and removed when it is not wanted. On the axis of the wheel *H* is fixed the bearer *N Ƭ Z*: and it is eaſy to ſee that when the winch *B* is turned, the wheels and bearers are put into a whirling motion.

Each bearer has two wires, *W*, *X*, and *Y*, *Z*, fixed and ſcrewed tight into them at the ends by nuts on the outſide. And when theſe nuts are unſcrewed, the wires may be drawn out in order to change the balls *U* and *V*, which ſlide upon the wires by means of braſs loops fixed into the balls, which keep the balls up from touching the wood below them. A ſtrong ſilk line goes through each ball, and is fixed to it at any length from the center of the bearer to its end as occaſion requires, by a nut-ſcrew at the top of the ball; the ſhank of the ſcrew goes into the center of the ball, and preſſes the line againſt the under ſide of the hole that it goes through. —The line goes from the ball, and under a ſmall pulley fixt in the middle of the bearer; then up through a ſocket in the round plate (ſee *S* and *Ƭ*) in the middle of each bearer; then through a ſlit in the middle of the ſquare top (*O* and *P*) of each tower, and going over a ſmall pulley on the top, comes down again the ſame way, and is at laſt faſtened to the upper end of the ſocket fixt in the middle of the above-mentioned round plate. Theſe plates *S* and *Ƭ* have each four round holes near their edges for letting them ſlide up and down upon the wires which make the corners of each tower. The balls and plates being thus connected, each by its particular line, it is plain that if the balls be drawn outward, or towards the ends *M* and *N* of their reſpective bearers, the round plates *S* and *T* will be drawn up to the top of their reſpective towers *O* and *P*.

There are ſeveral braſs weights, ſome of two ounces, ſome of three, and ſome of four, to be occaſionally put within the towers *O* and *P*, upon the round plates *S* and *T*: each weight having a round hole in the middle of it, for going upon the ſockets or axes of the plates, and is ſlit from the edge to the hole, for allowing it to be ſlipt over the aforeſaid line which comes from each ball to its reſpective plate. (See Fig. 2.)

The experiments to be made by this machine are as follows:

1. Take away the bearer *MX*, and take the Fig. 1. ivory ball *a*, to which the line or ſilk cord *b* is faſtened at one end; and having made a loop on the other end of the cord, put the loop over a pin fixt in the center of the board *d*. Then, turning the winch *B* to give the board a whirling motion, you will ſee that the ball does not immediately begin to move with the board, but, on account of its inactivity, it endeavours to continue in the ſtate of reſt which it was in before.— Continue turning, until the board communicates an equal degree of motion with its own to the ball, and then turning on, you will perceive that the ball will remain upon one part of the board, keeping the ſame velocity with it, and having no relative motion upon it, as is the caſe with every thing that lies looſe upon the planeſurface of the earth, which having the motion of the earth communicated to it, never endeavours to remove from

The propenſity of matter to keep the ſtate it is in.

from that place. But ſtop the board ſuddenly by hand, and the ball will go on, and continue to revolve upon the board, until the friction thereof ſtops its motion: which ſhews, that matter being once put into motion would continue to move for ever, if it met with no reſiſtance. In like manner, if a perſon ſtands upright in a boat before it begins to move, he can ſtand firm; but the moment the boat ſets off, he is in danger of falling towards that place which the boat departs from: becauſe, as matter, he has no natural propenſity to move. But when he acquires the motion of the boat, let it be ever ſo ſwift, if it be ſmooth and uniform, he will ſtand as upright and firm as if he was on the plain ſhore; and if the boat ſtrikes againſt any obſtacle, he will fall towards that obſtacle; on account of the propenſity he has, as matter, to keep the motion which the boat has put him into.

2. Take away this ball, and put a longer cord to it, which may be put down through the hollow axis of the bearer *MX*, and wheel *G*, and fix a weight to the end of the cord below the machine; which weight, if left at liberty, will draw the ball from the edge of the whirling-board to its center.

Bodies moving in orbits have a tendency to fly out of theſe orbits.

Draw off the ball a little from the center, and turn the winch; then the ball will go round and round with the board, and will gradually fly off farther and farther from the center, and raiſe up the weight below the machine: which ſhews that all bodies revolving in circles have a tendency to fly off from theſe circles, and muſt have ſome power acting upon them from the center of motion, to keep them from flying off. Stop the machine, and the ball will continue to revolve for ſome time upon the board; but as the friction gradually ſtops its motion, the weight acting upon it will bring it nearer and nearer to the center in every revolution, until it brings it quite thither. This ſhews, that if the planets met with any reſiſtance in going round the ſun, its attractive power would bring them nearer and nearer to it in every revolution, until they fell upon it.

Bodies move faſter in ſmall orbits than in large ones.

3. Take hold of the cord below the machine with one hand, and with the other throw the ball upon the round board as it were at right angles to the cord, by which means it will go round and round upon the board. Then obſerving with what velocity it moves, pull the cord below the machine, which will bring the ball nearer to the center of the board, and you will ſee that the nearer the ball is drawn to the center, the faſter it will revolve; as thoſe planets which are neareſt the ſun revolve faſter than thoſe which are more remote; and not only go round ſooner, becauſe they deſcribe ſmaller circles, but even move faſter in every part of their reſpective circles.

Their centrifugal forces ſhewn.

4. Take away this ball, and apply the bearer *MX*, whoſe center of motion is in its middle at *w*, directly over the center of the whirling-board *d*. Then put two balls (*V* and *U*) of equal weights upon their bearing wires, and having fixed them at equal diſtances from their reſpective centers of motion *w* and *x* upon their ſilk cords, by the ſcrew nuts, put equal weights in the towers *O* and *P*. Laſtly, put the catgut ſtrings *E* and *F* upon the grooves *G* and *H* of the ſmall wheels, which being of equal diameters, will give equal velocities to the bearers above, when the winch *B* is turned: and the balls *U* and *V* will

fly off towards *M* and *N*; and will raiſe the weights in the towers at the ſame inſtant. This ſhews, that when bodies of equal quantities of matter revolve in equal circles with equal velocities, their centrifugal forces are equal.

5. Take away theſe equal balls, and inſtead of them, put a ball of ſix ounces into the bearer *MX*, at a ſixth part of the diſtance *w z* from the center, and put a ball of one ounce into the oppoſite bearer, at the whole diſtance *x y*, which is equal to *w z* from the center of the bearer; and fix the balls at theſe diſtances on their cords, by the ſcrew nuts at top; and then the ball *U*, which is ſix times as heavy as the ball *V*, will be at only a ſixth part of the diſtance from its center of motion; and conſequently will revolve in a circle of only a ſixth part of the circumference of the circle in which *V* revolves. Now, let any equal weights be put into the towers, and the machine be turned by the winch; which (as the catgut ſtring is on equal wheels below) will cauſe the balls to revolve in equal times; but *V* will move ſix times as faſt as *U*, becauſe it revolves in a circle of ſix times its radius; and both the weights in the towers will riſe at once. This ſhews, that the centrifugal forces of revolving bodies (or their tendencies to fly off from the circles they deſcribe) are in direct proportion to their quantities of matter multiplied into their reſpective velocities; or into their diſtances from the centers of their reſpective circles. For, ſuppoſing *U*, which weighs ſix ounces, to be two inches from its center of motion *w*, the weight multiplied by the diſtance is 12: and ſuppoſing *V*, which weighs only one ounce, to be 12 inches diſtant from the center of motion *x*, the weight 1 ounce multiplied by the diſtance 12 inches

is 12. And as they revolve in equal times, their velocities are as their diſtances from the center, namely, as 1 to 6.

If theſe two balls be fixed at equal diſtances from their reſpective centers of motion, they will move with equal velocities; and if the tower *O* has 6 times as much weight put into it as the tower *P* has, the balls will raiſe their weights exactly at the ſame moment. This ſhews that the ball *U* being ſix times as heavy as the ball *V*, has ſix times as much centrifugal force, in deſcribing an equal circle with an equal velocity.

A double velocity in the ſame circle, is a balance to a quaduple power of gravity.

6. If bodies of equal weights revolve in equal circles with unequal velocities, their centrifugal forces are as the ſquares of the velocities. To prove this law by an experiment, let two balls *U* and *V* of equal weights be fixed on their cords at equal diſtances from their reſpective centers of motion *w* and *x*; and then let the catgut ſtring *E* be put round the wheel *K* (whoſe circumference is only one half of the circumference of the wheel *H* or *G*) and over the pulley *s* to keep it tight; and let four times as much weight be put into the tower *P*, as in the tower *O*. Then turn the winch *B*, and the ball *V* will revolve twice as faſt as the ball *U* in a circle of the ſame diameter, becauſe they are equidiſtant from the centers of the circles in which they revolve; and the weight in the towers will both riſe at the ſame inſtant, which ſhews that a double velocity in the ſame circle will exactly balance a quadruple power of attraction in the center of the circle. For the weights in the towers may be conſidered as the attractive forces in the centers, acting upon the revolving balls; which,

moving in equal circles, is the ſame thing as if they both moved in one and the ſame circle.

Kepler's Problem.

7. If bodies of equal weights revolve in unequal circles, in ſuch a manner that the ſquares of the times of their going round are as the cubes of their diſtances from the centers of the circles they deſcribe; their centrifugal forces are inverſely as the ſquares of their diſtances from thoſe centers. For, the catgut ſtring remaining as in the laſt experiment, let the diſtance of the ball *V* from the center *x* be made equal to two of the croſs diviſions on its bearer; and the diſtance of the ball *U* from the center *w* be three and a ſixth part; the balls themſelves being of equal weights, and *V* making two revolutions by turning the winch, in the time that *U* makes one: ſo that if we ſuppoſe the ball *V* to revolve in one ſecond, the ball *U* will revolve in two ſeconds, the ſquares of which are one and four: for the ſquare of 1 is only 1, and the ſquare of 2 is 4; therefore the ſquare of the period or revolution of the ball *V*, is contained four times in the ſquare of the period of the ball *U*. But the diſtance of *V* is 2, the cube of which is 8, and the diſtance of *U* is $3\frac{1}{6}$, the cube of which is 32 very nearly, in which 8 is contained four times; and therefore, the ſquares of the periods of *V* and *U* are to one another as the cubes of their diſtances from *x* and *w*, which are the centers of their reſpective circles. And if the weight in the tower *O* be four ounces, equal to the ſquare of 2, the diſtance of *V* from the center *x*; and the weight in the tower *P* be 10 ounces, nearly equal to the ſquare of $3\frac{1}{6}$, the diſtance of *U* from *w*; it will be found upon turning the machine by the winch, that the balls *U* and *V* will raiſe their reſpective weights at

the ſame inſtant of time. Which confirms that famous obſervation of KEPLER, viz. That the ſquares of the times in which the planets go round the ſun are in the ſame proportion as the cubes of their diſtances from him; and that the ſun's attraction is inverſely as the ſquare of the diſtance from his center: that is, at twice the diſtance, his attraction is four times leſs; and thrice the diſtance, nine times leſs; at four times the diſtance, ſixteen times leſs; and ſo on, to the remoteſt part of the ſyſtem.

8. Take off the catgut ſtring *E* from the great wheel *D* and the ſmall wheel *H*, and let the ſtring *F* remain upon the wheels *D* and *G*. Take away alſo the bearer *MX* from the whirling-board *d*, and inſtead thereof put the machine *AB* upon it, fixing this machine to the center of the board by the pins *c* and *d*, in ſuch a manner, that the end *ef* may riſe above the board to an angle of 30 or 40 degrees. In the upper ſide of this machine are two glaſs tubes *a* and *b*, cloſe ſtopt at both ends; and each tube is about three quarters full of water. In the tube *a* is a little quickſilver, which naturally falls down to the end *a* in the water, becauſe it is heavier than its bulk of water; and in the tube *b* is a ſmall cork which floats on the top of the water at *e*, becauſe it is lighter; and it is ſmall enough to have liberty to riſe or fall in the tube. While the board *b* with this machine upon it continues at reſt, the quickſilver lies at the bottom of the tube *a*, and the cork floats on the water near the top of the tube *b*. But, upon turning the winch, and putting the machine in motion, the contents of each tube will fly off towards the uppermoſt ends (which are fartheſt from the center of motion) the heavieſt

Fig. 3.

The abſurdity of the Carteſian vortexes.

with the greateſt force. Therefore the quickſilver in the tube *a* will fly off quite to the end *f*, and occupy its bulk of ſpace there, excluding the water from that place, becauſe it is lighter than quickſilver; but the water in the tube *b* flying off to its higher end *e*, will exclude the cork from that place, and cauſe the cork to deſcend towards the lowermoſt end of the tube, where it will remain upon the loweſt end of the water near *b*; for the heavier body having the greater centrifugal force, will therefore poſſeſs the uppermoſt part of the tube; and the lighter body will keep between the heavier and the lowermoſt part.

This demonſtrates the abſurdity of the Carteſian doctrine of the planets moving round the ſun in vortexes: for, if the planet be more denſe or heavy than its bulk of the vortex, it will fly off therein, farther and farther from the ſun; if leſs denſe, it will come down to the loweſt part of the vortex, at the ſun: and the whole vortex itſelf muſt be ſurrounded with ſomething like a great wall, otherwiſe it would fly quite off, planets and all together.—But while gravity exiſts, there is no occaſion for ſuch vortexes; and when it ceaſes to exiſt, a ſtone thrown upwards will never return to the earth again.

If one body moves round another, both of them muſt move round their common center of gravity.

9. If a body be ſo placed on the whirling-board of the machine (Fig. 1.) that the center of gravity of the body be directly over the center of the board, and the board be put into ever ſo rapid a motion by the winch *B*, the body will turn round with the board, but will not remove from the middle of it; for, as all parts of the body are in *equilibrio* round its center of gravity, and the center of gravity is at reſt in the center of motion, the centrifugal force of all parts of

the body will be equal at equal diſtances from its center of motion, and therefore the body will remain in its place. But if the center of gravity be placed ever ſo little out of the center of motion, and the machine be turned ſwiftly round, the body will fly off towards that ſide of the board on which its center of gravity lies. Thus, Fig. 4. if the wire *C* with its little ball *B* be taken away from the demi-globe *A*, and the flat ſide *e f* of this demi-globe be laid upon the whirling-board of the machine, ſo that their centers may coincide; if then the board be turned ever ſo quick by the winch, the demi-globe will remain where it was placed. But if the wire *C* be ſcrewed into the demi-globe at *d*, the whole becomes one body, whoſe center of gravity is now at or near *d*. Let the pin *c* be fixed in the center of the whirling-board, and the deep groove *b* cut in the flat ſide of the demi-globe be put upon the pin, ſo as the pin may be in the center of *A* [See Fig. 5. where this groove is repreſented at *b*] and let Fig. 5. the whirling-board be turned hy the winch, which will carry the little ball *B* (Fig. 4.) with its wire *C*, and the demi-globe *A*, all round the center-pin *c i*; and then, the centrifugal force of the little ball *B*, which weighs only one ounce, will be ſo great, as to draw off the demi-globe *A*, which weighs two pounds, until the end of the groove at *e* ſtrikes againſt the pin *c*, and ſo prevents the demi-globe *A* from going any farther: otherwiſe, the centrifugal force of *B* would have been great enough to have carried *A* quite off the whirling-board. Which ſhews, that if the ſun were placed in the very center of the orbits of the planets, it could not poſſibly remain there; for the centrifugal forces of the planets would carry them quite off, and the ſun with them; eſpecially when ſeveral of them happened to be in any one quarter of the heavens. For the ſun and planets are as much connected by the mutual attraction that ſubſiſts between them, as the bodies *A* and *B* are by the wire *C* which is fixed into them both. And even if there were but one ſingle planet in the whole heavens to go round ever ſo large a ſun in the center of its orbit, its centrifugal force would ſoon carry off both itſelf and the ſun. For, the greateſt body placed in any part of free ſpace might be eaſily moved: becauſe if there were no other body to attract it, it could have no weight or gravity of itſelf; and conſequently, though it could have no tendency of itſelf to remove from that part of ſpace, yet it might be very eaſily moved by any other ſubſtance.

10. As the centrifugal force of the light body *B* will not allow the heavy body *A* to remain in the center of motion, even though it be 24 times as heavy as *B*; let us now take the ball *A* (Fig. 6.) which weighs 6 ounces, and connect it by the wire *C* with the ball *B*, which weighs only one ounce; and let the fork *E* be fixed into the center of the whirling-board: then hang the balls upon the fork by the wire *C* in ſuch a mandr, that they may exactly balance each other; which will be when the center of gravity between them, in the wire at *d*, is ſupported by the fork. And this center of gravity is as much nearer to the center of the ball *A*, than to the center of the ball *B*, as *A* is heavier than *B*, allowing for the weight of the wire on each ſide of the fork. This done, let the machine be put into motion by the winch; and the balls *A* and *B* will go round their common center of gravity *d*, keeping their balance, becauſe either will not allow

the other to fly off with it. For, ſuppoſing the ball *B* to be only one ounce in weight, and the ball *A* to be ſix ounces; then, if the wire *C* were equally heavy on each ſide of the fork, the center of gravity *d* would be ſix times as far from the center of the ball *B* as from that of the ball *A*, and conſequently *B* will revolve with a velocity ſix times as great as *A* does; which will give *B* ſix times as much centrifugal force as any ſingle ounce of *A* has: but then, as *B* is only one ounce, and *A* ſix ounces, the whole centrifugal force of *A* will exactly balance the whole centrifugal force of *B*: and therefore, each body will detain the other ſo as to make it keep in its circle. This ſhews that the ſun and planets muſt all move round the common center of gravity of the whole ſyſtem, in order to preſerve that juſt balance which takes place among them. For, the planets being as unactive and dead as the above balls, they could no more have put themſelves into motion than theſe balls can; nor have kept in their orbits without being balanced at firſt with the greateſt degree of exactneſs upon their common center of gravity, by the Almighty hand that made them and put them in motion.

Perhaps it may be here aſked, that ſince the center of gravity between theſe balls muſt be ſupported by the fork *E* in this experiment, *what* prop it is that ſupports the center of gravity of the ſolar ſyſtem, and conſequently bears the weight of all the bodies in it; and by what is the prop itſelf ſupported? The anſwer is eaſy and plain; for the center of gravity of our balls muſt be ſupported, becauſe they gravitate towards the earth, and would therefore fall to it: but as the ſun and planets gravitate only towards one another, they have nothing elſe to fall to; and therefore have no occaſion for any thing to ſupport their common center of gravity: and if they did not move round that center, and conſequently acquire a tendency to fly off from it by their motions, their mutual attractions would ſoon bring them together; and ſo the whole would become one maſs in the ſun: which would alſo be the caſe if their velocities round the ſun were not quick enough to create a centrifugal force equal to the ſun's attraction.

But after all this nice adjuſtment, it appears evident that the Deity cannot withdraw his regulating hand from his works, and leave them to be ſolely governed by the laws which he has impreſt upon them at firſt. For if he ſhould once leave them ſo, their order would in time come to an end; becauſe the planets muſt neceſſarily diſturb one another's motions by their mutual attractions, when ſeveral of them are in the ſame quarter of the heavens; as is often the caſe: and then, as they attract the ſun more towards that quarter than when they are in a manner diſperſed equably around him, if he was not at that time made to deſcribe a portion of a larger circle round the common center of gravity, the balance would then be immediately deſtroyed; and as it could never reſtore itſelf again, the whole ſyſtem would begin to fall together, and would in time unite in a maſs at the ſun.—Of this diſturbance we have a very remarkable inſtance in the comet which appeared lately; and which, in going laſt up before from the ſun, went ſo near to Jupiter, and was ſo affected by his attraction, as to have the figure of its orbit much changed; and not only ſo, but to have its period altered, and its courſe to be different in the heavens from what it was laſt before.

11. Take away the fork and balls from the whirling-board, and place the trough *AB* thereon, fixing its center to the center of the whirling-board by the pin *H*. In this trough are two balls *D* and *E*, of unequal weights, connected by a wire *f*; and made to ſlide eaſily upon the wire *C* ſtretched from end to end of the trough, and made faſt by nut-ſcrews on the outſide of the ends. Let theſe balls be ſo placed upon the wire *C*, that their common center of gravity *g* may be directly over the center of the whirling-board. Then, turn the machine by the winch, ever ſo ſwiftly, and the trough and balls will go round their center of gravity, ſo as neither of the balls will fly off; becauſe, on account of the equilibrium, each ball detains the other with an equal force acting againſt it. But if the ball *E* be drawn a little more towards the end of the trough at *A*, it will remove the center of gravity towards that end from the center of motion; and then, upon turning the machine, the little ball *E* will fly off, and ſtrike with a conſiderable force againſt the end *A*, and draw the great ball *B* into the middle of the trough. Or, if the great ball *D* be drawn towards the end *B* of the trough, ſo that the center of gravity may be a little towards that end from the center of motion, and the machine be turned by the winch, the great ball *D* will fly off, and ſtrike violently againſt the end *B* of the trough, and will bring the little ball *E* into the middle of it. If the trough be not made very ſtrong, the ball *D* will break through it. Fig. 7.

12. The reaſon why the tides riſe at the ſame abſolute time on oppoſite ſides of the earth, and Of the tides.

conſequently in oppoſite directions, is made abundantly plain by a new experiment on the whirling-table. The cauſe of their riſing on the ſide next the moon every one underſtands to be owing to the moon's attraction: but why they ſhould riſe on the oppoſite ſide at the ſame time, where there is no moon to attract them, is perhaps not ſo generally underſtood. For it would ſeem that the moon ſhould rather draw the waters (as it were) cloſer to that ſide, than raiſe them upon it, directly contrary to her attractive force. Let
Fig. 8. the circle *abcd* repreſent the earth, with its ſide *c* turned toward the moon, which will then attract the waters ſo, as to raiſe them from *c* to *g*. But the queſtion is, why ſhould they riſe as high at that very time on the oppoſite ſide, from *a* to
Fig. 9. *e*? In order to explain this, let there be a plate *AB* fixed upon one end of the flat bar *DC*; with ſuch a circle drawn upon it as *abcd* (in Fig. 8.) to repreſent the round figure of the earth and ſea; and ſuch an ellipſis as *efgh* to repreſent the ſwelling of the tide at *e* and *g*, occaſioned by the influence of the moon. Over this plate *AB* let the three ivory balls *e*, *f*, *g*, be hung by the ſilk lines *h*, *i*, *k*, faſtened to the tops of the crooked wires *H*, *I*, *K*, in ſuch a manner, that the ball at *e* may hang freely over the ſide of the circle *e*, which is fartheſt from the moon *M* (at the other end of the bar); the ball at *f* may hang freely over the center, and the ball at *g* hang over the ſide of the circle *g*, which is neareſt the moon. The ball *f* may repreſent the center of the earth, the ball *g* ſome water on the ſide next the moon, and the ball *e* ſome water on the oppoſite ſide. On the back of the moon *M* is fixt the ſhort bar *N* parallel to the horizon, and there are three holes in it above the little weights *p*, *q*, *r*. A ſilk

ſilk thread *o* is tied to the line *k* cloſe above the ball *g*, and paſſing by one ſide of the moon *M*, goes through a hole in the bar *N*, and has the weight *p* hung to it. Such another thread *n* is tied to the line *i*, cloſe above the ball *f*, and paſſing through the center of the moon *M* and middle of the bar *N*, has the weight *q* hung to it, which is lighter than the weight *p*. A third thread *m* is tied to the line *h*, cloſe above the ball *e*, and paſſing by the other ſide of the moon *M*, through the bar *N*, has the weight *r* hung to it, which is lighter than the weight *q*.

The uſe of theſe three unequal weights is to repreſent the moon's unequal attraction at different diſtances from her. With whatever force ſhe attracts the center of the earth, ſhe attracts the ſide next her with a greater degree of force, and the ſide fartheſt from her with a leſs. So, if the weights are left at liberty, they will draw all the three balls towards the moon with different degrees of force, and cauſe them to make the appearance ſhewn in Fig. 10; by which Fig. 10. means they are evidently farther from each other than they would be if they hung at liberty by the lines *h*, *i*, *k*; becauſe the lines would then hang perpendicularly. This ſhews, that as the moon attracts the ſide of the earth which is neareſt her with a greater degree of force than ſhe does the center of the earth, ſhe will draw the water on that ſide more than ſhe draws the center, and ſo cauſe it to riſe on that ſide: and as ſhe draws the center more than ſhe draws the oppoſite ſide, the center will recede farther from the ſurface of the water on that oppoſite ſide, and ſo leave it as high there as ſhe raiſed it on the ſide next to her. For, as the center will be in the middle between the tops of the oppoſite elevations, they muſt of courſe be equally high on both ſides at the ſame time.

But upon this ſuppoſition the earth and moon would ſoon come together: and to be ſure they would, if they had not a motion round their common center of gravity, to create a degree of centrifugal force ſufficient to balance their mutual attraction. This motion they have; for as the moon goes round her orbit every month, at the diſtance of 240000 miles from the earth's center, and of 234000 miles from the center of gravity of the earth and moon, ſo does the earth go round the ſame center of gravity every month at the diſtance of 6000 miles from it; that is, from it to the center of the earth. Now as the earth is (in round numbers) 8000 miles in diameter, it is plain that its ſide next the moon is only 2000 miles from the common center of gravity of the earth and moon; its center 6000 miles diſtant therefrom; and its farther ſide from the moon 10000. Therefore the centrifugal forces of theſe parts are as 2000, 6000, and 10000; that is, the centrifugal force of any ſide of the earth, when it is turned from the moon, is five times as great as when it is turned towards the moon. And as the moon's attraction (expreſt by the numbers 6000) at the earth's center keeps the earth from flying out of this monthly circle, it muſt be greater than the centrifugal force of the waters on the ſide next her; and conſequently, her greater degree of attraction on that ſide is ſufficient to raiſe them; but as her attraction on the oppoſite ſide is leſs than the centrifugal force of the water there, the exceſs of this force is ſufficient to raiſe the water juſt as high on the oppoſite ſide.—To prove this experimentally, let the bar *D C* with its furniture be fixed upon the whirling-board of the machine (Fig. 1.) by puſhing the pin *P* into the center of the board; which pin is in the center of gravity of the whole bar with its three balls *e*, *f*, *g*, and moon *M*. Now if the whirling-board and bar be turned ſlowly round by the winch, until the ball *f* hangs over the center of the circle, as in Fig. 11. the ball *g* will be kept towards the moon by the heavieſt weight *p*, (Fig. 9.) and the ball *e*, on account of its greater centrifugal force, and the leſſer weight *r*, will fly off as far to the other ſide, as in Fig. 11. And ſo, whilſt the machine is kept turning, the balls *e* and *g* will hang over the ends of the ellipſis *l f k*. So that the centrifugal force of the ball *e* will exceed the moon's attraction juſt as much as her attraction exceeds the centrifugal force of the ball *g*, whilſt her attraction juſt balances the centrifugal force of the ball *f*, and makes it keep in its circle. And hence it is evident that the tides muſt riſe to equal heights at the ſame time on oppoſite ſides of the earth. This experiment, to the beſt of my knowledge, is entirely new. Fig. 9.

From the principles thus eſtabliſhed, it is evident that the earth moves round the ſun, and not the ſun round the earth; for the centrifugal law will never allow a great body to move round a ſmall one in any orbit whatever; ſpecially when we find that if a ſmall body moves round a great one, the great one muſt alſo move round the common center of gravity between them two. And it is well known that the quantity of matter in the ſun is 227000 times as great as the quantity of matter in the earth. Now, as the ſun's diſtance The earth's motion demonſtrated.

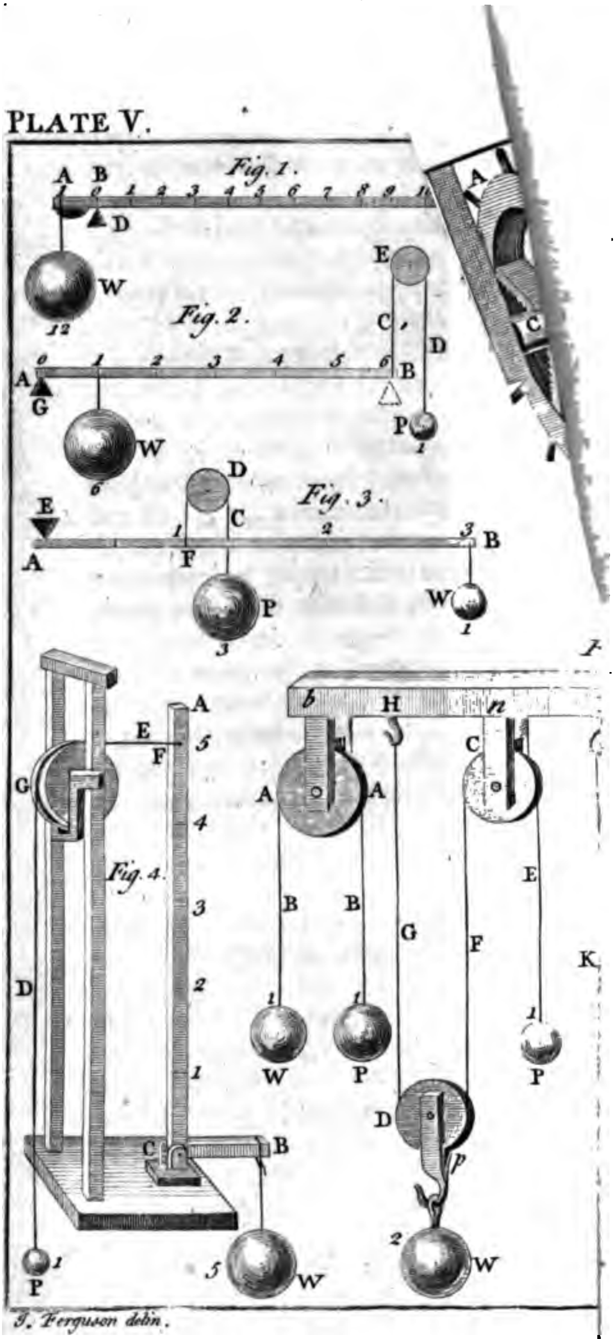
in a minute; and another body, which we ſhall call *B*, to weigh only four pounds, and to move 20 miles in a minute; the entire forces with which theſe two bodies would ſtrike againſt any obſtacle would be equal to each other, and therefore it would require equal powers to ſtop them. For 40 multiplied by 2 gives 80, the force of the body *A*: and 20 multiplied by 4 gives 80, the force of the body *B*.

Upon this eaſy principle depends the whole of mechanics: and it holds univerſally true, that when two bodies are ſuſpended on any machine, ſo as to act contrary to each other; if the machine be put into motion, and the perpendicular aſcent of one body multiplied into its weight, be equal to the perpendicular deſcent of the other body multiplied into its weight, thoſe bodies, how unequal ſoever in their weights, will balance one another in all ſituations: for, as the whole aſcent of one is performed in the ſame time with the whole deſcent of the other, their reſpective velocities muſt be directly as the ſpaces they move through; and the exceſs of weight in one body is compenſated by the exceſs of velocity in the other.—Upon this principle it is eaſy to compute the power of any mechanical engine, whether ſimple or compound; for it is but only finding how much ſwifter the power moves than the weight does (*i. e.* how much farther in the ſame time) and juſt ſo much is the power increaſed by the help of the engine.

How to compute the power of any mechanical engine.

In the theory of this ſcience, we ſuppoſe all planes perfectly even, all bodies perfectly ſmooth, levers to have no weight, cords to be extremely pliable, machines to have no friction; and in ſhort, all imperfections muſt be ſet aſide until the

PLATE V.



J. Ferguson delin.

he theory be eſtabliſhed; and then, proper llowances are to be made.

The mechanic powers, what.

The ſimple *machines*, uſually called *mechanical owers*, are ſix in number, viz. the *lever*, the *vheel and axle*, the *pulley*, the *inclined plane*, the *vedge*, and the *ſcrew*.—They are called mecha-ical powers, becauſe they help us mechanically o raiſe weights, move heavy bodies, and over-ome reſiſtances, which we could not effect with-ut them.

The *lever.*

1. A *lever* is a bar of iron or wood, one part f which being ſupported by a prop, all the ther parts turn upon that prop as their center f motion: and the velocity of every part or oint is directly as its diſtance from the prop. Therefore, when the weight to be raiſed at one nd is to the power applied at the other to raiſe t, as the diſtance of the power from the prop s to the diſtance of the weight from the prop, he power and weight will exactly balance or ounterpoiſe each other: and as a common lever has next to no friction on its prop, a very ittle additional power will be ſufficient to raiſe he weight.

There are four kinds of levers. 1. The ommon ſort, where the prop is placed between he weight and the power; but much nearer to he weight than to the power. 2. When the rop is at one end of the lever, the power at the ther, and the weight between them. 3. When he prop is at one end, the weight at the other, nd the power applied between them. 4. The ended lever, which differs only in form from he firſt ſort, but not in property. Thoſe of the firſt and ſecond kind are often uſed in mechanical engines; but there are few inſtances in which the third ſort is uſed.

A Com-

The *balance.*

A *common balance* is by ſome reckoned a lever of the firſt kind; but as both its ends are at equal diſtances from its center of motion, they move with equal velocities; and therefore, as it gives no mechanical advantage, it cannot properly be reckoned among the mechanical powers.

Plate V. Fig. 1. The firſt kind of lever.

A lever of the firſt kind is repreſented by the bar *A B C*, ſupported by the prop *D*. Its principal uſe is to looſen large ſtones in the ground, or raiſe great weights to ſmall heights, in order to have ropes put under them for raiſing them higher by other machines. The parts *A B* and *B C*, on different ſides of the prop *D*, are called the *arms* of the lever: the end *A* of the ſhorter arm *A B* being applied to the weight intended to be raiſed, or to the reſiſtance to be overcome; and the power applied to the end *C* of the longer arm *B C*.

In making experiments with this machine, the ſhorter arm *A B* muſt be as much thicker than the longer arm *B C*, as will be ſufficient to balance it on the prop. This ſuppoſed, let *P* repreſent a power, whoſe gravity is equal to 1 ounce, and *W* a weight, whoſe gravity is equal to 12 ounces. Then, if the power be 12 times as far from the prop as the weight is, they will exactly counterpoiſe; and a ſmall addition to the power *P* will cauſe it to deſcend, and raiſe the weight *W*; and the velocity with which the power deſcends will be to the velocity with which the weight riſes, as 12 to 1: that is, directly as their diſtances from the prop; and conſequently, as the ſpaces through which they move. Hence, it is plain that a man, who by his natural ſtrength, without the help of any machine, could ſupport an hundred weight, will by the help of this lever be enabled to ſupport twelve hundred. If the weight be leſs, or the power greater, the prop may be placed ſo much farther from the weight; and then it can be raiſed to a proportionably greater height. For univerſally, if the intenſity of the weight multiplied into its diſtance from the prop be equal to the intenſity of the power multiplied into its diſtance from the prop, the power and weight will exactly balance each other; and a little addition to the power will raiſe the weight. Thus, in the preſent inſtance, the weight *W* is 12 ounces, and its diſtance from the prop is 1 inch; and 12 multiplied by 1 is 12; the power *P* is equal to 1 ounce, and its diſtance from the prop is 12 inches, which multiplied by 1 is 12 again; and therefore there is an equilibrium between them. So, if a power equal to 2 ounces be applied at the diſtance of 6 inches from the prop, it will juſt balance the weight *W*; for 6 multiplied by 2 is 12, as before. And a power equal to 3 ounces placed at 4 inches diſtance from the prop would be the ſame; for 3 times 4 is 12; and ſo on, in proportion.

The *ſteel-yard.*

The *ſtatera* or Roman *ſteelyard* is a lever of this kind, and is uſed for finding the weights of different bodies by one ſingle weight placed at different diſtances from the prop or center of motion *D*. For, if a ſcale hangs at *A*, the extremity of the ſhorter arm *A B*, and is of ſuch a weight as will exactly counterpoiſe the longer arm *B C*; if this arm be divided into as many equal parts as it will contain, each equal to *A B*, the ſingle weight *P* (which we may ſuppoſe to be 1 pound) will ſerve for weighing any thing as heavy as itſelf, or as many times heavier as there are diviſions in the arm *B C*, or any quantity between its own weight and that quantity.

As for example, if *P* be 1 pound, and placed at the firſt diviſion 1 in the arm *BC*, it will balance 1 pound in the ſcale at *A*: if it be removed to the ſecond diviſion at 2, it will balance 2 pounds in the ſcale: if to the third, 3 pounds; and ſo on to the end of the arm *BC*. If each of theſe integral diviſions be ſubdivided into as many equal parts as a pound contains ounces, and the weight *P* be placed at any of theſe ſubdiviſions, ſo as to counterpoiſe what is in the ſcale, the pounds and odd ounces therein will by that means be aſcertained.

To this kind of lever may be reduced ſeveral ſorts of inſtruments, ſuch as ſciſſars, pinchers, ſnuffers; which are made of two levers acting contrary to one another: their prop or center of motion being the pin which keeps them together.

In common practice, the longer arm of this lever greatly exceeds the weight of the ſhorter: which gains great advantage, becauſe it adds ſo much to the power.

The ſecond kind of lever. A lever of the ſecond kind has the weight between the prop and the power. In this, as well as the former, the advantage gained is as the diſtance of the power from the prop to the diſtance of the weight from the prop: for the reſpective velocities of the power and weight are in that proportion; and they will balance each other when the intenſity of the power multiplied by its diſtance from the prop is equal to the intenſity of the weight multiplied by its diſtance from the prop. Thus, if *AB* be a lever on which the weight *W* of 6 ounces hangs at the diſtance of 1 inch from the prop *G*, and a power *P* equal to the weight of 1 ounce hangs at the end *B*, 6 inches from the prop, by the cord

Fig. 2.

C D going over the fixed pulley *E*, the power will juſt ſupport the weight: and a ſmall addition to the power will raiſe the weight, 1 inch for every 6 inches that the power deſcends.

This lever ſhews the reaſon why two men carrying a burden upon a ſtick between them, bear unequal ſhares of the burden in the inverſe proportion of their diſtances from it. For it is well known, that the nearer any of them is to the burden, the greater ſhare he bears of it: and if he goes directly under it, he bears the whole. So, if one man be at *G*, and the other at *P*, having the pole or ſtick *AB* reſting on their ſhoulders; if the burden or weight *W* be placed five times as near the man at *G*, as it is to the man at *P*, the former will bear five times as much weight as the latter. This is likewiſe applicable to the caſe of two horſes of unequal ſtrength to be ſo yoked, as that each horſe may draw a part proportionable to his ſtrength; which is done by ſo dividing the beam they pull, that the point of traction may be as much nearer to the ſtronger horſe than to the weaker, as the ſtrength of the former exceeds that of the latter.

To this kind of lever may be reduced oars, rudders of ſhips, doors turning upon hinges, cutting-knives which are fixed at the point of the blade, and the like.

The third kind of lever.

If in this lever we ſuppoſe the power and weight to change places, ſo that the power may be between the weight and the prop, it will become a lever of the third kind: in which, that there may be a balance between the power and the weight, the intenſity of the power muſt exceed the intenſity of the weight, juſt as much as the diſtance of the weight from the prop ex-

Fig. 3. ceeds the diſtances of the power from it. Thus, let *E* be the prop of the lever *A B*, and *W* a weight of 1 pound, placed 3 times as far from the prop, as the power *P* acts at *F*, by the cord *C* going over the fixed pulley *D*; in this caſe, the power muſt be equal to three pounds, in order to ſupport the weight.

To this ſort of lever are generally referred the bones of a man's arm: for when we lift a weight by the hand, the muſcle that exerts its force to raiſe that weight, is fixed to the bone about one tenth part as far below the elbow as the hand is. And the elbow being the center round which the lower part of the arm turns, the muſcle muſt therefore exert a force ten times as great as the weight that is raiſed.

As this kind of lever is a diſadvantage to the moving power, it is never uſed but in caſes of neceſſity; ſuch as that of a ladder, which being fixed at one end, is by the ſtrength of a man's arms reared againſt a wall. And in clock-work, where all the wheels may be reckoned levers of this kind, becauſe the power that moves every wheel, except the firſt, acts upon it near the center of motion by means of a ſmall pinion, and the reſiſtance it has to overcome, acts againſt the teeth round its circumference.

The fourth kind of lever. Fig. 4. The fourth kind of lever differs nothing from the firſt, but in being bended for the ſake of convenience. *ACB* is a lever of this ſort, bended at *C*, which is its prop, or center of motion. *P* is a power acting upon the longer arm *AC* at *F*, by means of the cord *D E* going over the pulley *G*; and *W* is a weight or reſiſtance acting upon the end *B* of the ſhorter arm *B C*. If the power is to the weight, as *C B* is to *C F*, they are in *equilibrio*. Thus, ſuppoſe *W* to be 5 pounds acting at the diſtance of one foot from the center of motion *C*, and *P* to be 1 pound acting at *F*, five feet from the center *C*, the power and weight will juſt balance each other. A hammer drawing a nail is a lever of this ſort.

The *wheel and axle.*

2. The ſecond mechanical power is the *wheel and axle*, in which the power is applied to the circumference of the wheel, and the weight is raiſed by a rope which coils about the axle as the wheel is turned round. Here it is plain that the velocity of the power muſt be to the velocity of the weight, as the circumference of the wheel is to the circumference of the axle: and conſequently, the power and weight will balance each other, when the intenſity of the power is to the intenſity of the weight, as the cicumference of the axle is to the circumference of the wheel. Let *A B* be a wheel, *C D* its axle, and ſuppoſe the circumference of the wheel to be 8 times as great as the circumference of the axle; then, a power *P* equal to 1 pound hanging by the cord *I*, which goes round the wheel, will balance a weight *W* of 8 pounds hanging by the rope *K*, which goes round the axle. And as the friction on the pivets or gudgeons of the axle is but ſmall, a ſmall addition to the power will cauſe it to deſcend, and raiſe the weight: but the weight will riſe with only an eighth part of the velocity wherewith the power deſcends, and conſequently, through no more than an eighth part of an equal ſpace, in the ſame time. If the wheel be pulled round by the handles *S*, *S*, the power will be increaſed in proportion to their length. And by this means, any weight may be raiſed as high as the operator pleaſes. Fig. 5.

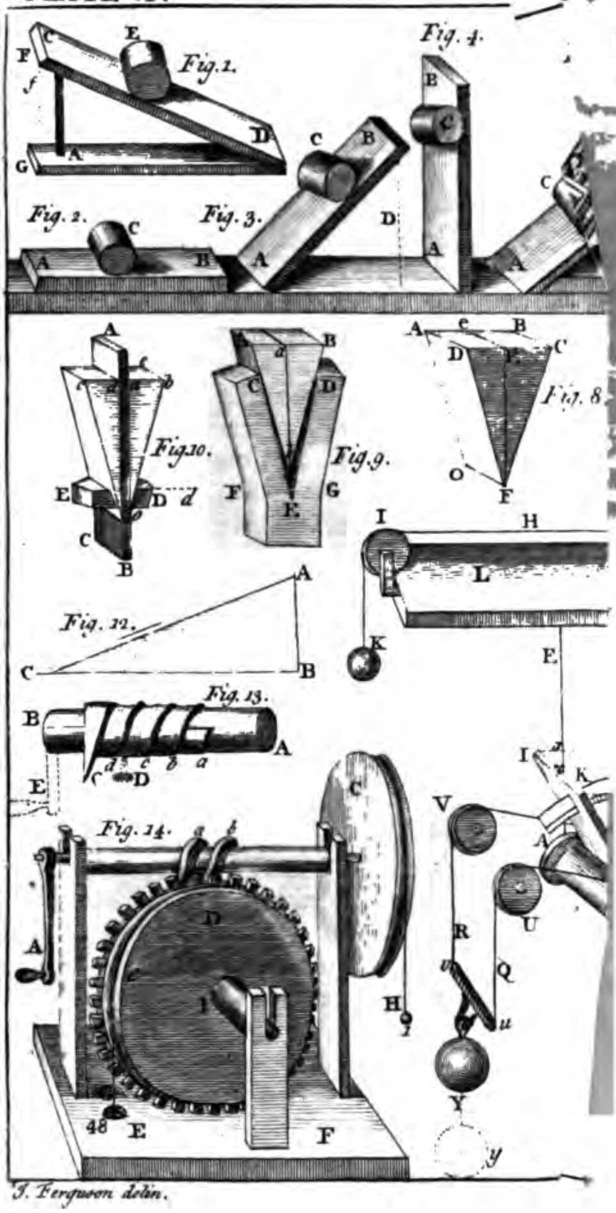
To this ſort of engine belong all cranes for raiſing great weights; and in this caſe, the wheel may have cogs all round it inſtead of handles, and a ſmall lantern or trundle may be made to work in the cogs, and be turned by a winch; which will make the power of the engine to exceed the power of the man who works it, as much as the number of revolutions of the winch exceed thoſe of the axle *D*, when multiplied by the exceſs of the length of the winch above the length of the ſemidiameter of the axle, added to the ſemidiameter or half thickneſs of the rope *K*, by which the weight is drawn up.—Thus, ſuppoſe the diameter of the rope and axle taken together, to be 13 inches, and conſequently, half their diameters to be 6½ inches; ſo that the weight *W* will hang at 6½ inches perpendicular diſtance from below the center of the axle. Now, let us ſuppoſe the wheel *AB*, which is fixt on the axle, to have 80 cogs, and to be turned by means of a winch 6½ inches long, fixt on the axis of a trundle of 8 ſtaves or rounds, working in the cogs of the wheel.—Here it is plain, that the winch and trundle would make 10 revolutions for one of the wheel *AB*, and its axis *D*, on which the rope *K* winds in raiſing the weight *W*; and the winch being no longer than the ſum of the ſemidiameters of the great axle and rope, the trundle could have no more power on the wheel, than a man could have by pulling it round by the edge, becauſe the winch would have no greater velocity than the edge of the wheel has, which we here ſuppoſe to be ten times as great as the velocity of the riſing weight: ſo that, in this caſe, the power gained would be as 10 to 1. But if the length of the winch be 13 inches, the power gained will be as 20 to 1: if 19 $\frac{1}{2}$ inches (which is long enough for any man to work by) the power gained would be as 30 to 1; that is, a man could raiſe 30 times as much by ſuch an engine, as he could do by his natural ſtrength without it, becauſe the velocity of the handle of the winch would be 30 times as great as the velocity of the riſing weight; the abſolute force of any engine being in proportion of the velocity of the power to the velocity of the weight raiſed by it.—But then, juſt as much power or advantage as is gained by the engine, ſo much time is loſt in working it. In this ſort of machines it is requiſite to have a ratchet-wheel *G* on one end of the axle, with a catch *H* to fall into its teeth; which will at any time ſupport the weight, and keep it from deſcending, if the perſon who turns the handle ſhould, through inadvertency or careleſſneſs, quit his hold whilſt the weight is raiſing. And by this means, the danger is prevented which might otherwiſe happen by the running down of the weight when left at liberty.

The pulley. 3. The third mechanical power or engine conſiſts either of one *moveable pully*, or a *ſyſtem of pulleys*; ſome in a block or caſe which is fixed, and others in a block which is moveable, and riſes with the weight. For though a ſingle pulley that only turns on its axis, and moves not out of its place, may ſerve to change the direction of the power, yet it can give no mechanical advantage thereto; but is only as the beam of a balance, whoſe arms are of equal length and weight. Thus, if the equal weights *W* and *P* hang by the cord *B B* upon the pulley *A*, whoſe frame *b* is fixed to the beam *H I*, they will counterpoiſe each other, juſt in the ſame manner as if the cord were cut in the middle, and its two Fig. 6.

ends hung upon the hooks fixt in the pulley at *A* and *A*, equally diſtant from its center.

But if a weight *W* hangs at the lower end of the moveable block *p* of the pulley *D*, and the cord *G F* goes under that pulley, it is plain that the half *G* of the cord bears one half of the weight *W*, and the half *F* the other; for they bear the whole between them. Therefore, whatever holds the upper end of either rope, ſuſtains one half of the weight: and if the cord at *F* be drawn up ſo as to raiſe the pulley *D* to *C*, the cord will then be extended to its whole length, all but that part which goes under the pulley: and conſequently, the power that draws the cord will have moved twice as far as the pulley *D* with its weight *W* riſes; on which account, a power whoſe intenſity is equal to one half of the weight will be able to ſupport it, becauſe if the power moves (by means of a ſmall addition) its velocity will be double the velocity of the weight; as may be ſeen by putting the cord over the fixt pulley *C* (which only changes the direction of the power, without giving any advantage to it) and hanging on the weight *P*, which is equal only to one half the weight *W*; in which caſe there will be an equilibrium, and a little addition to *P* will cauſe it to deſcend, and raiſe *W* through a ſpace equal to one half of that through which *P* deſcends.—Hence, the advantage gained will be always equal to twice the number of pulleys in the moveable or undermoſt block. So that, when the upper or fixt block *u* contains two pulleys, which only turn on their axes, and the lower or moveable block *U* contains two pulleys, which not only turn upon their axes, but alſo riſe with the block and weight; the advantage gained by this is as 4 to the

PLATE VI.



J. Ferguson delin.

; power. Thus, if one end of the rope *Q* be fixed to a hook at *I*, and the rope ver the pulleys *N* and *R*, and under the *L* and *P*, and has a weight *T*, of one hung to its other end at *T*, this weight ance and support a weight *W* of four hanging by a hook at the moveable , allowing the ſaid block as a part of the And if as much more power be added, fficient to overcome the friction of the the power will deſcend with four times velocity as the weight riſes, and conſe- through four times as much ſpace.

two pulleys in the fixed block *X*, and in the moveable block *Y*, are in the ſe with thoſe laſt-mentioned; and thoſe lower block give the ſame advantage to er.

ſyſtem of pulleys has no great weight, in a ſmall compaſs, it is eaſily carried and can be applied, in a great many or raiſing weights, where other engines But they have a great deal of friction accounts: 1. Becauſe the diameters of es bear a very conſiderable proportion to wn diameters; 2. Becauſe in working apt to rub againſt one another, or againſt s of the block; 3. Becauſe of the ſtiffneſs rope that goes over and under them.

The in-clined plane.

he fourth mechanical power is the *in-lane* and the advantage gained by it is as its length exceeds its perpendicular

Plat. VI. Fig. 1.

Let *A B* be a plane parallel to the hori- d *C D* a plane inclined to it; and ſuppoſe le length *C D* to be three times as great erpendicular height *G f F*: in this caſe, nder *E* will be ſupported upon the plane *C D*, plane *C B*; and, in that direction, fixed to a hook in the wall; which will ſupport the cylinder, and keep it from rolling off the plane. Let one end of the line *H* be tied to a hook at *C* in the machine, and the other end to a weight *K*, ſomewhat leſs than that which drew the cylinder up the plane before. If this line be put over the fixed pulley *I*, the weight *K* will draw the machine along the horizontal plane *L*, and under the cylinder *F*: and when the machine has been drawn a little more than the whole length *C A*, the cylinder will be raiſed to *d*, equal to the perpendicular height *A B* above the horizontal part at *A*. The reaſon why the machine muſt be drawn further than the whole length *C A* is, becauſe the weight *F* riſes perpendicular to *C B*.

To the inclined plane may be reduced all hatchets, chiſels, and other edge-tools which are chamfered only on one ſide.

The *wedge.* 5. The fifth mechanical power or machine is the *wedge*, which may be conſidered as two equally inclined planes *D E F* and *C E F*, joined together at their baſes *e E F O*: then *D C* is the whole thickneſs of the wedge at its back *ABCD*, where the power is applied: *E F* is the depth or heighth of the wedge: *D F* the length of one of its ſides, equal to *C F* the length of the other ſide; and *O F* is its ſharp edge, which is entered into the wood intended to be ſplit by the force of a hammer or mallet ſtriking perpendicularly on its back. Thus, *A B b* is a wedge driven into the cleft *C D E* of the wood *F G*.

Fig. 8.

Fig. 9.

When the wood does not cleave at any diſtance before the wedge, there will be an equilibrium between the power impelling the wedge downward, and the reſiſtance of the wood acting againſt the two ſides of the wedge when the power is to the reſiſtance, as half the thickneſs of the wedge at its back is to the length of either of its ſides; becauſe the reſiſtance then acts perpendicular to the ſides of the wedge. But, when the reſiſtance on each ſide acts parallel to the back, the power that balances the reſiſtances on both ſides will be as the length of the whole back of the wedge is to double its perpendicular height.

When the wood cleaves at any diſtance before the wedge (as it generally does) the power impelling the wedge will not be to the reſiſtance of the wood, as the length of the back of the wedge is to the length of both its ſides; but as half the length of the back is to the length of either ſide of the cleft, eſtimated from the top or acting part of the wedge. For, if we ſuppoſe the wedge to be lengthened down from *b* to the bottom of the cleft at *E*, the ſame proportion will hold; namely, that the power will be to the reſiſtance, as half the length of the back of the wedge is to the length of either of its ſides: or, which amounts to the ſame thing, as the whole length of the back is to the length of both the ſides.

In order to prove what is here advanced concerning the wedge, let us ſuppoſe the wedge to be divided lengthwiſe into two equal parts; and then it will become two equal inclined planes; one of which, as *a b c*, may be made uſe of as a half wedge for ſeparating the moulding *c d* from the wainſcot *A B*. It is evident, that when this half wedge has been driven its whole length *a c* between the wainſcot and moulding, its ſide *a c* will be at *e d*; and the moulding will be ſeparated to *f g* from the wainſcot. Now, from what has been already proved of the inclined plane, it appears, that to have an equilibrium between the power impelling the half wedge, and the reſiſtance of the moulding, the former muſt be to the latter, Fig. 7.

letter, as *a b* to *a c*; that is, as the thickneſs of the back which receives the ſtroke is to the length of the ſide againſt which the moulding acts. Therefore, ſince the power upon the half wedge is to the reſiſtance againſt its ſide, as the half back *a b* is to the whole ſide *a c*, it is plain, that the power upon which the whole wedge (where the whole back is double the half back) muſt be to the reſiſtance againſt both its ſides, as the thickneſs of the whole back is to the length of both the ſides; ſuppoſing the wedge at the bottom of the cleft: or as the thickneſs of the whole back to the length of both ſides of the cleft, when the wood ſplits at any diſtance before the wedge. For, when the wedge is driven quite into the wood, and the wood ſplits at ever ſo ſmall a diſtance before its edge, the top of the wedge then becomes the acting part, becauſe the wood does not touch it any where elſe. And ſince the bottom of the cleft muſt be conſidered as that part where the whole ſtickage or reſiſtance is accumulated, it is plain, from the nature of the lever, that the farther the power acts from the reſiſtance, the greater is the advantage.

Some writers have advanced, that the power of the wedge is to the reſiſtance to be overcome, as the thickneſs of the back of the wedge is to the length only of one of its ſides; which ſeems very ſtrange: for, if we ſuppoſe *A B* to be a ſtrong inflexible bar of wood or iron fixt into the ground at *C B*, and *D* and *E* to be two blocks of marble lying on the ground on oppoſite ſides of the bar; it is evident that the block *D* may be ſeparated from the bar to the diſtance *d*, equal to *a b*, by driving the inclined plane or half wedge *a b o* down between them; and the block *E* may be ſeparated to an equal diſtance on the other ſide, in like manner, by the half wedge *c d o*. Fig. 10.

But the power impelling each half wedge will be to the reſiſtance of the block againſt its ſide, as the thickneſs of that half wedge is to its perpendicular height, becauſe the block will be driven off perpendicular to the ſide of the bar *AB*. Therefore the power to drive both the half wedges is to both the reſiſtances, as both the half backs is to the perpendicular height of each half wedge. And if the bar be taken away, the blocks put cloſe together, and the two half wedges joined to make one; it will require as much force to drive it down between the blocks, as is equal to the ſum of the ſeparate powers acting upon the half wedges when the bar was between them.

To confirm this by an experiment, let two cylinders, as *AB* and *CD*, be drawn towards one another by lines running over fixed pulleys, and a weight of 40 ounces hanging at the lines belonging to each cylinder: and let a wedge of 40 ounces weight, having its back juſt as thick as either of its ſides is long, be put between the cylinders, which will then act againſt each ſide with a reſiſtance equal to 40 ounces, whilſt its own weight endeavours to bring it down and ſeparate them. And here, the power of the wedge's gravity impelling it downward, will be to the reſiſtance of both the cylinders againſt the wedge, as the thickneſs of the wedge is to double its perpendicular height; for there will then be an equilibrium between the weight of the wedge and the reſiſtance of the cylinders againſt it, and it will remain at any height between them; requiring juſt as much power to puſh it upward as to pull it downward.—If another wedge of equal weight and depth with this, and only half as thick, be put between the cylinders, it will require twice as much weight to be hung at the ends Fig. 11.

ends of the lines which draw them together, to keep the wedge from going down between them. That is, a wedge of 40 ounces, whoſe back is only equal to half its perpendicular height, will require 80 ounces to each cylinder, to keep it in an equilibrium between them: and twice 80 is 160, equal to four times 40. So that the power will be always to the reſiſtance, as the thickneſs of the back of the wedge is to twice its perpendicular height, when the cylinders move off in a line at right angles to that perpendicular.

The beſt way, though perhaps not the neateſt, that I know of, for making a wedge with its appurtenances for ſuch experiments, is as follows.
Fig. 11. Let *K I L M* and *L M N O* be two flat pieces of wood, each about fifteen inches long and three or four in breadth, joined together by a hinge at *L M*; and let *P* be a graduated arch of braſs, on which the ſaid pieces of wood may be opened to any angle not more than 60 degrees, and then fixt at the given angle by means of the two ſcrews *a* and *b*. Then, *I K N O* will repreſent the back of the wedge, *L M* its ſharp edge which enters the wood, and the outſides of the pieces *KILM* and *LMNO* the two ſides of the wedge againſt which the wood acts in cleaving. By means of the ſaid arch, the wedge may be opened ſo, as to adjuſt the thickneſs of its back in any proportion to the length of either of its ſides, but not to exceed that length: and any weight as *p* may be hung to the wedge upon the hook *M*, which weight, together with the weight of the wedge itſelf, may be conſidered as the impelling power; which is all the ſame in the experiment, whether it be laid upon the back of the wedge, to puſh it down, or hung to its edge to pull it down.—Let *AB* and *CD* be two wooden cylinders, each about two inches thick, where they

they touch the outſides of the wedge; and let their ends be made like two round flat plates, to keep the wedge from ſlipping off edgewiſe from between them. Let a ſmall cord with a loop on one end of it, go over a pivot in the end of each cylinder, and the cords *S* and *T* belonging to the cylinder *AB* go over the fixt pulleys *W* and *X*, and be faſtened at their other ends to the bar *w x*, on which any weight as *Z* may be hung at pleaſure. In like manner, let the cords *Q* and *R* belonging to the cylinder *CD* go over the fixt pulleys *V* and *U* to the bar *vu*, on which a weight *Y* equal to *Z* may be hung. Theſe weights, by drawing the cylinders towards one another, may be conſidered as the reſiſtance of the wood acting equally againſt oppoſite ſides of the wedge; the cylinders themſelves being ſuſpended near, and parallel to each other, by their pivots in loops on the lines *E*,*F*,*G*,*H*; which lines may be fixed to hooks in the ceiling of the room. The longer theſe lines are, the better; and they ſhould never be leſs than four feet each. The farther alſo the pulleys *V*,*U* and *X*,*W* are from the cylinders, the truer will the experiments be: and they may turn upon pins fixed into the wall.

In this machine, the weights *Y* and *Z*, and the weight *p*, may be varied at pleaſure, ſo as to be adjuſted in proportion of double the wedge's perpendicular height to the thickneſs of its back: and when they are ſo adjuſted, the wedge will be in *equilibrio* with the reſiſtance of the cylinders.

The wedge is a very great mechanical power, ſince not only wood but even rocks can be ſplit by it; which would be impoſſible to effect by the lever, wheel and axle, or pulley: for the force of the blow, or ſtroke, ſhakes the cohering parts, and thereby makes them ſeparate the more eaſily,

The *ſcrew.* 6. The ſixth and laſt mechanical power is the *ſcrew*; which cannot properly be called a ſimple machine, becauſe it is never uſed without the application of a lever or winch to aſſiſt in turning it: and then it becomes a compound engine of a very great force either in preſſing the parts of bodies cloſer together, or in raiſing great weights. It may be conceived to be made by cutting a piece of paper *ABC* (Fig. 12.) into the form of an inclined plane or half wedge, and then wrapping it round a cylinder *AB* (Fig. 13). And here it is evident, that the winch *E* muſt turn the cylinder once round before the weight of reſiſtance *D* can be moved from one ſpiral winding to another, as from *d* to *c*: therefore, as much as the circumference of a circle deſcribed by the handle of the winch is greater than the interval or diſtance between the ſpirals, ſo much is the force of the ſcrew. Thus, ſuppoſing the diſtance between the ſpirals to be half an inch, and the length of the winch to be twelve inches; the circle deſcribed by the handle of the winch where the power acts will be 76 inches nearly, or about 152 half inches, and conſequently 152 times as great as the diſtance between the ſpirals: and therefore a power at the handle, whoſe intenſity is equal to no more than a ſingle pound, will balance 152 pounds acting againſt the ſcrew; and as much additional force, as is ſufficient to overcome the friction, will raiſe the 152 pounds; and the velocity of the power will be to the velocity of the weight, as 152 to 1. Hence it appears, that the longer the winch is, and the nearer the ſpirals are to one another, ſo much the greater is the force of the ſcrew.

Fig. 12, 13.

A machine for ſhewing the force or power of the ſcrew may be contrived in the following manner

Fig. 14.

manner. Let the wheel *C* have a ſcrew *a b* on its axis, working in the teeth of the wheel *D*, which ſuppoſe to be 48 in number. It is plain, that for every time the wheel *C* and ſcrew *a b* are turned round by the winch *A*, the wheel *D* will be moved one tooth by the ſcrew; and therefore, in 48 revolutions of the winch, the wheel *D* will be turned once round. Then, if the circumference of a circle deſcribed by the handle of the winch *A* be equal to the circumference of a groove *e* round the wheel *D*, the velocity of the handle will be 48 times as great as the velocity of any given point in the groove. Conſequently, if a line *G* (above number 48) goes round the groove *e*, and has a weight of 48 pounds hung to it below the pedeſtal *E F*, a power equal to one pound at the handle will balance and ſupport the weight.—To prove this by experiment, let the circumferences of the grooves of the wheels *C* and *D* be equal to one another; and then if a weight *H* of one pound be ſuſpended by a line going round the groove of the wheel *C*, it will balance a weight of 48 pounds hanging by the line *G*; and a ſmall addition to the weight *H* will cauſe it to deſcend, and ſo raiſe up the other weight.

If the line *G*, inſtead of going round the groove *e* of the wheel *D*, goes round its axle *I*; the power of the machine will be as much increaſed, as the circumference of the groove *e* exceeds the circumference of the axle: which, ſuppoſing it to be ſix times, then one pound at *H* will balance 6 times 48, or 288 pounds hung to the line on the axle: and hence the power or advantage of this machine will be as 288 to 1. That is to ſay, a man, who by his natural ſtrength could lift an hundred weight, will be able to raiſe 288 hundred, or $14\frac{8}{20}$ ton weight by this engine.

But the following engine is ſtill more powerful, on account of its having the addition of four pulleys: and in it we may look upon all the mechanical powers as combined together, even if we take in the balance. For, as the axis *D* of the bar *AB* is in its middle at *C*, it is plain that if equal weights are ſuſpended upon any two pins equi-diſtant from the axis *C*, they will counterpoiſe each other.—It becomes a lever by hanging a ſmall weight *P* upon the pin *n*, and a weight as much heavier upon either of the pins *b*, *c*, *d*, *e*, or *f*, as is in proportion to the pins being ſo much nearer the axis. The wheel and axle *FG* is evident; ſo is the ſcrew *E* which takes in the inclined plane, and with it the half wedge. Part of a cord goes round the axle, the reſt under the lower pulleys *K*, *m*, over the upper pulleys *L*, *n*, and then it is tied to a hook at *m* in the lower or moveable block, on which the weight *W* hangs.

Plate VII. Fig. 1.

A combination of all the mechanical powers.

In this machine, if the wheel *F* has 30 teeth, it will be turned once round in thirty revolutions of the bar *AB*, which is fixt on the axis *D* of the ſcrew *E*: if the length of the bar is equal to twice the diameter of the wheel, the pins *a* and *n* at the ends of the bar will move 60 times as faſt as the teeth of the wheel do: and conſequently, one ounce at *P* will balance 60 ounces hung upon a tooth at *q* in the horizontal diameter of the wheel. Then, if the diameter of the wheel *F* is ten times as great as the diameter of the axle *G*, the wheel will have 10 times the velocity of the axle; and therefore one ounce *P* at the end of the lever *AC* will balance 10 times 60 or 600 ounces hung to the rope *H* which goes round the axle. Laſtly, if four pulleys be added, they will make the velocity of the lower block *K*, and weight *W*, four times leſs than the velocity of the axle: and this being the laſt power in the machine, which is four times as great as that gained by the axle, it makes the whole power of the machine 4 times 600, or 2400. So that a man who could lift one hundred weight in his arms by his natural ſtrength, would be able to raiſe 2400 times as much by this engine.—But it is here as in all other mechanical caſes; for the time loſt is always as much as the power gained, becauſe the velocity with which the power moves will ever exceed the velocity with which the weight riſes, as much as the intenſity of the weight exceeds the intenſity of the power.

The friction of the ſcrew itſelf is very conſiderable; and there are few compound engines, but what, upon account of the friction of the parts againſt one another, will require a third part more of power to work them when loaded, than what is ſufficient to conſtitute a balance between the weight and the power.

LECT. IV.

Of mills, cranes, wheel-carriages, and the engine for driving piles.

AS theſe engines are ſo univerſally uſeful, it would be needleſs to make any apology for deſcribing them.

In a common *breaſt mill*, where the fall of water may be about ten feet, *A A* is the great wheel, which is generally about 17 or 18 feet in diameter,

Plate VII. Fig. 2.

A common mill.

diameter, reckoned from the outermoſt edge of any float board at *a* to that of its oppoſite float at *b*. To this wheel the water is conveyed through a channel, and by falling upon the wheel, turns it round.

On the axis *B B* of this wheel, and within the mill houſe, is a wheel *D*, about 8 or 9 feet diameter, having 61 cogs, which turn a trundle *E* containing ten upright ſtaves or rounds; and when theſe are the number of cogs and rounds, the trundle will make 6$\frac{1}{10}$ revolutions for one revolution of the wheel.

The trundle is fixt upon a ſtrong iron axis called the ſpindle, the lower end of which turns in a braſs foot, fixt at *F*, in the horizontal beam *S T* called the bridge-tree; and the upper part of the ſpindle turns in a wooden buſh fixt into the nether millſtone which lies upon beams in the floor *Y Y*. The top part of the ſpindle above the buſh is ſquare, and goes into a ſquare hole in a ſtrong iron croſs *a b c d*, (ſee Fig. 3.) called the rynd; under which, and cloſe to the buſh, is a round piece of thick leather upon the ſpindle, which it turns round at the ſame time as it does the rynd.

The rynd is let into grooves in the under ſurface of the running millſtone *G* (Fig. 2.) and ſo turns it round in the ſame time that the trundle *E* is turned round by the cog-wheel *D*. This millſtone has a large hole quite through its middle, called the eye of the ſtone, through which the middle part of the rynd and upper end of the ſpindle may be ſeen; whilſt the four ends of the rynd lie hid below the ſtone in their grooves.

The end *T* of the bridge-tree *T S* (which ſupports the upper millſtone *G* upon the ſpindle) is fixed into a hole in the wall; and the end *S* is let into a beam *Q R* called the brayer, whoſe end *R* remains fixt in a mortiſe: and its other end *Q* hangs by a ſtrong iron rod *P* which goes through the floor *YY*, and has a ſcrew-nut on its top at *O*; by the turning of which nut, the end *Q* of the brayer is raiſed or depreſſed at pleaſure; and conſequently the bridge-tree *TS* and upper millſtone. By this means, the upper millſtone may be ſet as cloſe to the under one, or raiſed as high from it, as the miller pleaſes. The nearer the millſtones are to one another, the finer they grind the corn, and the more remote from one another, the coarſer.

The upper millſtone *G* is incloſed in a round box *H*, which does not touch it any where; and is about an inch diſtant from its edge all around. On the top of this box ſtands a frame for holding the hopper *k k*, to which is hung the ſhoe *I* by two lines faſtened to the hind-part of it, fixed upon hooks in the hopper, and by one end of the crook-ſtring *K* faſtened to the fore-part of it at *i*; the other end being twiſted round the pin *L*. As the pin is turned one way, the ſtring draws up the ſhoe cloſer to the hopper, and ſo leſſens the aperture between them; and as the pin is turned the other way, it lets down the ſhoe, and enlarges the aperture.

If the ſhoe be drawn up quite to the hopper, no corn can fall from the hopper into the mill; if it be let a little down, ſome will fall: and the quantity will be more or leſs, according as the ſhoe is more or leſs let down. For the hopper is open at bottom, and there is a hole in the bottom of the ſhoe, not directly under the bottom of the hopper, but forwarder towards the end *i*, over the middle of the eye of the millſtone.

There is a ſquare hole in the top of the ſpindle, Fig. 3. in which is put the feeder *e*: this feeder (as the ſpindle turns round) jogs the ſhoe three times in each revolution, and ſo cauſes the corn to run conſtantly down from the hopper through the ſhoe, into the eye of the millſtone, where it falls upon the top of the rynd, and is, by the motion of the rynd, and the leather under it, thrown below the upper ſtone, and ground between it and the lower one. The violent motion of the ſtone creates a centrifugal force in the corn going round with it, by which means it gets farther and farther from the center, as in a ſpiral, in every revolution, until it be thrown quite out; and, being then ground, it falls through a ſpout *M*, called the mill-eye, into the trough *N*.

When the mill is fed too faſt, the corn bears up the ſtone, and is ground too coarſe; and beſides, it clogs the mill ſo as to make it go too ſlow. When the mill is too ſlowly fed, it goes too faſt, and the ſtones by their attrition are apt to ſtrike fire againſt one another. Both which inconveniences are avoided by turning the pin *L* backwards or forwards, which draws up or lets down the ſhoe; and ſo regulates the feeding as the miller ſees convenient.

The heavier the running millſtone is, and the greater the quantity of water that falls upon the wheel, ſo much the faſter will the mill bear to be fed; and conſequently ſo much the more it will grind. And on the contrary, the lighter the ſtone, and the leſs the quantity of water, ſo much ſlower muſt the feeding be. But when the ſtone is conſiderably wore, and become light, the mill muſt be fed ſlowly at any rate; otherwiſe the ſtone will be too much born up by the corn under it, which will make the meal coarſe.

The quantity of power required to turn a heavy millſtone is but very little more than what is ſufficient to turn a light one: for as it is ſupported upon the ſpindle by the bridge-tree *S T*, and the end of the ſpindle that turns in the braſs foot therein being but ſmall, the odds ariſing from the weight is but very inconſiderable in its action againſt the power or force of the water. And beſides, a heavy ſtone has the ſame advantage as a heavy fly; namely, that it regulates the motion much better than a light one.

In order to cut and grind the corn, both the upper and under millſtones have channels or furrows cut into them, proceeding obliquely from the center towards the circumference. And theſe furrows are cut perpendicularly on one ſide and obliquely on the other into the ſtone, which gives each furrow a ſharp edge, and in the two ſtones they come, as it were, againſt one another like the edges of a pair of ſciſſars: and ſo cut the corn, to make it grind the eaſier when it falls upon the places between the furrows. Theſe are cut the ſame way in both ſtones when they lie upon their backs, which makes them run croſs ways to each other when the upper ſtone is inverted by turning its furrowed ſurface towards that of the lower. For, if the furrows of both ſtones lay the ſame way, a great deal of the corn would be driven onward in the lower furrows, and ſo come out from between the ſtones without being either cut or bruiſed.

When the furrows become blunt and ſhallow by wearing, the running ſtone muſt be taken up, and both ſtones new dreſt with a chiſel and hammer. And every time the ſtone is taken up, there muſt be ſome tallow put round the ſpindle upon the buſh, which will ſoon be melted by the heat the ſpindle acquires from its turning and rubbing againſt the buſh, and ſo will get in betwixt them: otherwiſe the buſh would take fire in a very little time.

The buſh muſt embrace the ſpindle quite cloſe, to prevent any ſhake in the motion, which would make ſome parts of the ſtones grate and fire againſt each other; whilſt other parts of them would be too far aſunder, and by that means ſpoil the meal in grinding.

Whenever the ſpindle wears the buſh ſo as to begin to ſhake in it, the ſtone muſt be taken up, and a chiſel drove into ſeveral parts of the buſh; and when it is taken out, wooden wedges muſt be driven into the holes; by which means the buſh will be made to embrace the ſpindle cloſe all around it again. In doing this, great care muſt be taken to drive equal wedges into the buſh on oppoſite ſides of the ſpindle; otherwiſe it will be thrown out of the perpendicular, and ſo hinder the upper ſtone from being ſet parallel to the under one, which is abſolutely neceſſary for making good work. When any accident of this kind happens, the perpendicular poſition of the ſpindle muſt be reſtored by adjuſting the bridge-tree *S T* by proper wedges put between it and the brayer *Q R*.

It often happens, that the rynd is a little wrenched in laying down the upper ſtone upon it; or is made to ſink a little lower upon one ſide of the ſpindle than on the other; and this will cauſe one edge of the upper ſtone to drag all around upon the other, whilſt the oppoſite edge will not touch. But this is eaſily ſet to rights, by raiſing the ſtone a little with a lever, and putting bits of paper, cards or thin chips, between the rynd and the ſtone.

The diameter of the upper ſtone is generally about ſix feet, the lower ſtone about an inch more: and the upper ſtone when new contains about 22½ cubc feet, which weighs ſomewhat more than 19000 pounds. A ſtone of this diameter ought never to go more than 60 times round in a minute; for if it turns faſter, it will heat the meal.

The grinding ſurface of the under ſtone is a little convex from the edge to the center, and that of the upper ſtone a little more concave: ſo that they are fartheſt from one another in the middle, and come gradually nearer towards the edges. By this means, the corn at its firſt entrance between the ſtones is only bruiſed; but as it goes farther on towards the circumference or edge, it is cut ſmaller and ſmaller; and at laſt finely ground juſt before it comes out from between them.

The water-wheel muſt not be too large, for if it be, its motion will be too ſlow; nor too little, for then it will want power. And for a mill to be in perfection, the floats of the wheel ought to move with a third part of the velocity of the water, and the ſtone to turn round once in a ſecond of time.

In order to conſtruct a mill in this perfect manner, obſerve the following rules:

1. Meaſure the perpendicular height of the fall of water, in feet, above that part of the wheel on which the water begins to act; and call that, the height of the fall.

2. Multiply this conſtant number 64.2882 by the height of the fall in feet, and the ſquare root of the product ſhall be the velocity of the water at the bottom of the fall, or the number of feet that the water there moves *per* ſecond.

3. Divide the velocity of the water by 3, and the quotient ſhall be the velocity of the float-boards of the wheel; or the number of feet they

muſt each go through in a ſecond, when the water acts upon them ſo, as to have the greateſt power to turn the mill.

4. Divide the circumference of the wheel in feet by the velocity of its floats in feet *per* ſecond, and the quotient ſhall be the number of ſeconds in which the wheel turns round.

5. By this laſt number of ſeconds divide 60; and the quotient ſhall be the number of turns of the wheel in a minute.

6. Divide 60 (the number of revolutions the millſtone ought to have in a minute) by the number of turns of the wheel in a minute, and the quotient ſhall be the number of turns the millſtone ought to have for one turn of the wheel.

7. Then, as the number of turns of the wheel in a minute is to the number of turns of the millſtone in a minute, ſo muſt the number of ſtaves in the trundle be to the number of cogs in the wheel, in the neareſt whole numbers that can be found.

By theſe rules I have calculated the following table to a water wheel 18 feet diameter, which I apprehend may be a good ſize in general.

To conſtruct a mill by this table, find the height of the fall of water in the firſt column, and againſt that height, in the ſixth column, you have the number of cogs in the wheel, and ſtaves in the trundle, for cauſing the millſtone to make about 60 revolutions in a minute, as near as poſſible, when the wheel goes with a third part of the velocity of the water. And it appears by the 7th column, that the number of cogs in the wheel, and ſtaves in the trundle, are ſo near the truth for the required purpoſe, that the leaſt number of revolutions of the millſtone in a minute is between 59 and 60, and the greaeſt number never amounts to 61.

The

The MILL-WRIGHT's TABLE.

Height of the fall of water.	Velocity of the water *per* ſecond.		Velocity of the wheel *per* ſecond.		Revolutions of the wheel *per* minute.		Revolutions of the mill-ſtone for one of the wheel.		Cogs in the wheel and ſtaves in the trundle.		Rev. of the mill-ſtone *per* min. by theſe ſtaves and cogs.	
Feet.	Feet.	100 parts of a foot.	Feet.	100 parts of a foot.	Rev.	100 parts of a Rev.	Rev.	100 parts of a Rev.	Cogs.	Staves.	Rev.	100 parts of a Rev.
1	8	.02	2	.67	2	.83	21	.20	127	6	59	.92
2	11	.34	3	.78	4	.00	15	.00	105	7	60	.00
3	13	.89	4	.63	4	.91	12	.22	98	8	60	.14
4	16	.04	5	.35	5	.67	10	.58	95	9	59	.87
5	17	.93	5	.98	6	.34	9	.46	85	9	59	.84
6	19	.64	6	.55	6	.94	8	.64	78	9	60	.10
7	21	.21	7	.07	7	.50	8	.00	72	9	60	.00
8	22	.68	7	.56	8	.02	7	.48	67	9	59	.67
9	24	.05	8	.02	8	.51	7	.05	70	10	59	.57
10	25	.35	8	.45	8	.97	6	.69	67	10	60	.09
11	26	.59	8	.86	9	.40	6	.38	64	10	60	.16
12	27	.77	9	.26	9	.82	6	.11	61	10	59	.90
13	28	.91	9	.64	10	.22	5	.87	59	10	60	.18
14	30	.00	10	.00	10	.60	5	.66	56	10	59	.36
15	31	.05	10	.35	10	.99	5	.46	55	10	60	.48
16	32	.07	10	.69	11	.34	5	.29	53	10	60	.10
17	33	.06	11	.02	11	.70	5	.13	51	10	59	.67
18	34	.02	11	.34	12	.02	4	.99	50	10	60	.10
19	34	.95	11	.65	12	.37	4	.85	49	10	60	.61
20	35	.86	11	.95	12	.68	4	.73	47	10	59	.59
1	2		3		4		5		6		7	

Such a mill as this, with a fall of water about $7\frac{1}{2}$ feet, will require about 32 hogſheads every minute to turn the wheel with a third part of the velocity with which the water falls; and to overcome the reſiſtance ariſing from the friction of the geers and attrition of the ſtones in grinding the corn.

The greater fall the water has, the leſs quantity of it will ſerve to turn the mill. The water is kept up in the mill-dam, and let out by a ſluice called the penſtock, when the mill is to go. When the penſtock is drawn up by means of a lever, it opens a paſſage through which the water flows to the wheel: and when the mill is to be ſtopt, the penſtock is let down, which ſtops the water from falling upon the wheel.

A leſs quantity of water will turn an overſhot-mill (where the wheel has buckets inſtead of float-boards) than a breaſt-mill where the fall of the water ſeldom exceeds half the height *Ab* of the wheel. So that, where there is but a ſmall quantity of water, and a fall great enough for the wheel to lie under it, the bucket (or overſhot) wheel is always uſed. But where there is a large body of water, with a little fall, the breaſt or float-board wheel muſt take place. Where the water runs only upon a little declivity, it can act but ſlowly upon the under part of the wheel at *b*; in which caſe, the motion of the wheel will be very ſlow: and therefore, the floats ought to be very long, though not high, that a large body of water may act upon them; ſo that what is wanting in velocity may be made up in power: and then the cog-wheel may have a greater number of cogs in proportion to the rounds in the trundle, in order to give the millſtone a ſufficient degree of velocity.

They who have read what is ſaid in the firſt lecture, concerning the acceleration of bodies falling freely by the power of gravity acting conſtantly and uniformly upon them, may perhaps aſk, Why ſhould the motion of the wheel be equable, and not accelerated, ſeeing the water acts conſtantly and uniformly upon it? The plain anſwer is, That the velocity of the wheel can never be ſo great as the velocity of the water that turns it; for, if it ſhould become ſo great, the power of the water would be quite loſt upon the wheel, and then there would be no proper force to overcome the friction of the geers and attrition of the ſtones. Therefore, the velocity with which the wheel begins to move, will increaſe no longer than till its *momentum* or force is balanced by the reſiſtance of the working parts of the mill; and then the wheel will go on with an equable motion.

A *hand-mill.*

[If the cog-wheel *D* be made about 18 inches diameter, with 30 cogs, the trundle as ſmall in proportion, with 10 ſtaves, and the millſtones be each about two feet in diameter, and the whole work be put into a ſtrong frame of wood, as repreſented in the figure, the engine will be a hand-mill for grinding corn or malt in private families. And then, it may be turned by a winch inſtead of the wheel *A A*: the millſtone making three revolutions for every one of the winch. If a heavy fly be put upon the axle *B*, near the winch, it will help to regulate the motion.]

If the cogs of the wheel and rounds of the trundle could be put in as exactly as the teeth are cut in the wheels and pinions of a clock, then the trundle might divide the wheel exactly: that is to ſay, the trundle might make a given number of revolutions for one of the wheel, without a fraction. But as any exact number is not neceſſary in mill-work, and the cogs and rounds cannot be ſet in ſo truly as to make all the intervals between them equal; a ſkilful mill-wright will always give the wheel what he calls a *hunting cog*; that is, one more than what will anſwer to an exact diviſion of the wheel by the trundle. And then, as every cog comes to the trundle, it will take the next ſtaff or round behind the one which it took in the former revolution: and by that means, will wear all the parts of the cogs and rounds which work upon one another equally, and to equal diſtances from one another in a little time; and ſo make a true uniform motion throughout the whole work. Thus, in the above water-mill, the trundle has 10 ſtaves, and the wheel 61 cogs.

Sometimes, where there is a ſufficient quantity of water, the cog-wheel *A A* turns a large Fig. 4. trundle *B B*, on whoſe axis *C* is fixed the horizontal wheel *D*, with cogs all around its edge, turning two trundles *E* and *F* at the ſame time; whoſe axes or ſpindles *G* and *H* turn two mill-ſtones *I* and *K*, upon the fixed ſtones *L* and *M*. And when there is not work for them both, either may be made to lie quiet, by taking out one of the ſtaves of its trundle, and turning the vacant place towards the cog wheel *D*. And there may be a wheel fixt on the upper end of the great upright axle *C* for turning a couple of boulting-mills; and other work for drawing up the ſacks, fanning and cleaning the corn, ſharpening of tools, &c.

A *Horſe-mill.* If, inſtead of the cog-wheel *A A* and trundle *B B*, horizontal levers be fixed into the axle *C*, below the wheel *D*; then, horſes may be put to theſe levers for turning the mill: which is often done where water cannot be had for that purpoſe.

A *wind-mill.* The working parts of a wind-mill differ very little from thoſe of a water-mill; only the former is

PLATE . VIII.

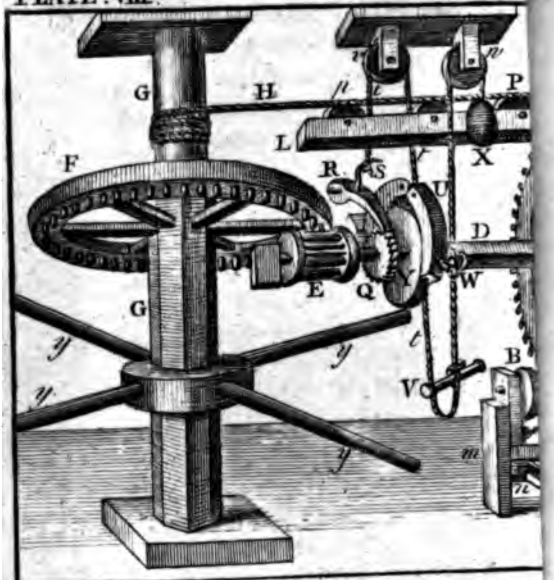
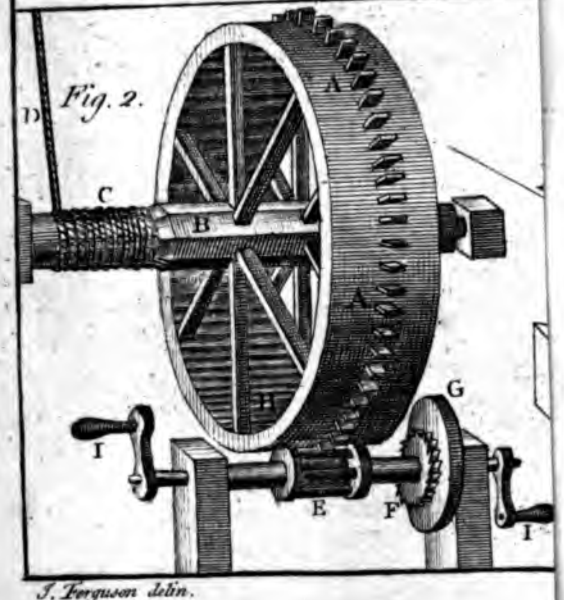


Fig. 2.



J. Ferguson delin.

is turned by the action of the wind upon four ſails, every one of which ought (as is generally believed) to make an angle of $54\frac{2}{3}$ degrees with a plane perpendicular to the axis on which the arms are fixt for carrying them. It being demonſtrable, that when the ſails are ſet to ſuch an angle, and the axis turned end-ways toward the wind, the wind has the greateſt power upon the ſails. But this angle anſwers only to the caſe of a vane or ſail juſt beginning to move*: for, when the vane has a certain degree of motion, it yields to the wind; and then that angle muſt be increaſed to give the wind its full effect.

Again, the increaſe of this angle ſhould be different, according to the different velocities from the axis to the extremity of the vane. At the axis it ſhould be $54\frac{2}{3}$ degrees, and thence continually decreaſe, giving the vane a twiſt, and ſo cauſing all the ribs of the vane to lie in different planes

Laſtly, Theſe ribs ought to decreaſe in length from the axis to the extremity, giving the vane a curvilineal form,; ſo that no part of the force of any one rib be ſpent upon the reſt, but all move on independent of each other. All this is required to give the ſails of a wind-mill their true form: and we ſee both the twiſt and the diminution of the ribs exemplified in the wings of birds.

It is almoſt incredible to think with what velocity the tips of the ſails move when acted upon by a moderate gale of wind. I have ſeveral times counted the number of revolutions made by the ſails in ten or fifteen minutes; and from the length of the arms from tip to tip, have computed, that if a hoop of that diameter was to run upon the ground with the ſame velocity that it would move if put upon the ſail-arms, it would go upwards of 30 miles in an hour.

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* See MACLAURIN's Fluxions, near the end.

As the ends of the ſails neareſt the axis cannot move with the ſame velocity that the tips or fartheſt ends do, although the wind acts equally ſtrong upon them; perhaps a better poſition than that of ſtretching them along the arms directly from the center of motion, might be to have them ſet perpendicularly acroſs the farther ends of the arms, and there adjuſted lengthwiſe to the proper angle. For, in that caſe, both ends of the ſails would move with the ſame velocity; and being farther from the center of motion, they would have ſo much the more power: and then, there would be no occaſion for having them ſo large as they are generally made; which would render them lighter, and conſequently, there would be ſo much the leſs friction on the thick neck of the axle where it turns in the wall.

A *crane.*

Plate VIII. Fig. 1.

A *crane* is an engine by which great weights are raiſed to certain heights, or let down to certain depths. It conſiſts of wheels, axles, pulleys, ropes, and a gib or gibbet. When the rope *H* is hooked to the weight *K*, a man turns the winch *A*, on the axis whereof is the trundle *B*, which turns the wheel *C*, on whoſe axis *D* is the trundle *E*, which turns the wheel *F* with its upright axis *G*, on which the great rope *H H* winds as the wheel turns; and going over a pulley *I* at the end of the arm *d* of the gib *c c d e*, it draws up the heavy weight *K*; which, being raiſed to a proper height, as from a ſhip to the quay, is then brought over the quay by pulling the wheel *Z* round by the handles *z*, *z*, which turns the gib by means of the half wheel *b* fixt on the gib-poſt *c c*, and the ſtrong pinion *a* fixt on the axis of the wheel *Z*. This wheel gives the man that turns it an abſolute command over the

the gib, ſo as to prevent it from taking any unlucky ſwing, ſuch as often happens when it is only guided by a rope tied to its arm *d*; and people are frequently hurt, ſometimes killed, by ſuch accidents.

The great rope goes between two upright rollers *i* and *k*, which turn upon gudgeons in the fixed beams *f* and *g*; and as the gib is turned towards either ſide, the rope bends upon the roller next that ſide. Were it not for theſe rollers, the gib would be quite unmanageable; for the moment it were turned ever ſo little towards any ſide, the weight *K* would begin to deſcend, becauſe the rope would be ſhortened between the pulley *I* and axis *G*; and ſo the gib would be pulled violently to that ſide, and either be broke to pieces, or break every thing that came in its way. Theſe rollers muſt be placed ſo, that the ſides of them, round which the rope bends, may keep the middle of the bended part directly even with the center of the hole in which the upper gudgeon of the gib turns in the beam *f*. The truer theſe rollers are placed, the eaſier the gib is managed, and the leſs apt to ſwing either way by the force of the weight *K*.

A ratchet-wheel *Q* is fixt upon the axis *D*, near the trundle *E*; and into this wheel the catch or click *R* falls. This hinders the machinery from runing back by the weight of the burden *K*, if the man who raiſes it ſhould happen to be careleſs, and ſo leave off working at the winch *A* ſooner than he ought to do.

When the weight *K* is raiſed to its proper height from the ſhip, and brought over the quay by turning the gib about, it is let down gently upon the quay, or into a cart ſtanding thereon, in the following manner: A man takes hold of the rope *t t* (which goes over the pulley *v*, and is tied to a hook at *S* in the catch *R*) and ſo diſengages the catch from the ratchet-wheel *Q*; and then, the man at the winch *A* turns it backward, and lets down the weight *K*. But if the weight pulls too hard againſt this man, another lays hold of the ſtick *V*, and by pulling it downward, draws the gripe *U* cloſe to the wheel *Y*, which, by rubbing hard againſt the gripe, hinders the too quick deſcent of the weight; and not only ſo, but even ſtops it at any time, if required. By this means, heavy goods may be either raiſed or let down at pleaſure without any danger of hurting the men who work the engine.

When part of the goods are craned up, and the rope is to be let down for more, the catch *R* is firſt diſengaged from the ratchet-wheel *Q*, by pulling the cord *t*; then the handle *q* is turned half round backward, which, by the crank *n n* in the piece *o*, pulls down the frame *b* between the guides *m* and *m* (in which it ſlides in a groove) and ſo diſengages the trundle *B* from the wheel *C*: and then, the heavy hook *β* at the end of the rope *H* deſcends by its own weight, and turns back the great wheel *F* with its trundle *E*, and the wheel *C*; and this laſt wheel acts like a fly againſt the wheel *F* and hook *β*; and ſo hinders it from going down too quick; whilſt the weight *X* keeps up the gripe *U* from rubbing againſt the wheel *Y*, by means of a cord going from the weight, over the pulley *w* to the hook *W* in the gripe; ſo that the gripe never touches the wheel, unleſs it be pulled down by the handle *V*.

When the crane is to be ſet at work again, for drawing up another burden, the handle *q* is turned half round forwards; which, by the crank *n n*, raiſes up the frame *b*, and cauſes the trundle *B* to lay hold of the wheel *C*; and then, by turning the winch *A*, the burden of goods *K* is drawn up as before.

The crank *n n* turns pretty ſtiff in the mortiſe near *o*, and ſtops againſt the farther end of it when it has got juſt a little beyond the perpendicular; ſo that it can never come back of itſelf: and therefore, the trundle *B* can never come away from the wheel *C*, until the handle *q* be turned half round, backwards.

The great rope runs upon rollers in the lever *L M*, which keeps it from bending between the axle at *G* and the pulley *I*. This lever turns upon the axis *N* by means of the weight *O*, which is juſt ſufficient to keep its end *L* up to the rope; ſo that, as the great axle turns, and the rope coils round it, the lever riſes with the rope, and prevents the coilings from going over one another.

The power of this crane may be eſtimated thus; ſuppoſe the trundle *B* to have 13 ſtaves or rounds, and the wheel *C* to have 78 ſpur cogs; the trundle *E* to have 14 ſtaves, and the wheel *F* 56 cogs. Then, by multiplying the ſtaves of the trundles, 13 and 14, into one another, their product will be 182; and by multiplying the cogs of the wheels, 78 and 56, into one another, their product will be 4368, and dividing 4368 by 182, the quotient will be 24; which ſhews that the winch *A* makes 24 turns for one turn of the wheel *F* and its axle *G* on which the great rope or chain *H I H* winds. So that, if the length or radius of the winch *A* were only equal to half the diameter of the great axle *G*, added to half the thickneſs of the rope *H*, the power of the crane would be as 24 to 1: but the radius of the winch being double the above length, it doubles the ſaid power, and ſo makes it as 48 to 1: in which caſe, a man may raiſe 48 times as much weight by this engine as he could do by his natural ſtrength without it, making proper allowance for the friction of the working parts.—Two men may work at once, by having another winch on the oppoſite end of the axis of the trundle under *B*; and this will make the power double.

If this power be thought greater than what may be generally wanted, the wheels may be made with fewer cogs in proportion to the ſtaves in the trundles; and ſo the power may be of whatever degree is judged to be requiſite. But if the weight be ſo great as will require yet more power to raiſe it (ſuppoſe a double quantity) then the rope *H* may be put under a moveable pulley, as δ, and the end of it tied to a hook in the gib at ϵ; which will give a double power to the machine, and ſo raiſe a double weight hooked to the block of the moveable pulley.

When only ſmall burthens are to be raiſed, this may be quickly done by men puſhing the axle *G* round by the long ſpokes *y*, *y*, *y*, *y*; having firſt diſengaged the trundle *B* from the wheel *C*: and then, this wheel will only act as a fly upon the wheel *F*; and the catch *R* will prevent its running back, if the men ſhould inadvertently leave off puſhing before the burthen be unhooked from β.

Laſtly, When very heavy burthens are to be raiſed, which might endanger the breaking of the cogs in the wheel *F*; their force againſt theſe cogs may be much abated by men puſhing round the long ſpokes *y*, *y*, *y*, *y*, whilſt the man at *A* turns the winch.

I have only ſhewn the working parts of this crane, without the whole of the beams which ſupport them; knowing that theſe are eaſily ſuppoſed, and that if they had been drawn, they would have hid a great deal of the working parts from ſight, and alſo confuſed the figure.

Another *crane.* Fig. 2.

Another very good *crane* is made in the following manner. *A A* is a great wheel turned by men walking within it at *H.* On the part *C*, of its axle *B C*, the great rope *D* is wound as the wheel turns; and this rope draws up goods in the ſame way as the rope *H H* does in the above-mentioned crane, the gib-work here being ſuppoſed to be of the ſame ſort. But theſe cranes are very dangerous to the men in the wheel; for, if any of the men ſhould chance to fall, the burthen will make the wheel run back and throw them all about within it; which often breaks their limbs, and ſometimes kills them. The late ingenious Mr. *Padmore* of Briſtol, (whoſe contrivance the forementioned crane is, ſo far as I can remember its conſtruction, after ſeeing it once about twelve years ago *,) obſerving this dangerous conſtruction, contrived a method for remedying it, by putting cogs all around the outſide of the wheel, and applying a trundle *E* to turn it; which increaſes the power as much as the number of cogs in the wheel is greater than the number of ſtaves in the trundle: and by putting a ratchet-wheel *F* on the axis of the trundle, (as in the above-mentioned crane) with a catch to fall into it, the great wheel is ſtopt from running back by the force of the weight, even if all the men in it ſhould leave off walking. And by one man working at the winch *l*, or two men at the oppoſite winches when needful, the men in the wheel are much aſſiſted, and much greater weights are raiſed, than could be by men only within the wheel. Mr. *Padmore* put alſo a gripe-wheel *G* upon the axis of the trundle, which being pinched in the ſame manner as deſcribed in the former crane, heavy burthens may be let down without the leaſt danger. And before this contrivance, the lowering of goods was always attended with the utmoſt danger to the men in the wheel; as every one muſt be ſenſible of, who has ſeen ſuch engines at work.

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* Since the firſt edition of this book was printed, I have ſeen the ſame crane again; and do find, that though the working parts are much the ſame as above deſcribed, yet the method of raiſing or lowering the trundle *B*, and the catch *R*, are better contrived than I had deſcribed them.

And it is ſurpriſing that the maſters of wharfs and cranes ſhould be ſo regardleſs of the limbs, or even lives of their workmen, that excepting the late Sir *James Creed* of Greenwich, and ſome gentlemen at Briſtol, there is ſcarce an inſtance of any who has uſed this ſafe contrivance.

Wheel-carriages. The ſtructure of *wheel-carriages* is generally ſo well known, that it would be needleſs to deſcribe them. And therefore, we ſhall only point out ſome inconveniencies attending the common method of placing the wheels, and loading the waggons.

In coaches, and all other four-wheeled carriages, the fore-wheels are made of a leſs ſize than the hind ones, both on account of turning ſhort, and to avoid cutting the braces: otherwiſe, the carriage would go much eaſier if the fore-wheels were as high as the hind ones, and the higher the better, becauſe they would ſink to leſs depths in little hollowings in the roads, and be the more eaſily drawn out of them. But carriers and coachmen give another reaſon for making the fore-wheels much lower than the hind-wheels; namely, that when they are ſo, the hind-wheels help to puſh on the fore ones: which is too unphiloſophical and abſurd to deſerve a refutation, and yet for their ſatisfaction we ſhall ſhew by experiment that it has no exiſtence but in their own imaginations.

It is plain that the ſmall wheels muſt turn as much oftener round than the great ones, as their circumferences are leſs. And therefore, when the carriage is loaded equally heavy on both axles, the fore-axle muſt ſuſtain as much more friction, and conſequently wear out as much ſooner, than the hind-axle, as the fore-wheels are leſs than the hind-ones. But the great misfortune is, that all the carriers to a man do obſtinately perſiſt, againſt the cleareſt reaſon and demonſtration, in putting the heavier part of the load upon the fore-axle of the waggon; which not only makes the friction greateſt where it ought to be leaſt, but alſo preſſeth the fore-wheels deeper into the ground than the hind-wheels, notwithſtanding the fore-wheels, being leſs than the hind ones, are with ſo much the greater difficulty drawn out of a hole or over an obſtacle, even ſuppoſing the weights on their axles were equal. For the difficulty, with equal weights, will be as the depth of the hole or height of the obſtacle is to the ſemidiameter of the wheel. Thus, if we ſuppoſe the ſmall wheel *D* of the waggon *AB* to fall into a hole of the depth *EF*, which is equal to the ſemidiameter of the wheel, and the waggon to be drawn horizontally along; it is evident, that the point *E* of the ſmall wheel will be drawn directly againſt the top of the hole; and therefore, Fig. 3.

fore, all the power of horſes and men will not be able to draw it out, unleſs the ground gives way before it. Whereas, if the hind-wheel *G* falls into ſuch a hole, it ſinks not near ſo deep in proportion to its ſemidiameter; and therefore, the point *G* of the large wheel will not be drawn directly, but obliquely, againſt the top of the hole; and ſo will be eaſily got out of it. Add to this, that as a ſmall wheel will often ſink to the bottom of a hole, in which a great wheel will go but a very little way, the ſmall wheels ought in all reaſon to be loaded with leſs weight than the great ones: and then the heavier part of the load would be leſs jolted upward and downward, and the horſes tired ſo much the leſs, as their draught raiſed the load to leſs heights.

It is true, that when the waggon-road is much up-hill, there may be danger in loading the hind part much heavier than the fore part; for then the weight would overhang the hind-axle, eſpecially if the load be high, and endanger tilting up the fore-wheels from the ground. In this caſe, the ſafeſt way would be to load it equally heavy on both axles; and then, as much more of the weight would be thrown upon the hind-axle than upon the fore one, as the ground riſes from a level below the carriage. But as this ſeldom happens, and when it does, a ſmall temporary weight laid upon the pole between the horſes would overbalance the danger; and this weight might be thrown into the waggon when it comes to level ground; it is ſtrange that an advantage ſo plain and obvious as would ariſe from loading the hind-wheels heavieſt, ſhould not be laid hold of, by complying with this method.

To confirm theſe reaſonings by experiment, let a ſmall model of a waggon be made, with its fore-wheels 2½ inches in diameter, and its hind-wheels 4½; the whole model weighing about 20 ounces. Let this little carriage be loaded any how with weights, and have a ſmall cord tied to each of its ends, equally high from the ground it reſts upon; and let it be drawn along a horizontal board, firſt by a weight in a ſcale hung to the cord at the fore-part; the cord going over a pulley at the end of the board to facilitate the draught, and the weight juſt ſufficient to draw it along. Then, turn the carriage, and hang the ſcale and weight to the hind cord, and it will be found to move along with the ſame velocity as at firſt: which ſhews, that the power required to draw the carriage is all the ſame, whether the great or ſmall wheels are foremoſt; and therefore the great wheels do not help in the leaſt to puſh on the ſmall wheels in the road.

Hang the ſcale to the fore-cord, and place the fore-wheels (which are the ſmall ones) in two holes, cut three eight parts of an inch deep into the board; then put a weight of 32 ounces into the carriage, over the fore-axle, and an equal weight over the hind one: this done, put 44 ounces into the ſcale, which will be juſt ſufficient to draw out the fore-wheels: but if this weight be taken out of the ſcale, and one of 16 ounces put into its place, if the hind-wheels are placed in the holes, the 16 ounce weight will draw them out; which is little more than a third part of what was neceſſary to draw out the fore-wheels. This ſhews, that the larger the wheels are, the leſs power will draw the carriage, eſpecially on rough ground.

Put 64 ounces over the axle of the hind-wheels, and 32 over the axle of the fore-ones, in the carriage; and place the fore wheels in the holes: then, put 38 ounces into the ſcale, which will juſt draw out the fore-wheels; and when the hind ones come to the hole, they will find but very little reſiſtance, becauſe they ſink but a little way into it.

But ſhift the weights in the carriage, by putting the 32 ounces upon the hind-axle, and the 64 ounces upon the fore one; and place the fore-wheels in the holes: then, if 76 ounces be put into the ſcale, it will be found no more than ſufficient to draw out theſe wheels; which is double the power required to draw them out, when the lighter part of the load was put upon them: which is a plain demonſtration of the abſurdity of putting the heavieſt part of the load in the fore-part of the waggon.

Every one knows what an out-cry was made by the generality, if not the whole body, of the carriers, againſt the broad-wheel act; and how hard it was to perſuade them to comply with it, even though the government allowed them to draw with more horſes, and carry greater loads, than uſual. Their principal objection was, that as a broad wheel muſt touch the ground in a great many more points than a narrow wheel, the friction muſt of courſe be juſt ſo much the greater; and conſequently, there muſt be ſo many more horſes than uſual, to draw the waggon. I believe that the majority of people were of the ſame opinion, not conſidering, that if the whole weight of the waggon and load in it bears upon a great many points, each ſuſtains a proportionably leſs degree of weight and friction, than when it bears only upon a few points; ſo that what is wanting in one, is made up in the other; and therefore will be juſt equal under equal degrees of weight, as may be ſhewn by the following plain and eaſy experiment.

Let one end of a piece of packthread be faſtened to a brick, and the other end to a common ſcale for holding weights: then, having laid the brick edgewiſe on a table, and let the ſcale hang under the edge of the table, put as much weight into the ſcale as will juſt draw the brick along the table. Then taking back the brick to its former place, lay it flat on the table, and leave it to be acted upon by the ſame weight in the ſcale as before, which will draw it along with the ſame eaſe as when it lay upon its edge. In the former caſe, the brick may be conſidered as a narrow wheel on the ground; and in the latter as a broad wheel. And ſince the brick is drawn along with equal eaſe, whether its broad ſide or narrow edge touches the table, it ſhews that a broad wheel might be drawn along the ground with the ſame eaſe as a narrow one (ſuppoſing them equally heavy) even though they ſhould drag, and not roll, as they go along.

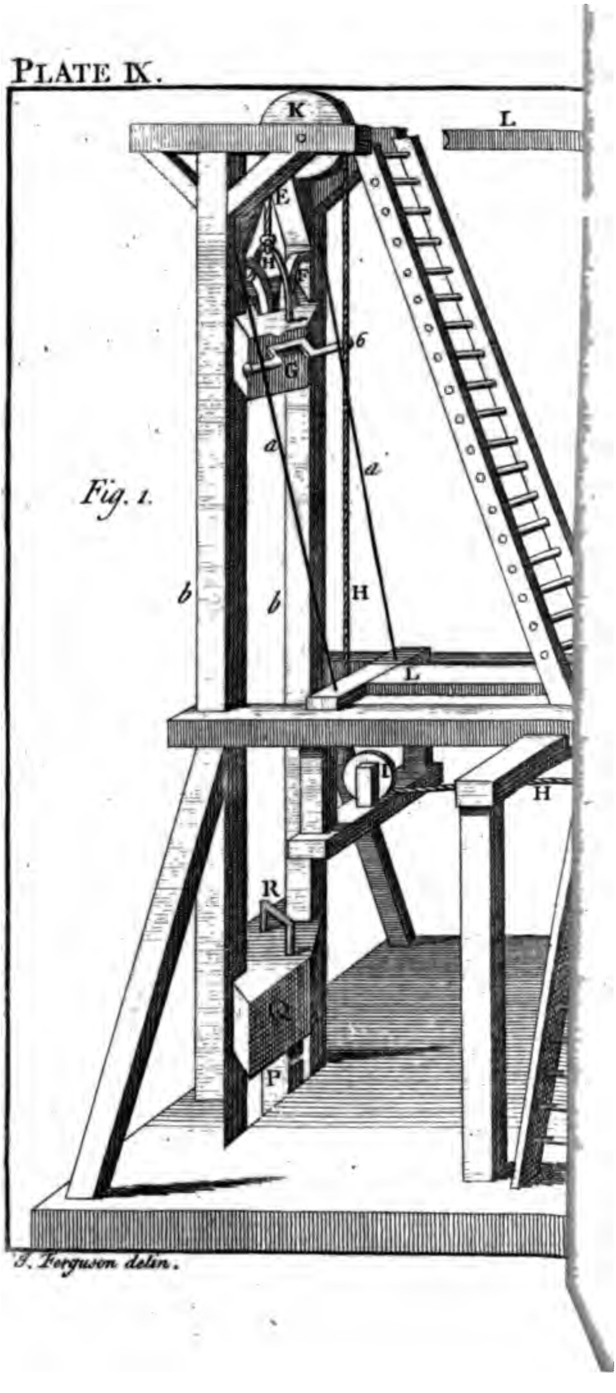
As narrow wheels are always ſinking into the ground, eſpecially when the heavieſt part of the load lies upon them, they muſt be conſidered as going conſtantly up hill, even on level ground. And their ſides muſt ſuſtain a great deal of friction by rubbing againſt the ruts made by them. But both theſe inconveniencies are avoided by broad wheels; which, inſtead of cutting and ploughing up the roads, roll them ſmooth, and harden them; as experience teſtifies in places where they have been uſed, eſpecially either on wettiſh or ſandy ground: though after all it muſt be confeſſed, that they will not do in ſtiff clayey croſs roads; becauſe they would ſoon gather up as much clay as would be almoſt equal to the weight of an ordinary load.

If the wheels were always to go upon ſmooth and level ground, the beſt way would be to make the ſpokes perpendicular to the naves; that is, to ſtand at right angles to the axles; becauſe they would then bear the weight of the load perpendicularly, which is the ſtrongeſt way for wood. But becauſe the ground is generally uneven, one wheel often falls into a cavity or rut when the other does not; and then it bears much more of the weight than the other does: in which caſe, concave or diſhing wheels are beſt, becauſe when one falls into a rut, and the other keeps upon high ground, the ſpokes become perpendicular in the rut, and therefore have the greateſt ſtrength when the obliquity of the load throws moſt of its weight upon them; whilſt thoſe on the high ground have leſs weight to bear, and therefore need not be at their full ſtrength. So that the uſual way of making the wheels concave is by much the beſt.

The axles of the wheels ought to be perfectly ſtraight, that the rim of the wheels may be parallel to each other; for then they will move eaſieſt, becauſe they will be at liberty to go on ſtraight forwards. But in the uſual way of practice, the axles are bent downward at their ends; which brings the ſides of the wheels next the ground nearer to one another than their oppoſite or higher ſides are: and this not only makes the wheels to drag ſidewiſe as they go along, and gives the load as much greater power of cruſhing them than when they are parallel to each other, but alſo endangers the over-turning of the carriage when any wheel falls into a hole or rut; or when

PLATE IX.

Fig. 1.



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when the carriage goes in a road which has one ſide lower than the other, as along the ſide of a hill. Thus (in the hind view of a waggon or cart) let *A E* and *B F* be the great wheels parallel to each other, on their ſtraight axle *K*, and Fig. 4. *H C I* the carriage loaded with heavy goods from *C* to *G*. Then, as the carriage goes on in the oblique road *A a B*, the center of gravity of the whole machine and load will be at *C**; and the line of direction *C d D* falling within the wheel *B F*, the carriage will not overſet. But if the wheels be inclined to each other on the ground, Fig. 5. as *A E* and *B F* are, and the machine be loaded as before, from *C* to *G*, the line of direction *C d D* falls without the wheel *B F*, and the whole machine tumbles over. When it is loaded with heavy goods (ſuch as lead or iron) which lie low, Fig. 4. it may travel ſafely upon an oblique road ſo long as the center of gravity is at *C*, and the line of direction *C d* falls within the wheels; but if it be loaded high with lighter goods (ſuch as woolpacks) from *C* to *L*, the center of gravity is raiſed Fig. 6. from *C* to *K*, which throws the line of direction *K k* without the loweſt edge of the wheel *B F*, and then the load overſets the waggon.

* See page 13.

If there be ſome advantage from ſmall fore-wheels, on account of the carriage turning more eaſily and ſhort than it can be made to do when they are large; there is at leaſt as great a diſadvantage attending them, which is, that as their axle is below the level of the horſes breaſts, the horſes not only have the loaded carriage to draw along, but alſo part of its weight to bear; which tires them ſooner, and makes them grow much ſtiffer in their hams, than they would be if they drew on a level with the fore-axle. And for this reaſon, we find coach horſes ſoon become unfit for riding. So that on all accounts it is plain, that the fore-wheels of all carriages ought to be ſo high, as to have their axles even with the breaſt of the horſes; which would not only give the horſes a fair draught, but likewiſe keep them longer fit for drawing the carriage.

Plate IX. Fig. 1, 2. We ſhall conclude this lecture with a deſcription of Mr. *Vauloue*'s curious engine, which was made uſe of for driving the piles of Weſtminſter-bridge: and the reader may caſt his eyes upon the firſt and ſecond figures of the plate, in which the ſame letters of reference are annexed to the ſame parts, in order to explain thoſe in the ſecond, which are either partly or wholly hid in the firſt.

The *pile-engine.* *A* is the great upright ſhaft or axle, on which are the great wheel *B* and drum *C*, turned by horſes joined to the bars *S*, *S*. The wheel *B* turns the trundle *X*, on the top of whoſe axis is the fly *O*, which ſerves to regulate the motion, and alſo to act againſt the horſes, and keep them from falling when the heavy ram *Q* is diſcharged to drive the pile *P* down into the mud in the bottom of the river. The drum *C* is looſe upon the ſhaft *A*, but is locked to the wheel *B* by the bolt *Y*. On this drum the great rope *H H* is wound; one end of the rope being fixed to the drum, and the other to the follower *G*, to which it is conveyed over the pulleys *I* and *K*. In the follower *G* is contained the tongs *F* (ſee Fig. 3.) that takes hold of the ram *Q* by the ſtaple *R* for drawing it up. *D* is a ſpiral or fuſy fixt to the drum, on which is wound the ſmall rope *T* that goes over the pulley *U*, under the pulley *V*, and is faſtened to the top of the frame at 7. To the pulley block *V* is hung the counterpoiſe *W*, which hinders the follower from accelerating as it goes down to take hold of the ram: for, as the follower tends to acquire velocity in its deſcent, the line *T* winds downwards upon the fuſy, on a larger and larger radius, by which means the counterpoiſe *W* acts ſtronger and ſtronger againſt it; and ſo allows it to come down with only a moderate and uniform velocity. The bolt *Y* locks the drum to the great wheel, being puſhed upward by the ſmall lever 2, which goes through a mortiſe in the ſhaft *A*, turns upon a pin in the bar 3 fixt to the great wheel *B*, and has a weight 4, which always tends to puſh up the bolt *Y* through the wheel into the drum. *L* is the great lever turning on the axis *m*, and reſting upon the forcing bar 5, 5, which goes down through a hollow in the ſhaft *A*, and bears up the little lever 2.

By the horſes going round, the great rope *H* is wound about the drum *C*, and the ram *Q* is drawn up by the tongs *F* in the follower *G*, until the tongs comes between the inclined planes *E*; which, by ſhutting the tongs at the top, opens it at the foot, and diſcharges the ram, which falls down between the guides *b b* upon the pile *P*, and drives it by a few ſtrokes as far into the mud as it can go; after which, the top part is ſawed off cloſe to the mud, by an engine for that purpoſe. Immediately after the ram is diſcharged, the piece 6 upon the follower *G* takes hold of the ropes *a*, *a*, which raiſe the end of the lever *L*, and cauſe its end *N* to deſcend and preſs down the forcing bar 5 upon the little lever 2, which by pulling down the bolt *Y*, unlocks the drum *C* from the great wheel *B*; and then, the follower, being at liberty, comes down by its own weight to the ram; and the lower ends of the tongs ſlip over the ſtaple *R*, and the weight of their heads cauſes them to fall outward, and ſhuts upon it. Then the weight 4 puſhes up the bolt *Y* into the drum, which locks it to the great wheel, and ſo the ram is drawn up as before.

As the follower comes down, it cauſes the drum to turn backward, and unwinds the rope from it, whilſt the horſes, great wheel, trundle and fly, go on with an uninterrupted motion: and as the drum is turning backward, the counterpoiſe *W* is drawn up, and its rope *Y* wound upon the ſpiral fuſy *D*.

There are ſeveral holes in the under ſide of the drum, and the bolt *Y* always takes the firſt one that it finds when the drum ſtops by the falling of the follower upon the ram; until which ſtoppage, the bolt has not time to ſlip into any of the holes.

This engine was placed upon a barge on the water, and ſo was eaſily conveyed to any place deſired.—I never had the good fortune to ſee it, but drew this figure from a model which I made from a print of it; being not quite ſatisfied with the view which the print gives. I have been told that the ram was a ton weight, and that the guides *b b*, between which it was drawn up and let fall down, were 30 feet high. I ſuppoſe the great wheel may have had 100 cogs, and the trundle 10 ſtaves or rounds; ſo that the fly would make 10 revolutions for one of the great wheel.

F
E
7.

LECT. V.

Of hydroſtatics, and hydraulic machines.

THE ſcience of *hydroſtatics* treats of the nature, gravity, preſſure, and motion of fluids in general; and of weighing ſolids in them.

Definition of a fluid.

A fluid is a body that yields to the leaſt preſſure or difference of preſſures. Its particles are ſo ſmall, that they cannot be diſcerned by the beſt microſcopes; they are hard, ſince no fluid, except air or ſteam, can be preſſed into a leſs ſpace than it naturally poſſeſſes; and they muſt be round and ſmooth, ſeeming they are ſo eaſily moved among one another.

All bodies, both fluid and ſolid, preſs downwards by the force of gravity: but fluids have this wonderful property, that their preſſure upwards and ſidewiſe is equal to their preſſure downwards; and this is always in proportion to their perpendicular height, without any regard to their quantity; for, as each particle is quite free to move, it will move towards that part or ſide on which the preſſure is leaſt. And hence, no particle or quantity of a fluid can be at reſt, till it is every way equally preſſed.

Plate X. Fig. 1. Fluids preſs as much upward as downward.

To ſhew by experiment that fluids preſs upward as well as downward, let *A B* be a long upright tube filled with water near to its top; and *C D* a ſmall tube open at both ends, and immerſed into the water in the large one; if the immerſion be quick, you will ſee the water riſe in the ſmall tube to the ſame height that it ſtands in the great one, or until the ſurfaces of the water in both are on the ſame level: which ſhews that the water is preſſed upward into the ſmall tube by the weight of what is in the great one; otherwiſe it could never riſe therein, contrary to its natural gravity; unleſs the diameter of the bore were ſo ſmall, that the attraction of the tube would raiſe the water; which will never happen, if the tube be as wide as that in a common barometer. And, as the water riſes no higher in the ſmall tube than till its ſurface be on a level with the ſurface of the water in the great one, this ſhews that the preſſure is not in proportion to the quantity of water in the great tube, but in proportion to its perpendicular height therein: for there is much more water in the great tube all around the ſmall one, than what is raiſed to the ſame height in the ſmall one, as it ſtands within the great.

Take out the ſmall tube, and let the water run out of it; then it will be filled with air. Stop its upper end with the cork *C*, and it will be full of air all below the cork: this done, plunge it again to the bottom of the water in the great tube, and you will ſee the water riſe up in it only to the height *D*; which ſhews that the air is a body, otherwiſe it could not hinder the water from riſing up to the ſame height as it did before, namely, to *A*; and in ſo doing, it drove the air out at the top; but now the air is confined by the cork *C*: and it alſo ſhews that the air is a compreſſible body, for if it were not ſo, a drop of water could not enter into the tube.

The preſſure of fluids being equal in all directions, it follows that the ſides of a veſſel are as much preſſed by a fluid in it, all around in any given ring of points, as the fluid below that ring is preſſed by the weight of all that ſtands above it. Hence the preſſure upon every point in the ſides, immediately above the bottom, is equal to the preſſure upon every point of the bottom. To ſhew this by experiment, let a hole be made at *e* in the ſide of the tube *A B* cloſe by the bottom; and another hole of the ſame ſize in the bottom at *C*; then pour water into the tube, keeping it full as long as you chooſe the holes ſhould run, and have two baſons ready to receive the water that runs through the two holes, until you think there is enough in each baſon; and you will find by meaſuring the quantities, that they are equal; which ſhews that the water run with equal ſpeed through both holes: which it could not have done, if it had not been equally preſſed through them both. For, if a hole of the ſame ſize be made in the ſide of the tube, as about *f*, and if all three are permitted to run together, you will find that the quantity run through the hole at *f* is much leſs than what has run in the ſame time through either of the holes *C* or *e*. Fig. 2.

In the ſame figure, let a tube be turned up from the bottom at *C* into the ſhape *D E*, and the hole at *C* be ſtopt with a cork. Then, pour water into the tube to any height, as *Ag*, and it will ſpout up in a jet *E F G*, nearly as high as it is kept in the tube *A B*, by continuing to pour in as much there as runs through the hole *E*; which will be the caſe whilſt the ſurface *Ag* keeps at the ſame height. And if a little ball of cork *G* be laid upon the top of the jet, it will be ſupported thereby, and dance upon it. The reaſon why the jet riſes not quite ſo high as the ſurface of the water *Ag*, is owing to the reſiſtance it meets with in the open air: for, if a tube, either great or ſmall, was ſcrewed upon the pipe at *E*, the water would riſe in it until the ſurface of the water in both tubes were on the ſame level; as will be ſhewn by the next experiment.

The *hy- droſtatic paradox.*

Any quantity of a fluid, how ſmall ſoever, may be made to balance and ſupport any quantity, how great ſoever. This is deſervedly termed *the hydroſtatical paradox*, which we ſhall firſt ſhew by an experiment, and then account for it upon the principle above mentioned; namely, that *the preſſure of fluids is directly as their perpendicular height*, *without any regard to their quantity.*

Fig. 3.

Let a ſmall glaſs tube *DCG*, open throughout, and bended at *B*, be joined to the end of a great one *AI* at *c d*, where the great one is alſo open; ſo that theſe tubes in their openings may freely communicate with each other. Then pour water through a ſmall necked funnel into the ſmall tube at *H*; this water will run through the joining of the tubes at *c d*, and riſe up into the great tube; and if you continue pouring until the ſurface of the water comes to any part, as *A*, in the great tube, and then leave off, you will ſee that the ſurface of the water in the ſmall tube will be juſt as high, at *D*; ſo that the perpendicular height of the water will be the ſame in both tubes, however ſmall the one be in proportion to the other. This ſhews, that the ſmall column *D C G* balances and ſupports the great column *A c d*: which it could not do if their preſſures were not equal againſt one another in the recurved bottom at *B*.—If the ſmall tube be made longer, and inclined in the ſituation *G E F*, the ſurface of the water in it will ſtand at *F*, on the ſame level with the ſurface *A* in the great tube; that is, the water will have the ſame perpendicular height in both tubes, although the column in the ſmall tube is longer than that in the great one;

the former being oblique, and the latter perpendicular.

Since then the preſſure of fluids is directly as their perpendicular heights, without any regard to their quantities, it appears that whatever the figure or ſize of veſſels be, if they are of equal heights, and if the areas of their bottoms are equal, the preſſures of equal heights of water are equal upon the bottoms of theſe veſſels; even though the one ſhould hold a thouſand or ten thouſand times as much water as would fill the other. To confirm this part of the hydroſtatical paradox by an experiment, let two veſſels be prepared of equal heights but very unequal contents, ſuch as *A B* in Fig. 4. and *AB* in Fig. 5. Let each veſſel be open at both ends, and their bottoms *D d*, *D d* be of equal widths. Let a braſs bottom *C C* be exactly fitted to each veſſel, not to go into it, but for it to ſtand upon; and let a piece of wet leather be put between each veſſel and its braſs bottom, for the ſake of cloſeneſs. Join each bottom to its veſſel by a hinge *D*, ſo that it may open like the lid of a box; and let each bottom be kept up to its veſſel by equal weights *E* and *E* hung to lines which go over the pulleys *F* and *F* (whoſe blocks are fixed to the ſides of the veſſels at *f*) and the lines tied to hooks at *d* and *d*, fixed in the braſs bottoms oppoſite to the hinges *D* and *D*. Things being thus prepared and fitted, hold the veſſel *AB* (Fig. 5.) upright in your hands over a baſon on a table, and cauſe water to be poured into the veſſel ſlowly, till the preſſure of the water bears down its bottom at the ſide *d*, and raiſes the weight *E*; and then part of the water will run out at *d*. Mark the height at which the ſurface *H* of the water ſtood in the veſſel, when the bot- Fig. 4, 5.

tom began to give way at *d*; and then, holding up the other veſſel *A B* (Fig. 4.) in the ſame manner, cauſe water to be poured into it at *H*; and you will ſee that when the water riſes to *A* in this veſſel, juſt as high as it did in the former, its bottom will alſo give way at *d*, and it will loſe part of the water.

The natural reaſon of this ſurpriſing phenomenon is, that ſince all parts of a fluid at equal depths below the ſurface are equally preſſed in all manner of directions, the water immediately below the fixed part *B f* (Fig. 4.) will be preſſed as much upward againſt its lower ſurface within the veſſel, by the action of the column *A g*, as it would be by a column of the ſame height, and of any diameter whatever; (as was evident by the experiment with the tube, Fig. 3.) and therefore, ſince action and reaction are equal and contrary to each other, the water immediately below the ſurface *B f* will be preſſed as much downward by it, as if it was immediately touched and preſſed by a column of the height *g A*, and of the diameter *B f*: and therefore, the water in the cavity *B D d f* will be preſſed as much downward upon its bottom *C C*, as the bottom of the other veſſel (Fig. 5.) is preſſed by all the water above it.

Fig. 4. To illuſtrate this a little farther, let a hole be made at *f* in the fixed top *B f*, and let a tube *G* be put into it; then, if water be poured into the tube *A*, it will (after filling the cavity *B d*) riſe up into the tube *G*, until it comes to a level with that in the tube *A*, which is manifeſtly owing to the preſſure of the water in the tube *A*, upon that in the cavity of the veſſel below it. Conſequently, that part of the top *B f*, in which the hole is now made, would, if corked up, be

preſſed upward with a force equal to the weight of all the water which is ſupported in the tube *G*: and the ſame thing would hold at *g*, if a hole were made there. And ſo if the whole cover or top B *f* were full of holes, and had tubes as high as the middle one *Ag* put into them, the water in each tube would riſe to the ſame height as it is kept into the tube *A*, by pouring more into it, to make up the deficiency that it ſuſtains by ſupplying the others, until they are all full: and then the water in the tube *A* would ſupport equal heights of water in all the reſt of the tubes. Or, if all the tubes except *A*, or any other one, were taken away, and a large tube equal in diameter to the whole top *Bf* were placed upon it, and cemented to it, and then if water were poured into the tube that was left in either of the holes, it would aſcend through all the reſt of the holes, until it filled the large tube to the ſame height that it ſtands in the ſmall one, after a ſufficient quantity had been poured into it: which ſhews, that the top *Bf* was preſſed upward by the water under it, and before any hole was made in it, with a force equal to that wherewith it is now preſſed downward by the weight of all the water above it in the great tube. And therefore, the reaction of the fixed top *Bf* muſt be as great, in preſſing the water downward upon the bottom *CC*, as the whole preſſure of the water in the great tube would have been, if the top had been taken away, and the water in that tube left to preſs directly upon the water in the cavity *B D d f*.

Fig. 6. The *hydroſtatic bellows.*

Perhaps the beſt machine in the world for demonſtrating the upward preſſure of fluids, is the hydroſtatic bellows *A*; which conſiſts of two thick oval boards, each about 16 inches broad, and 18 inches long, covered with leather, to open and ſhut like a common bellows, but without valves; only a pipe *B*, about three feet high, is fixed into the bellows at *e*. Let ſome water be poured into the pipe at *c*, which will run into the bellows, and ſeparate the boards a little. Then lay three weights *b*, *c*, *d*, each weighing 100 pounds, upon the upper board; and pour more water into the pipe *B*, which will run into the bellows, and raiſe up the board with all the weights upon it; and if the pipe be kept full, until the weights are raiſed as high as the leather which covers the bellows will allow them, the water will remain in the pipe, and ſupport all the weights, even though it ſhould weigh no more than a quarter of a pound, and they 300 pounds: nor will all their force be able to cauſe them to deſcend and force the water out at the top of the pipe.

The reaſon of this will be made evident, by conſidering what has been already ſaid of the reſult of the preſſure of fluids of equal heights without any regard to the quantities. For, if a hole be made in the upper board, and a tube be put into it, the water will riſe in the tube to the ſame height that it does in the pipe; and would riſe as high (by ſupplying the pipe) in as many tubes as the board could contain holes. Now, ſuppoſe only one hole to be made in any part of the board, of an equal diameter with the bore of the pipe *B*; and that the pipe holds juſt a quarter of a pound of water; if a perſon claps his finger upon the hole, and the pipe be filled with water, he will find his finger to be preſſed upward with a force equal to a quarter of a pound. And as the ſame preſſure is equal upon all equal parts of the board, each part whoſe area is equal to the area of the hole, will be preſſed upward with a force equal to that of a quarter of a pound: the ſum of all which preſſures againſt the under ſide of an oval board 16 inches broad, and 18 inches long, will amount to 300 pounds; and therefore ſo much weight will be raiſed up and ſupported by a quarter of a pound of water in the pipe.

How a man may raiſe himſelf upward by his breath.

Hence, if a man ſtands upon the upper board, and blows into the bellows through the pipe *B*, he will raiſe himſelf upward upon the board: and the ſmaller the bore of the pipe is, the eaſier he will be able to raiſe himſelf. And then, by clapping his finger upon the top of the pipe, he can ſupport himſelf as long as he pleaſes; provided the bellows be air-tight ſo as not to loſe what is blown into it.

This figure, I confeſs, ought to have been much larger than any other upon the plate; but it was not thought of, until all the reſt were drawn; and it could not ſo properly come into any other plate.

How ſolid lead may be made to ſwim in water.

Fig. 7.

Upon this principle of the upward preſſure of fluids, a piece of lead may be made to ſwim in water, by immerſing it to a proper depth, and keeping the water from getting above it. Let *CD* be a glaſs tube, open throughout, and *EFG* a flat piece of lead, exactly fitted to the lower end of the tube, not to go within it, but for it to ſtand upon; with a wet leather between the lead and the tube to make cloſe work. Let this leaden bottom be half an inch thick, and held cloſe to the tube by pulling the packthread *IHL* upward at *L* with one hand, whilſt the tube is held in the other by the upper end *C*. In this ſituation, let the tube be immerſed in water in the glaſs veſſel *AB*, to the depth of ſix inches below the ſurface of the water at *K*; and then, the leaden bottom *EFG* will be plunged to the depth of ſomewhat more than eleven times its own thickneſs: holding the tube at that depth, you may let go the thread at *L*; and the lead will not fall from the tube, but will be kept to it by the upward preſſure of the water below it, occaſioned by the height of the water at *K* above the level of the lead. For as lead is 11.33 times as heavy as its bulk of water, and is in this experiment immerſed to a depth ſomewhat more than 11.33 times its thickneſs, and no water getting into the tube between it and the lead, the column of water *E a b c G* below the lead is preſſed upward againſt it by the water *K D E G L* all around the tube; which water being a little more than 11.33 times as high as the lead is thick, is ſufficient to balance and ſupport the lead at the depth *K E.* If a little water be poured into the tube upon the lead, it will increaſe the weight upon the column of water under the lead, and cauſe the lead to fall from the tube to the bottom of the glaſs veſſel, where it will lie in the ſituation *b d.* Or, if the tube be raiſed a little in the water, the lead will fall by its own weight, which will then be too great for the preſſure of the water around the tube upon the column of water below it.

How light wood may be made to lie at the bottom of water.

Let two pieces of wood be plained quite flat, ſo as no water may get in between them when they are put together: let one of the pieces, as *b d*, be cemented to the bottom of the veſſel *A B* (Fig. 7.) and the other piece be laid flat and cloſe upon it, and held down to it by a ſtick, whilſt water is poured into the veſſel; then remove the ſtick, and the upper piece of wood will not riſe from the lower one: for, as the upper one is preſſed down both by its own weight and the weight of all the water over it, whilſt the contrary preſſure of the water is kept off by the wood under it, it will lie as ſtill as a ſtone would do in its place. But if it be raiſed ever ſo little at any edge, ſome water will then get under it; which being acted upon by the water above, will immediately preſs it upward; and as it is lighter than its bulk of water, it will riſe, and float upon the ſurface of the water.

All fluids weigh juſt as much in their own elements as they do in open air. To prove this by experiment, let as much ſhot be put into a phial, as, when corked, will make it ſink in water: and being thus charged, let it be weighed, firſt in air, and then in water, and the weights in both caſes wrote down. Then, as the phial hangs ſuſpended in water, and counterpoiſed, pull out the cork, that water may run into it, and it will deſcend, and pull down that end of the beam. This done, put as much weight into the oppoſite ſcale as will reſtore the equipoiſe; which weight will be found to anſwer exactly to the additional weight of the phial when it is again weighed in air, with the water in it.

The velocity of ſpouting water.

The velocity with which water ſpouts out at a hole in the ſide or bottom of a veſſel, is as the * ſquare root of the depth or diſtance of the hole below the ſurface of the water. For, in order to make double the quantity of a fluid run through one hole as through another of the ſame ſize, it will require four times the preſſure of the other, and therefore muſt be four times the depth of the other below the ſurface of the water: and for the ſame reaſon, three times the quantity running in an equal time through the

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* The ſquare root of any number is that which being multiplied by itſelf produces the ſaid number. Thus, 2 is the ſquare root of 4, and 3 is the ſquare root of 9: for 2 multiplied by 2 produces 4, and 3 multiplied by 3 produces 9, &c.

ſame ſort of hole, muſt run with three times the velocity, which will require nine times the preſſure; and conſequently muſt be nine times as deep below the ſurface of the fluid: and ſo on.—To prove this by an experiment, let two pipes, as *C* and *g*, of equal ſized bores, be fixed into the ſide of the veſſel *A B*; the pipe *g* being four times as deep below the ſurface of the water at *b* in the veſſel as the pipe *C* is: and whilſt theſe pipes run, let water be conſtantly poured into the veſſel, to keep the ſurface ſtill at the ſame height. Then, if a cup that holds a pint be ſo placed as to receive the water that ſpouts from the pipe *C*, and at the ſame moment a cup that holds a quart be ſo placed as to receive the water that ſpouts from the pipe *g*, both cups will be filled at the ſame time by their reſpective pipes. Fig. 8.

The horizontal diſtance to which water will ſpout from pipes.

The horizontal diſtance, to which a fluid will ſpout from a horizontal pipe, in any part of the ſide of an upright veſſel below the ſurface of the fluid, is equal to twice the length of a perpendicular to the ſide of the veſſel, drawn from the mouth of the pipe to a ſemicircle deſcribed upon the altitude of the fluid: and therefore, the fluid will ſpout to the greateſt diſtance poſſible from a pipe, whoſe mouth is at the center of the ſemicircle; becauſe a perpendicular to its diameter (ſuppoſed parallel to the ſide of the veſſel) drawn from that point, is the longeſt that can poſſibly be drawn from any part of the diameter to the circumference of the ſemicircle. Fig. 8. Thus, if the veſſel *A B* be full of water, the horizontal pipe *D* be in the middle of its ſide, and the ſemicircle *N d c b* be deſcribed upon *D* as a center, with the radius or ſemidiameter *D g N*, or *D f b*, the perpendicular *D d* to the diameter *N D b* is the longeſt that can be drawn from any part of the diameter to the circumference *N d c b*. And if the veſſel be kept full, the jet *G* will ſpout from the pipe *D*, to the horizontal diſtance *N M*, which is double the length of the perpendicular *D d*. If two other pipes, as *C* and *E*, be fixed into the ſide of the veſſel at equal diſtances above and below the pipe *D*, the perpendiculars *C c* and *E e*, from theſe pipes to the ſemicircle, will be equal; and the jets *F* and *H* ſpouting from them will each go to the horizontal diſtance *N K*; which is double the length of either of the equal perpendiculars *C c* or *E H*.

How water may be conveyed over hills and vallies.

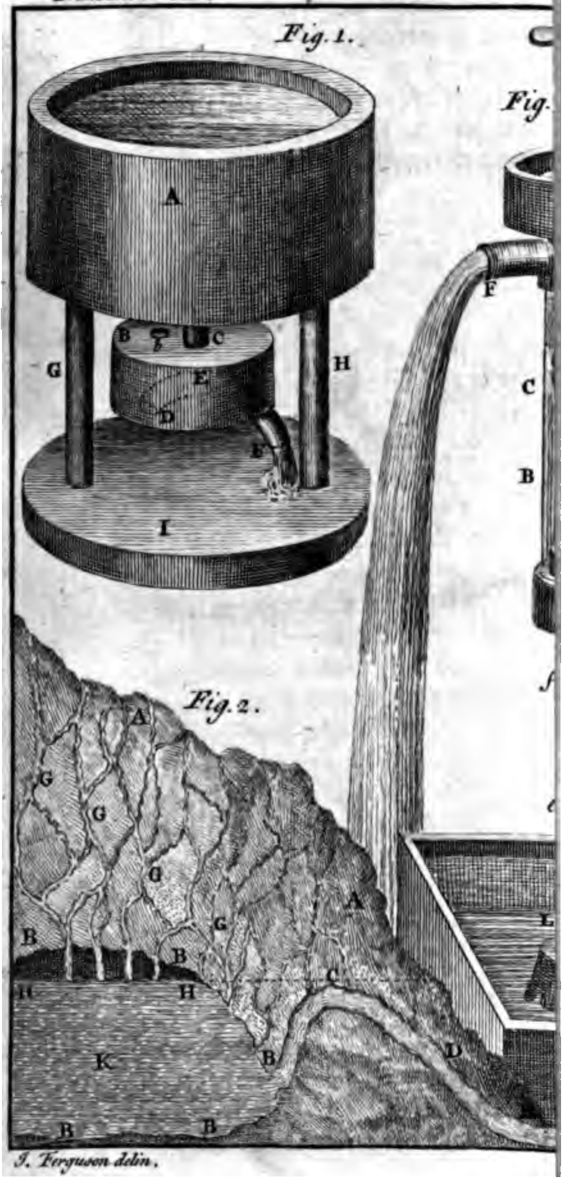
Fluids by their preſſure may be conveyed over hills and vallies in bended pipes, to any height not greater than the level of the ſpring from whence they flow. But when they are deſigned to be raiſed higher than the ſprings, forcing engines muſt be uſed; which ſhall be deſcribed when we come to treat of pumps.

A *ſyphon*, generally uſed for decanting liquors, is a bended pipe, whoſe legs are of unequal lengths; and the ſhorteſt leg muſt always be put into the liquor intended to be decanted, that the perpendicular altitude of the column of liquor in the other leg may be longer than the column in the immerſed leg, eſpecially above the ſurface of the water. For, if both columns were equally high in that reſpect, the atmoſphere, which preſſes as much upward as downward, and therefore acts as much upward againſt the column in the leg that hangs without the veſſel, as it acts downward upon the ſurface of the liquor in the veſſel, would hinder the running of the liquor through the ſyphon, even though it were brought over the bended part by ſuction. So that there is nothing left to cauſe the motion of the liquor, but the ſuperior weight of the column, in the longer leg, on account of its having the greater perpendicular height.

Fig. 9. Let *D* be a cup filled with water to *C*, and *A B C* a ſyphon, whoſe ſhorter leg *B C F* is immerſed in the water from *C* to *F*. If the end of the other leg were no lower than the line *A C*, which is level with the ſurface of the water, the ſyphon would not run, even though the air ſhould be drawn out of it at the mouth *A*. For although the ſuction would draw ſome water at firſt, yet the water would ſtop at the moment the ſuction ceaſed; becauſe the air would act as much upward againſt the water at *A*, as it acted downward for it by preſſing on the ſurface at *C*. But if the leg *A B* comes down to *G*, and the air be drawn out at *G* by ſuction, the water will immediately follow, and continue to run, until the ſurface of the water in the cup comes down to *F*; becauſe, till then, the perpendicular height of the column *B A G* will be greater than that of the column *C B*; and conſequently, its weight will be greater, until the ſurface comes down to *F*; and then the ſyphon will ſtop, though the leg *C F* ſhould reach to the bottom of the cup. For which reaſon, the leg that hangs without the cup is always made long enough to reach below the level of its bottom; as from *d* to *E*: and then, when the ſyphon is emptied of air by ſuction at *E*, the water immediately follows, and by its continuity brings away the whole from the cup; juſt as pulling one end of a thread will make the whole clue follow.

If the perpendicular height of a ſyphon, from the ſurface of the water to its bended top at *B*, be

PLATE XI.



J. Ferguson delin.

e than 33 feet, it will draw no water, ough the other leg were much longer, ſyphon quite emptied of air; becauſe ght of a column of water 33 feet high l to the weight of as thick a column of ching from the ſurface of the earth to of the atmoſphere; ſo that there will e an equilibrium, and conſequently, there would be weight enough of air e ſurface *C* to make the water aſcend in *C B* almoſt to the height *B*, if the ſy- ere emptied of air, yet that weight would ſufficient to force the water over the and therefore, it could never be brought o the leg *B A G*.

Fig. 10. *Tantalus's cup.*

hole be made quite through the bottom up *A*, and the longer leg of the bended *D E B G* be cemented into the hole, ſo end *D* of the ſhorter leg *D E* may al- ouch the bottom of the cup within. if water be poured into this cup, it will the ſhorter leg by its upward preſſure, out the air all the way before it through ger leg: and when the cup is filled above d of the ſyphon at *F*, the preſſure of er in the cup will force it over the bend yphon; and it will deſcend in the longer *G*, and run through the bottom, until be emptied.

is generally called *Tantalus's cup*, and of the ſyphon in it are almoſt cloſe to- and a little hollow ſtatue, or figure of is ſometimes put over the ſyphon to con- the bend *E* being within the neck of re as high as the chin. So that poor *Tantalus* ſtands up to the chin in water, ng it will riſe a little higher, and he may drink; but inſtead of that, when the water comes up to his chin, it immediately begins to deſcend, and ſo, as he cannot ſtoop to follow it, he is left as much pained with thirſt as ever.

The fountain at command. Plate XI. Fig. 1.

The device called *the fountain at command* acts upon the ſame principle with the ſyphon in the cup. Let two veſſels *A* and *B* be joined together by the pipe *C* which opens into them both. Let *A* be open at top, *B* cloſe both at top and bottom (ſave only a ſmall hole at *b* to let the air get out of the veſſel *B*) and *A* be of ſuch a ſize, as to hold about ſix times as much water as *B*. Let a ſyphon *D E F* be ſoldered to the veſſel *D*, ſo that the part *D E e* may be within the veſſel, and *F* without it; the end *D* almoſt touching the bottom of the veſſel, and the end *F* below the level of *D*: the veſſel *B* hanging to *A* by the pipe *C* (ſoldered into both) and the whole ſupported by the pillars *G* and *H* upon the ſtand *I*. The bore of the pipe muſt be conſiderably leſs than the bore of the ſyphon.

The whole being thus conſtructed, let the veſſel *A* be filled with water, which will run through the pipe *C*, and fill the veſſel *B*. When *B* is filled above the top of the ſyphon at *E*, the water will run through the ſyphon, and be diſcharged at *F*. But as the bore of the ſyphon is larger than the bore of the pipe, the ſyphon will run faſter than the pipe, and will ſoon empty the veſſel *B*; upon which the water will ceaſe from running through the ſyphon at *F*, until the pipe *C* re-fills the veſſel *B*, and then it will begin to run as before. And thus the ſyphon will continue to run and ſtop alternately, until all the water in the veſſel *A* has run through the pipe *C*.—So that after a few trials, one may eaſily gueſs about what time the ſyphon will ſtop, and when it will begin to run; and then, to amuſe others, he may call out *ſtop*, or *run*, accordingly.

Intermitting ſprings. Fig. 2.

Upon this principle, we may eaſily account for *intermitting* or *reciprocating ſprings*. Let *A A* be part of a hill, within which there is a cavity *B B*; and from this cavity a vein or channel running in the direction *BCDE*. The rain that falls upon the ſide of the hill will ſink and ſtrain through the ſmall pores and cranies *G, G, G, G*; and fill the cavity with water *K*. When the water riſes to the level *HHC*, the vein *BCDE* will be filled to *C*, and the water will run through *CDF* as through a ſyphon; which running will continue until the cavity be emptied, and then it will ſtop until the cavity be filled again.

The *common pump.*

The *common pump* (improperly called the *ſucking pump*), with which we draw water out of wells, is an engine both pneumatic and hydraulic. It conſiſts of a pipe open at both ends, in which is a moveable piſton or bucket, as big as the bore of the pipe in that part wherein it works; and is leathered round, ſo as to fit the bore exactly; and may be moved up and down, without ſuffering any air to come between it and the pipe or pump barrel.

We ſhall explain the conſtruction both of this and the forcing-pump by pictures of glaſs models, in which both the action of the piſtons and motion of the valves are ſeen.

Fig. 3.

Hold the model *DCBL* upright in the veſſel of water *K*, the water being deep enough to riſe at leaſt as high as from *A* to *L*. The valve *a* on the moveable bucket *G*, and the valve *b*

So, at every time the bucket is raiſed, the valve *b* riſes, and the valve *a* falls; and at every time the bucket is depreſſed, the valve *b* falls, and *a* riſes.

As it is the preſſure of the air or atmoſphere which cauſes the water to riſe, and follow the piſton or bucket *G* as it is drawn up; and ſince a column of water 33 feet high is of equal weight with as thick a column of the atmoſphere, from the earth to the very top of the air; therefore, the perpendicular height of the piſton or bucket from the ſurface of the water in the well muſt always be leſs than 33 feet; otherwiſe the water will never get above the bucket. But, when the height is leſs, the preſſure of the atmoſphere will be greater than the weight of the water in the pump, and will therefore raiſe it above the bucket: and when the water has once got above the bucket, it may be lifted thereby to any height, if the rod *D* be made long enough, and a ſufficient degree of ſtrength be employed, to raiſe it with the weight of the water above the bucket.

The force required to work a pump, will be as the height to which the water is raiſed, and as the ſquare of the diameter of the pump-bore, in that part where the piſton works. So that, if two pumps be of equal heights, and one of them be twice as wide in the bore as the other, the wideſt will raiſe four times as much water as the narroweſt; and will therefore require four times as much ſtrength to work it.

The wideneſs or narrowneſs of the pump, in any other part beſides that in which the piſton works, does not make the pump either more or leſs difficult to work, except what difference may ariſe from the friction of the water in the bore; which is always greater in a narrow bore than in a wide one, becauſe of the greater velocity of the water.

The pump-rod is never raiſed directly by ſuch a handle as *E* at the top, but by means of a lever, whoſe longer arm (at the end of which the power is applied) generally exceeds the length of the ſhorter arm five or ſix times; and, by that means, it gives five or ſix times as much advantage to the power. Upon theſe principles, it will be eaſy to find the dimenſions of a pump that ſhall work with a given force, and draw water from any given depth. But, as theſe calculations have been generally neglected by pump-makers (either for want of ſkill or induſtry) the following table was calculated by the late ingenious Mr. *Booth* for their benefit*. In this calculation, he ſuppoſed the handle of the pump to be a lever increaſing the power five times; and had often found that a man can work a pump four inches diameter, and 30 feet high, and diſcharge 27½ gallons of water (Engliſh wine meaſure) in a minute. Now, if it be required to find the diameter of a pump, that ſhall raiſe water with the ſame eaſe from any other height above the ſurface of the *well*; look for that height in the firſt column, and over-againſt it in the ſecond you have the diameter or width of the pump; and in the third, you find the quantity of water which a man of ordinary ſtrength can diſcharge in a minute.

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* I have taken the liberty to make a few alterations in Mr. *Booth*'s numbers in the table, and to lengthen it out from 80 feet to 100.

Height of the pump above the ſurface of the well.	Diameter of the bore where the bucket works.		Water diſcharged in a minute, Engliſh wine meaſure.	
Feet.	Inches.	100 parts.	Gallons.	Pints.
10	6	.93	81	6
15	5	.66	54	4
20	4	.90	40	7
25	4	.38	32	6
30	4	.00	27	2
35	3	.70	23	3
40	3	.46	20	3
45	3	.27	18	1
50	3	.10	16	3
55	2	.95	14	7
60	2	.84	13	5
65	2	.72	12	4
70	2	.62	11	5
75	2	.53	10	7
80	2	.45	10	2
85	2	.38	9	5
90	2	.31	9	1
95	2	.25	8	5
100	2	.19	8	1

Fig. 4. The *forcing-pump.*

The *forcing pump* raiſes water through the box *H* in the ſame manner as the ſucking-pump does, when the plunger or piſton *g* is lifted up by the rod *D d*. But this plunger has no hole through it, to let the water in the barrel *BC* get above it, when it is depreſſed to *B*, and the valve *b* (which roſe by the aſcent of the water through the box *H* when the plunger *g* was drawn up) falls down and ſtops the hole in *H*, the moment that the plunger is raiſed to its greateſt height. Therefore, as the water between the plunger *g* and box *H* can neither get through the plunger upon its deſcent, nor back again into the lower part of the pump *L e*, but has a free paſſage by the cavity around *H* into the pipe *M M*, which opens into the air-veſſel *K K* at *P*; the water is forced through the pipe *M M* by the deſcent of the plunger, and driven into the air-veſſel; and in running up through the pipe at *P*, it opens the valve *a*; which ſhuts at the moment the plunger begins to be raiſed, becauſe the action of the water againſt the under ſide of the valve then ceaſes.

The water, being thus forced into the air-veſſel *KK* by repeated ſtrokes of the plunger, gets above the lower end of the pipe *G H I*, and then begins to condenſe the air in the veſſel *K K*. For, as the pipe *G H* is fixed air-tight into the veſſel below *F*, and the air has no way to get out of the veſſel but through the mouth of the pipe at *I*, and cannot get out when the mouth *I* is covered with water, and is more and more condenſed as the water riſes upon the pipe, the air then begins to act forcibly by its ſpring againſt the ſurface of the water at *H*: and this action drives the water up through the pipe *I H G F*, from whence it ſpouts in a jet *S* to a great height; and is ſupplied by alternately raiſing and depreſſing of the plunger *g*, which conſtantly forces the water that it raiſes through the valve *H*, along the pipe *M M*, into the air-veſſel *K K*.

The higher that the ſurface of the water *H* is raiſed in the air-veſſel, the leſs ſpace will the air be condenſed into, which before filled that veſſel; and therefore the force of its ſpring will be ſo much the ſtronger upon the water, and will drive it with the greater force through the pipe at *F*: and as the ſpring of the air continues whilſt the plunger *g* is riſing, the ſtream or jet *S* will be uniform, as long as the action of the plunger continues: and when the valve *b* opens, to let the water follow the plunger upward, the valve *a* ſhuts, to hinder the water, which is forced into the air veſſel, from running back by the pipe *MM* into the barrel of the pump.

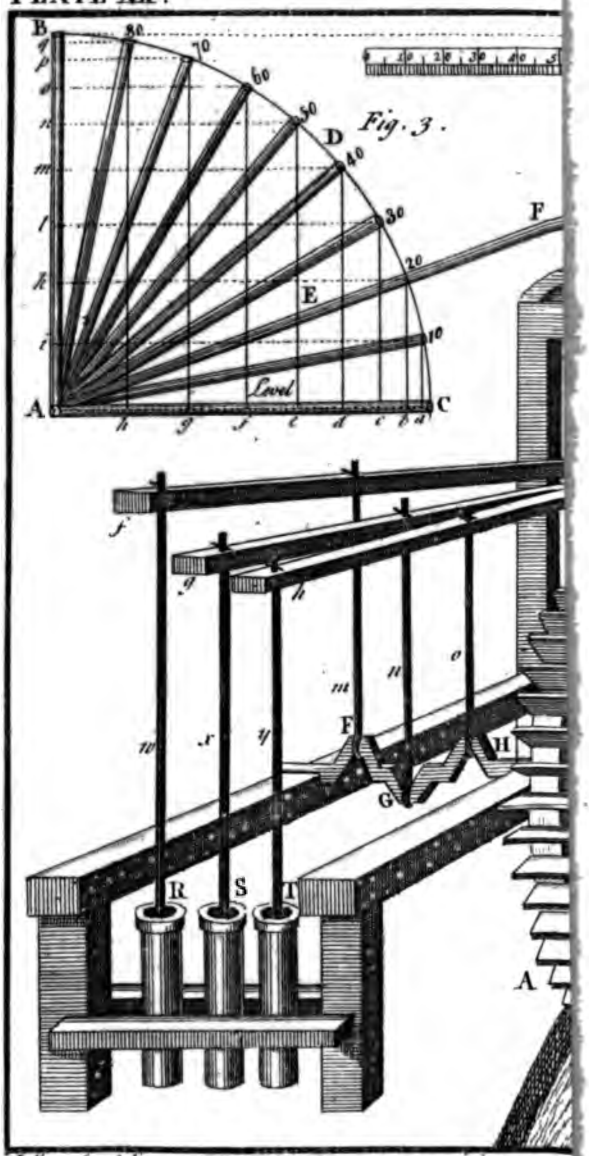
If there was no air-veſſel to this engine, the pipe *GHI* would be joined to the pipe *MMN* at *P*; and then, the jet *S* would ſtop every time the plunger is raiſed, and run only when the plunger is depreſſed.

Mr. *Newſham*'s water-engine, for extinguiſhing fire, conſiſts of two forcing-pumps, which alternately drive water into a cloſe veſſel of air; and by forcing the water into that veſſel, the air in it is thereby condenſed, and compreſſes the water ſo ſtrongly, that it ruſhes out with great impetuoſity and force through a pipe that comes down into it; and makes a continued uniform ſtream by the condenſation of the air upon its ſurface in the veſſel.

By means of forcing-pumps, water may be raiſed to any height above the level of a river or ſpring; and machines may be contrived to work theſe pumps, either by a running ſtream, a fall of water, or by horſes. An inſtance in each ſort will be ſufficient to ſhew the method.

Firſt,

PLATE XII.



J. Ferguson delin.

Plate XII. Fig. 1.

Firſt, by a running ſtream, or a fall of water. Let *A A* be a wheel, turned by the fall of water *B B*; and have any number of cranks (ſuppoſe ſix) as *C, D, E, F, G, H,* on its axis, according to the ſtrength of the fall of water, and the height to which the water is intended to be raiſed by the engine. As the wheel turns round, theſe cranks move the levers *c, d, e, f, g, h* up and down, by the iron rods *i, k, l, m, n, o*; which alternately raiſe and depreſs the piſtons by the other iron rods *p, q, r, ſ, t, u, w, x, y,* in twelve pumps; nine whereof, as *L, M, N, O, P, Q, R, S, T*, appear in the plate; the other three being hid behind the work at *V*. And as pipes may go from all theſe pumps, to convey the water (drawn up by them to a ſmall height) into a cloſe ciſtern, from which the main pipe goes off, the water will be forced into this ciſtern by the deſcent of the piſtons. And as each pipe, going from its reſpective pump into the ciſtern, has a valve at its end in the ciſtern, theſe valves will hinder the return of the water by the pipes; and therefore, when the ciſtern is once full, each piſton upon its deſcent will force the water (conveyed into the ciſtern by a former ſtroke) up the main pipe, to the height the engine was intended to raiſe it: which height depends upon the quantity raiſed, and the power that turns the wheel. When the power upon the wheel is leſſened by any defect of the quantity of water turning it, a proportionable number of the pumps may be ſet aſide, by diſengaging their rods from the vibrating levers.

A pump engine to go by water.

This figure is a repreſentation of the engine erected at *Blenheim* for the Duke of *Marlborough*, by the late ingenious Mr. *Alderſea*. The water-

wheel

wheel is 7½ feet in diameter, according to Mr. *Switzer's* account in his Hydraulics.

When ſuch a machine is placed in a ſtream that runs upon a ſmall declivity, the motion of the levers and action of the pumps will be but ſlow; ſince the wheel muſt go once round for each ſtroke of the pumps. But, when there is a large body of ſlow running water, a cog or ſpur-wheel may be placed upon each ſide of the water-wheel *AA*, upon its axis, to turn a trundle upon each ſide; the cranks being upon the axis of the trundle. And by proportioning the cog-wheels to the trundles, the motion of the pumps may be made quicker, according to the quantity and ſtrength of the water upon the firſt wheel; which may be as great as the workman pleaſes; according to the length and breadth of the float-boards or wings of the wheel. In this manner, the engine for raiſing water at *London-Bridge* is conſtructed; in which, the water-wheel is 20 feet diameter, and the floats 14 feet long.

A pump-engine to go by horſes.

Fig. 2.

Where a ſtream or fall of water cannot be had, and gentlemen want to have water raiſed, and brought to their houſes from a rivulet or ſpring; this may be effected by a horſe-engine, working three forcing pumps which ſtand in a reſervoir filled by the ſpring or rivulet: the piſtons being moved up and down in the pumps by means of a triple crank *ABC*, which, as it is turned round by the trundle *G*, raiſes and depreſſes the rods *D,E,F.* The trundle may be turned by ſuch a wheel as *F* in Fig. 1. of Plate VIII, having levers *y,y,y,y*, on its upright axle, to which horſes may be joined for working the engine. And if the wheel has three times as many cogs as the trundle has ſtaves or rounds, the trundle and cranks will make three revolutions for every one of the wheel: and as each crank will fetch a ſtroke in the time it goes round, the three cranks will make nine ſtrokes for every turn of the great wheel.

The cranks ſhould be made of caſt iron, becauſe *that* will not bend; and they ſhould each make an angle of 120 with both of the others, as at *a*,*b*,*c*; which is (as it were) a view of their *radii*, in looking endwiſe at the axis: and then there will be always one or other of them going downward, which will puſh the water forward with a continued ſtream into the main pipe. For, when *b* is almoſt at its loweſt poſiion, and is therefore juſt beginning to loſe its action upon the piſton which it moves, *c* is beginning to move downward, which will by its piſton continue the propelling force upon the water: and when *c* is come down to the poſition of *b*, *a* will be in the poſition of *c*.

Plate XII. Fig. 2.

The more perpendicularly the piſton rods move up and down in the pumps, the freer and better will their ſtrokes be: but a little deviation from the perpendicular will not be material. Therefore, when the pump-rods *D*, *E*, and *F* go down into a deep well, they may be moved directly by the cranks, as is done in a very good horſe-engine of this ſort at the late Sir *James Creed's* at *Greenwich*, which forces up water about 64 feet from a well under ground, to a reſervoir on the top of his houſe. But when the cranks are only at a ſmall height above the pumps, the piſtons muſt be moved by vibrating levers, as in the above engine at *Blenheim:* and the longer the levers are, the nearer will the ſtrokes be to a perpendicular.

A calculation of the quantity of

Let us ſuppoſe, that in ſuch an engine as Sir *James Creed's*, the great wheel is 12 feet diameter, the trundle 4 feet, and the radius or length

of

water that may be raiſed by a horſe-engine.

of each crank 9 inches, working a piſton in its pump. Let ther be three pumps in all, and the bore of each pump be four inches diameter. Then, if the great wheel has three times as many cogs as the trundle has ſtaves, the trundle and cranks will go three times round for each revolution of the horſes and wheel, and the three cranks will make nine ſtrokes of the pumps in that time, each ſtroke being 18 inches (or double the length of the crank) in a four-inch bore. Let the diameter of the horſe-walk be 18 feet, and the perpendicular height to which the water is raiſed above the ſurface of the well be 64 feet.

If the horſes go at the rate of two miles an hour (which is very moderate walking) they will turn the great wheel 187 times round in an hour.

In each turn of the wheel the piſtons make 9 ſtrokes in the pumps, which amount to 1683 in an hour.

Each ſtroke raiſes a column of water 18 inches long, and four inches thick, in the pump-barrels; which column, upon the deſcent of the piſton, is forced into the main pipe, whoſe perpendicular altitude above the ſurface of the well is 64 feet.

Now, ſince a column of water 18 inches long, and 4 inches thick, contains 226.18 cubic inches, this number multiplied by 1683 (the ſtrokes in an hour) gives 380661 for the number of cubic inches of water raiſed in an hour.

A gallon, in wine meaſure, contains 231 cubic inches, by which divide 380661, and it quotes 1468 in round numbers, for the number of gallons raiſed in an hour; which, divided by 63, gives 26½ hogſheads.—If the horſes go faſter, the quantity raiſed will be ſo much the greater.

In this calculation it is ſuppoſed that no water is s waſted by the engine. But as no forcing ngine can be ſuppoſed to loſe leſs than a fifth art of the calculated quantity of water, between he piſtons and barrels, and by the opening and hutting of the valves, the horſes ought to walk lmoſt 2½ miles *per* hour, to fetch up this loſs.

A column of water 4 inches thick, and 64 feet high, weighs 349$\frac{9}{16}$ pounds avoirdupoiſe, or 424$\frac{5}{12}$ pounds troy; and this weight together with the friction of the engine, is the reſiſtance that muſt be overcome by the ſtrength of the horſes.

The horſe-tackle ſhould be ſo contrived, that the horſes may rather puſh on than drag the levers after them. For if they draw, in going round the walk, the outſide leather ſtraps will rub againſt their ſides and hams; which will hinder them from drawing at right angles to the levers, and ſo make them pull at a diſadvantage. But if they puſh the levers before their breaſts, inſtead of dragging them, they can always walk at right angles to theſe levers.

It is no ways material what the diameter of the main or conduct pipe be: for the whole reſiſtance of the water therein, againſt the horſes will be according to the height to which it is raiſed, and the diameter of that part of the pump in which the piſton works, as we have already obſerved. So that by the ſame pump, an equal quantity of water may be raiſed in (and conſequently made to run from) a pipe of a foot diameter, with the ſame eaſe as in a pipe of five or ſix inches: or rather with more eaſe, becauſe its velocity in a large pipe will be leſs than in a ſmall one; and therefore its friction againſt the ſides of the pipe will be leſs alſo.

And the force required to raiſe water depends not upon the length of the pipe, but upon the perpendicular height to which it is raiſed therein above the level of the ſpring. So that the ſame force, which would raiſe water to the height *AB* Fig. 3. in the upright pipe *Aiklmnopq B*, will raiſe it to the ſame height or level *B I H* in the oblique pipe *AEFGH*. For the preſſure of the water at the end *A* of the latter, is no more that its preſſure againſt the end *A* of the former.

The weight or preſſure of water at the lower end of the pipe, is always as the ſine of the angle to which the pipe is elevated above the level parallel to the horizon. For, although the water in the upright pipe *AB* would require a force applied immediately to the lower end *A* equal to the weight of all the water in it, to ſupport the water, and a little more to drive it up, and out of the pipe; yet, if that pipe be inclined from its upright poſition to an angle of 80 degrees (as in *A* 80) the force required to ſupport or to raiſe the ſame cylinder of water will then be as much leſs, as the ſine 80 *b* is leſs than the radius *AB*; or as the ſine of 80 degrees is leſs than the ſine of 90. And ſo, decreaſing as the ſine of the angle of elevation leſſens, until it arrives at its level *AC* or place of reſt, where the force of the water is nothing at either end of the pipe. For, although the abſolute weight of the water is the ſame in all poſitions, yet its preſſure at the lower end decreaſes, as the ſine of the angle of elevation decreaſes; as will appear plainly by a farther conſideration of the figure.

Let two pipes, *AB* and *AC*, of equal lengths and bores, join each other at *A*; and let the pipe *AB* be divided into 100 equal parts, as the ſcale *S* is; whoſe length is equal to the length of the pipe.—Upon this length, as a radius, deſcribe the quadrant *BCD*, and divide it into 90 equal parts or degrees.

Let the pipe *AC* be elevated to 10 degrees upon the quadrant, and filled with water; then, part of the water that is in it will riſe in the pipe *AB*, and if it be kept full of water, it will raiſe the water in the pipe *AB* from *A* to *i*; that is, to a level *i* 10 with the mouth of the pipe at 10: and the upright line *a* 10, equal to *Ai*, will be the ſine of 10 degrees elevation; which being meaſured upon the ſcale *S*, will be about 17.4 of ſuch parts as the pipe contains 100 in length: and therefore, the force or preſſure of the water at *A*, in the pipe *A* 10, will be to the force or preſſure at *A* in the pipe *A B*, as 17.4 to 100.

Let the ſame pipe be elevated to 20 degrees in the quadrant, and if it be kept full of water, part of that water will run into the pipe *A B*, and riſe therein to the height *A k*, which is equal to the length of the upright line *b* 20, or to the ſine of 20 degrees elevation; which, being meaſured upon the ſcale *S*, will be 34.2 of ſuch parts as the pipe contains 100 in length. And therefore, the preſſure of the water at *A*, in the full pipe *A* 20, will be to its preſſure, if that pipe were raiſed to the perpendicular ſituation *A B*, as 34.2 to 100.

Elevate the pipe to the poſition *A* 30 on the quadrant, and if it be ſupplied with water, the water will riſe from it, into the pipe *AB*, to the height *A l*, or to the ſame level with the mouth of the pipe at 30. The ſine of this elevation, or of the angle of 30 degrees, is *c* 30; which is juſt equal to half the length of the pipe, or to 50 of ſuch parts of the ſcale, as the length of the pipe contains 100. Therefore, the preſſure of the water at *A*, in a pipe elevated 30 degrees above the horizontal level, will be equal to one half of what it would be, if the ſame pipe ſtood upright in the ſituation *A B*.

a half; and of any height from one foot to two hundred: together with the weight of the ſaid number of cubic inches, both in troy and avoirdupoiſe ounces. The number of cubic inches divided by 231, will reduce the water to gallons in wine meaſure; and divided by 282, will reduce it to the meaſure of ale gallons. Alſo, the troy ounces divided by 12, will reduce the weight to troy pounds; and the avoirdupoiſe ounces divided by 16, will reduce the weight to avoirdupoiſe pounds.

And here I muſt repeat it again, that the weight or preſſure of the water acting againſt the power that works the engine, muſt always be eſtimated according to the perpendicular height to which it is to be raiſed, without any regard to the length of the conduct-pipe, when it has an oblique poſition; and as if the diameter of that pipe were juſt equal to the diameter of that part of the pump in which the piſton works. Thus, by the following tables, the preſſure of the water, againſt an engine whoſe pump is of a 4½ inch bore, and the perpendicular height of the water in the conduct-pipe is 80 feet, will be equal to 8057.5 troy ounces, and to 8848.2 avoirdupoiſe ounces; which makes 671.4 troy pounds, and 553 avoirdupoſe.

For any bore whoſe diameter exceeds 6½ inches, multiply the numbers on the following page, againſt any height, (belonging to 1 inch diameter) by the ſquare of the diameter of the given bore, and the products will be the number of cubic inches, troy ounces, and avoirdupoiſe ounces of water, that the given bore will contain.

1 Inch diameter.

Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoir-dupoiſe ounces.
1	9.42	4.97	5.46
2	11.85	9.95	10.92
3	28,27	14.92	16.38
4	37.70	19.89	21.85
5	47.12	24.87	27.31
6	56.55	29.84	32.77
7	65.97	34.82	38.23
8	75.40	39.79	43.69
9	84.82	44.76	49.16
10	94.25	49.74	54.62
20	188.49	99.48	109.24
30	282.74	149.21	163.86
40	376.99	198.95	218.47
50	471.24	248.69	273.09
60	565.49	298.43	327.71
70	659.73	348.17	382.33
80	753.98	397.90	436.95
90	848.23	447.64	491.57
100	942.48	497.38	546.19
200	1884.96	994.76	1092.38

Example, *Required the number of cubic inches, and the weight of the water, in an upright pipe* 278 *feet high, and* 1½ *inch diameter?*

Here the neareſt ſingle decimal figure is only taken into the account: and the whole being reduced by diviſion, amounts to 25½ wine gallons in meaſure; to 259½ pounds troy, and to 213½ pounds avoirdupoiſe.

	Feet	Cubic inches	Troy oz.	Avoird. oz.
	200	4241.1	2238.2	2457.8
	70	1484.4	783.3	860.2
	8	169.6	89.5	98.3
Anſ.	278	5895.1	3111.0	3416.3

$1\frac{1}{2}$ Inch diameter.

Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupoiſe ounces.
1	21.21	11.19	12.29
2	42.41	22.38	24.58
3	63.62	33.57	36.87
4	84.82	44.76	49.16
5	106.03	55.95	61.45
6	127.23	67.15	73.73
7	147.44	78.34	86.02
8	169.65	89.53	98.31
9	190.85	100.72	110.60
10	212.06	111.91	122.89
20	424.12	223.82	245.78
30	636.17	335.73	368.68
40	848.23	447.64	491.57
50	1060.29	559.55	614.46
60	1272.35	671.46	737.35
70	1484.40	783.37	860.24
80	1696.46	895.28	983.14
90	1908.52	1007.19	1106.03
100	2120.58	1119.10	1228.92
200	4241.15	2238.20	2457.84

Theſe tables were at firſt calculated to fi decimal places for the ſake of exactneſs; bu in tranſcribing them there are no more tha two decimal figures taken into the account, an ſometimes but one; becauſe there is no neceſſit

fo

2 Inches diameter.

Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupoiſe ounces.
1	37.70	19.89	21.85
2	75.40	39.79	43.69
3	113.10	59.68	65.54
4	150.80	79.58	87.39
5	188.50	99.47	109.24
6	226.19	119.37	131.08
7	263.89	139.26	152.93
8	301.59	159.16	174.78
9	339.29	179.06	196.63
10	376.99	198.95	218.47
20	753.98	397.90	436.95
30	1130.97	596.85	655.42
40	1507.97	795.80	873.90
50	1884.96	994.75	1092.37
60	2261.95	1193.70	1310.85
70	2638.94	1392.65	1529.32
80	3015.93	1591.60	1747.80
90	3392.92	1790.56	1966.27
100	3769 91	1989.51	2184.75
200	7539.82	3979.00	4369.50

for computing to hundredth parts of an inch or of an ounce in practice. And as they never appeared in print before, it may not be amiſs to give the reader an account of the principles upon which they were conſtructed.

The

2½ Inches diameter.			
Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoir-dupoiſe ounces.
1	58.90	31.08	34.14
2	117.81	62.17	68.27
3	176.71	93.26	102.41
4	235.62	124.34	136.55
5	294.52	155.43	170.68
6	353.43	186.52	204.82
7	412.33	217.60	238.96
8	471.24	248.69	273.09
9	530.14	279.77	307.23
10	589.05	310.86	341.37
20	1178.10	621.72	682.73
30	1767.15	932.58	1024.10
40	2356.20	1243.44	1365.47
50	2545.25	1554.30	1706.83
60	3534.29	1865.16	2048.20
70	4123.34	2176.02	2389.57
80	4712.39	2486.88	2730.94
90	5301.44	2797.74	3072.30
100	5890.49	3108.60	2413.67.
200	11780.98	6217.20	4827.34

The ſolidity of cylinders are found by multiplying the areas of their baſes by their altitudes. And ARCHIMEDES gives the followin proportion for finding the area of a circle, an the ſolidity of a cylinder raiſed upon that circle:

A

3 Inches diameter.

Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupoiſe ounces.
1	84.8	44.76	49.16
2	169.6	89.53	98.31
3	254.5	134.29	147.47
4	239.3	179.06	196.63
5	424.1	223.82	245.78
6	508.9	268.58	294.94
7	593.7	313.35	344.10
8	698.6	358.11	393.25
9	763.4	402.87	442.41
10	848.2	447.64	491.57
20	1696.5	895.28	983.14
30	2544.7	1342.92	1474.70
40	3392.9	1790.56	1966.27
50	4241.1	2238.19	2457.84
60	5089.4	2685.83	2949.41
70	5937.6	3133.47	3440.98
80	6785.8	3581.11	3932.55
90	7634.1	4028.75	4424.12
100	8482.3	4476.39	4915.68
200	16964.6	8952.78	9831.36

As 1 is to 0.785399, ſo is the ſquare of the diameter to the area of the circle. And as 1 is to 0.785399, ſo is the ſquare of the diameter multiplied by the height to the ſolidity of the cylinder. By this analogy the ſolid inches and parts

3½ Inches diameter.

Feet high.	Quantity in cubic inches,	Weight in troy ounces.	In avoirdupoiſe ounces.
1	115.4	60.9	66.9
2	230.9	121.8	133.8
3	346.4	182.8	200.7
4	461.8	243.7	267.6
5	577.3	304.6	334.5
6	692.7	365.6	401.4
7	808.2	426.5	468.4
8	923.6	487.4	535.3
9	1039.1	548.4	602.2
10	1154.5	609.3	669.1
20	2309.1	1218.6	1338.2
30	3463.6	1827.9	2007.2
40	4618.1	2437.1	2676.3
50	5772.7	3046.4	3345.4
60	6927.2	3655.7	4014.5
70	8081.8	4265.0	4683.6
80	9236.3	4874.3	5352.6
90	10390.8	5483.6	6021.7
100	11545.4	6092.9	6690.8
200	23090.7	12185.7	13381.5

parts of an inch in the tables are calculated to cylinder 200 feet high, of any diameter from 1 inch to 6½, and may be continued at pleaſure.

And as to the weight of a cubic foot of runnin water, it has been often found upon trial, b

Dr

4 Inches diameter.			
Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoir-dupoiſe ounces.
1	150.8	79.6	87.4
2	301.6	159.2	174.8
3	452.4	238.7	262.2
4	603.2	318.3	349.6
5	754.0	397.9	436.9
6	904.8	477.3	524.3
7	1055.6	557.1	611.7
8	1206.4	636.6	699.1
9	1357.2	716.2	786.5
10	1508.0	795.8	873.9
20	3115.9	1591.6	1747.8
30	4523.9	2387.4	2621.7
40	6031.9	3183.2	3495.6
50	7539.8	3997.0	4369.5
60	9047.8	4774.8	5243.4
70	10555.8	5570.6	6117.3
80	12063.7	6366.4	6991.2
90	13571.7	7162.2	7865.1
100	15079.7	7958 0	8739.0
200	30159.3	15916 0	17478.0

Dr. *Wyberd* and others, to be 76 pounds troy, which is equal to 62.5 pounds avoirdupoiſe. Therefore, ſince there are 1728 cubic inches in a cubic foot, a troy ounce of water contains 1.8949 cubic inch; and an avoirdupoiſe ounce

The weight of running water.

4½ Inches diameter.

Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupoiſe ounces.
1	190.8	100.7	110.6
2	381.7	201.4	221.2
3	572.6	302.2	331.8
4	763.4	402.9	442.4
5	954.3	503.6	553.0
6	1145.1	604.3	663.6
7	1338.0	705.0	774.2
8	1526.8	805.7	884.8
9	1717.7	906.5	995.4
10	1908.5	1007.2	1106.0
20	3817.0	2014.4	2212.1
30	5725.6	3021.6	3818.1
40	7634.1	4028.7	4424.1
50	9542.6	5035.9	5530.1
60	11451.1	6043.1	6636.2
70	13359.6	7050.3	7742.2
80	15268.2	8057.5	8848.2
90	17176.7	9064.7	9954.3
100	19085.2	10071.9	11060.3
200	38170.4	20143.8	22120.6

ounce of water 1.72556 cubic inch. Conſequently, if the number of cubic inches contained in any given cylinder, be divided by 1.8949, it will give the weight in troy ounces; and divided by 1.72556, will give the weight in

5 Inches diameter.

Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoir-dupoiſe ounces.
1	235.6	124.3	136.5
2	471.2	248.7	273.1
3	706.6	373.0	409.6
4	942.5	497.4	546.2
5	1178.1	621.7	682.7
6	1413.7	746.1	819.3
7	1649.3	870.4	955.8
8	1885.0	994.8	1092.4
9	2120.6	1119.1	1228.9
10	2356.2	1243.4	1365.5
20	4712.4	2486.9	2730.9
30	7068.6	3730.3	4096.4
40	9424.8	4973.8	5461.9
50	11780.0	6217.2	6827.3
60	14137.2	7460.6	8192.8
70	16493.4	8704.1	9558.3
80	18849.6	9947.5	10923.7
90	21205.8	11191.0	12289.2
100	23562.0	12434.4	13654.7
200	47124.0	24868.8	27309.3

n avoirdupoiſe ounces. By this method, the veights ſhewn in the tables were calculated; nd are near enough for any common practice.

The *fire-engine* comes next in order to be ex- lained; but as it would be difficult, even by the The *fire-engine.*

5½ Inches diameter.			
Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupoiſe ounces.
1	285.1	150.5	164.3
2	570.2	300.9	328.5
3	855.3	451.4	492.8
4	1140.4	601.8	657.1
5	1425.5	752.3	821.3
6	1710.6	902.7	985.6
7	1995.7	1053.2	1149.9
8	2280.8	1203.6	1314.2
9	2565.9	1354.1	1478.4
10	2851.0	1504.6	1642.7
20	5702.0	3009.1	3285.4
30	8553.0	4513.7	4928.1
40	11404.0	6018.2	6570.8
50	14255.0	7522.8	8213.5
60	17106.0	9027.4	9856.2
70	19957.0	10531.9	11498.9
80	22808.0	12036.5	13141.6
90	25659.0	13541.1	14784.3
100	28510.0	15045.6	16426.9
200	57020.0	30091.2	32853.9

the beſt plates, to give a particular deſcription of its ſeveral parts, ſo as to make the whole intelligible, I ſhall only explain the principles upon which it is conſtructed.

1. What-

6 Inches diameter.			
Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupoiſe ounces.
1	339.3	179.1	196.6
2	678.6	358.1	393.3
3	1017.9	537.2	589.9
4	1357.2	716.2	786.5
5	1696.5	895.3	983.1
6	2035.7	1074.3	1179.8
7	2375.0	1253.4	1376.4
8	2714.3	1432.4	1573.0
9	3053.6	1611.5	1769.6
10	3392.9	1790.6	1966.3
20	6785.8	3581.1	3932.5
30	10178.8	5371.7	5898.8
40	13571.7	7162.2	7865.1
50	16964.6	8952.8	9831.4
60	20357.5	10743.3	11797.6
70	23750.5	12533.9	13763.9
80	27143.4	14324.4	15730.2
90	30536.3	16115.0	17696.5
100	33929.2	17905.6	19662.7
200	67858.4	35811.2	39325.4

1. Whatever weight of water is to be raiſed, he pump-rod muſt be loaded with weights ſuficient for that purpoſe, if it be done by a orcing-pump, as is generally the caſe; and the power

6½ Inches diameter.

Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupoiſe ounces.
1	398.2	210.1	230.7
2	797.4	420.3	461.4
3	1195.6	630.4	692.1
4	1593.8	840.6	922.8
5	1991.9	1050.8	1153.6
6	2390.1	1260.9	1384.3
7	2788.3	1471.1	1615.0
8	3186.5	1681.2	1845.7
9	3584.7	1891.3	2076.4
10	3982.9	2101.5	2307.1
20	7965.8	4202.9	4614.3
30	11948.8	6304.4	6921.4
40	15931.7	8405.9	9228.6
50	19914.6	10507.4	11535.7
60	23897.9	12608.9	13842.9
70	27880.5	14710.4	16150.0
80	31863.4	16811.8	18457.2
90	35846.3	18913.3	20764.3
100	39829.3	21014.8	23071.5
200	79658.6	42029.6	46143.0

power of the engine muſt be ſufficient for th weight of the rod, in order to bring it up.

2. It is known, that the atmoſphere preſſe upon the ſurface of the earth with a force equa to 15 pounds upon every ſquare inch.

3. Whe

3. When water is heated to a certain degree, the particles thereof repel one another, and conſtitute an elaſtic fluid, which is generally called *ſteam* or *vapour*.

4. Hot ſteam is very elaſtic; and when it is cooled by any means, particularly by its being mixed with cold water, its elaſticity is deſtroyed immediately, and it is reduced to water again.

5. If a veſſel be filled with hot ſteam, and then cloſed, ſo as to keep out the external air, and all other fluids; when that ſteam is by any means condenſed, cooled, or reduced to water, *that* water will fall to the bottom of the veſſel; and the cavity of the veſſel will be almoſt a perfect vacuum.

6. Whenever a vacuum is made in any veſſel, the air by its weight will endeavour to ruſh into the veſſel, or to drive in any other body that will give way to its preſſure; as may be eaſily ſeen by a common ſyringe. For, if you ſtop the bottom of a ſyringe, and then draw up the piſton, if it be ſo tight as to drive out all the air before it, and leave a vacuum within the ſyringe, the piſton being let go will be driven down with a great force.

7. The force with which the piſton is driven down, when there is a vacuum under it, will be as the ſquare of the diameter of the bore in the ſyringe. That is to ſay, it will be driven down with four times as much force in a ſyringe of a two-inch bore, as in a ſyringe of one inch: for the areas of circles are always as the ſquares of their diameters.

8. The preſſure of the atmoſphere being equal to 15 pounds upon a ſquare inch, it will be almoſt equal to 12 pounds upon a circular inch. So that if the bore of the ſyringe

be round, and one inch in diameter, the piſton will be preſt down into it by a force nearly equal to 12 pounds: but if the bore be two inches diameter, the piſton will be preſt down with four times that force.

And hence it is eaſy to find with what force the atmoſphere preſſes upon any given number either of ſquare or circular inches.

Theſe being the principles upon which this engine is conſtructed, we ſhall next deſcribe the chief working parts of it: which are, 1. A boiler. 2. A cylinder and piſton. 3. A beam or lever.

The *boiler* is a large veſſel made of iron or copper; and commonly ſo big as to contain about 2000 gallons.

The *cylinder* is about 40 inches diameter, bored ſo ſmooth, and its leathered piſton fitting ſo cloſe, that little or no water can get between the piſton and ſides of the cylinder.

Things being thus prepared, the cylinder is placed upright, and the ſhank of the piſton is fixed to one end of the *beam*, which turns on a center like a common balance.

The boiler is placed under the cylinder, with a communication between them, which can be opened and ſhut occaſionally.

The boiler is filled about half full of water, and a ſtrong fire is placed under it: then, if the communication between the boiler and the cylinder be opened, the cylinder will be filled with hot ſteam; which would drive the piſton quite out at the top of it. But there is a contrivance by which the beam, when the piſton is near the top of the cylinder, ſhuts the communication at the top of the boiler within.

This is no ſooner ſhut, than another is opened by which a little cold water is thrown upwards in a jet into the cylinder, which mixing with the hot ſteam, condenſes it immediately; by which means a vacuum is made in the cylinder, and the piſton is preſſed down by the weight of the atmoſphere; and ſo lifts up the loaded pump-rod at the other end of the beam.

If the cylinder be 42 inches in diameter, the piſton will be preſſed down with a force greater than 20000 pounds, and will conſequently lift up that weight at the oppoſite end of the beam: and as the pump-rod with its plunger is fixed to that end, if the bore where the plunger works were 10 inches diameter, the water would be forced up through a pipe of 180 yards perpendicular height.

But, as the parts of this engine have a good deal of friction, and muſt work with a conſiderable velocity, and there is no ſuch thing as making a perfect vacuum in the cylinder, it is found that no more than 8 pounds of preſſure muſt be allowed for, on every circular inch of the piſton in the cylinder, that it may make about 16 ſtrokes in a minute, about 6 feet each.

Where the boiler is very large, the piſton will make between 20 and 25 ſtrokes in a minute, and each ſtroke 7 or 8 feet; which, in a pump of 9 inches bore, will raiſe upwards of 300 hogſheads of water in an hour.

It is found by experience that a cylinder, 40 inches diameter, will work a pump 10 inches diameter, and 100 yards long: and hence we can find the diameter and length of a pump, that can be worked by any other cylinder.

For the convenience of thoſe who would make uſe of this engine for raiſing water, we ſhall ſubjoin part of a table calculated by Mr. *Beighton*, ſhewing how any given quantity of water may be raiſed in an hour, from 48 to 440 hogſheads; at any given depth, from 15 to 100 yards; the machine working at the rate of 16 ſtrokes *per* minute, and each ſtroke being 6 feet long.

One example of the uſe of this table will make the whole plain. Suppoſe it were required to draw 150 hogſheads *per* hour, at 90 yards depth; in the ſecond column from the right hand, I find the neareſt number, viz. 149 hogſheads 40 gallons, againſt which, on the right hand, I find the diameter of the bore of the pump muſt be 7 inches; and in the ſame collateral line, under the given depth 90, I find 27 inches, the diameter of the cylinder fit for that purpoſe.—And ſo for any other.

A Table ſhewing the power of the engine for raiſing water by fire.

The depth to be drawn in yards.													In one hour.		Diam. of pump.
15	20	25	30	35	40	45	50	60	70	80	90	100	Hogſh.	Gal.	Inches.
18½	21¾	24	26¼	28½	30½	32½	34¼	37¼	40	—	—	—	440		12
17	19¾	22	24	26¼	28	29¾	31¼	34¼	37	39½	—	—	369	33	11
15½	18	20	22	23¾	25¼	27	28¾	31¼	33¾	36	38¼	40	304	48	10
14	16¼	18	20	21½	23	24¼	25	28	30½	33	35	36½	247	7	9
13½	15¼	17¼	19	20¼	21¾	23	24	26¼	28½	31	32½	35½	221	15	8½
12½	14½	16½	18½	19	20½	21½	23	25	27	29	30½	32½	195	22	8
12	14	15½	17½	18¾	19¼	21	22	24¼	26	28	29½	31½	182	13	7¾
11	13¾	15	16½	18	19	20	21¼	23¾	25	27	28½	30¼	172	30	7½
10¾	13	14	15½	16¾	18¾	19	20½	22	24	25½	27	28½	149	40	7
10	12	13	14	15½	16¼	18	19	20	22	23	24½	26¼	128	54	6½
9½	11	12	13	14	15½	16	17	19	20½	22	23	24½	110	1	6
	10	11	12	13	14	15	15¾	17	19	20	21	22¼	94	30	5½
		10	11	11¾	13	13¾	14	15½	16¾	18½	19¼	20¼	66	61	5
			10	10	11¼	12	13½	14	15	16	17	18¼	60	60	4½
				9	10	11	11½	12	13½	14	15	16	48	51	4

Diameter of the cylinder in inches.

This table is calculated to the meaſure of ale gallons, at 282 cubic inches *per* gallon.

Plate XIII. The Perſian wheel.

Water may be raiſed by means of a ſtream *AB* turning a wheel *CDE*, according to the order of the letters, with buckets *a*, *a*, *a*, *a*, &c. hung upon the wheel by ſtrong pins *b*, *b*, *b*, *b*, &c. fixed in the ſide of the rim: but the wheel muſt be made as high as the water is intended to be raiſed above the level of that part of the ſtream in which the wheel is placed. As the wheel turns, the buckets on the right-hand go down into the water, and are filled therewith, and go up full on the left-hand, until they come to the top at *K*; where they ſtrike againſt the end *n* of the fixed trough *M*, and are thereby overſet, and empty the water into the trough; from which it may be conveyed in pipes to the place which it is deſigned for: and as each bucket gets over the trough, it falls into a perpendicular poſition again, and goes down empty, until it comes to the water at *A*, where it is filled as before. On each bucket is a ſpring *r*, which going over the top or crown of the bar *m* (fixed to the trough *M*) raiſes the bottom of the bucket above the level of its mouth, and ſo cauſes it to empty all its water into the trough.

Sometimes this wheel is made to raiſe water no higher than its axle; and then, inſtead of buckets hung upon it, its ſpokes *C*, *d*, *e*, *f*, *g*, *h*, are made of a bent form, and hollow within; theſe hollows opening into the holes *C*, *D*, *E*, *F*, in the outſide of the wheel, and alſo into thoſe at *O* in the box *N* upon the axle. So that, as the holes *C*, *D*, &c. dip into the water, it runs into them; and as the wheel turns, the water riſes in the hollow ſpokes, *c*, *d*, &c. and runs out in a ſtream *P* from the holes at *O*, and falls into the trough *Q*, from whence it is conveyed by pipes. And this is a very eaſy way of raiſing water, becauſe the engine requires no animal power to turn it.

Of the ſpecific gravities of bodies.

The art of weighing different bodies in water, and thereby finding their ſpecific gravities, or weights, bulk for bulk, was invented by ARCHIMEDES; of which we have the following account:

Hiero king of *Syracuſe*, having employed a goldſmith to make a crown, and given him a maſs of pure gold for that purpoſe, ſuſpected that the workman had kept back part of the gold for his own uſe, and made up the weight by allaying the crown with copper. But the king not knowing how to find out the truth of that matter, referred it to *Archimedes*; who having ſtudied a long time in vain, found it out at laſt by chance. For, going into a bathing-tub of water, and obſerving that he thereby raiſed the water higher in the tub than it was before, he concluded inſtantly that he had raiſed is juſt as high as any thing elſe could have done, that was exactly of his bulk: and conſidering that any other body of equal weight, and of leſs bulk than himſelf, could not have raiſed the water ſo high as he did; he immediately told the king, that he had found a method by which he could diſcover whether there were any cheat in the crown. For, ſince gold is the heavieſt of all known metals, it muſt be of leſs bulk, according to its weight, than any other metal. And therefore he deſired that a maſs of pure gold, equally heavy with the crown when weighed in air, ſhould be weighed againſt it in water; and if the crown was not allayed, it would counterpoiſe the maſs of gold when they were both immerſed in water, as well as it did when they were weighed in air. But upon making the trial, he found that the maſs of gold weighed much heavier in water than the crown did. And not only ſo, but that, when the maſs and crown were immerſed ſeparately in one veſſel of water, the crown raiſed the water much higher than the maſs did; which ſhewed it to be allayed with ſome ligher metal that increaſed its bulk. And ſo, by making trials with different metals, all equally heavy with the crown when weighed in air, he found out the quantity of alloy in the crown.

The ſpecific gravities of bodies are as their weights, bulk for bulk; thus a body is ſaid to have two or three times the ſpecific gravity of another, when it contains two or three times as much matter in the ſame ſpace.

A body immerſed in a fluid will ſink to the bottom, if it be heavier than its bulk of the fluid. If it be ſuſpended therein, it will loſe as much of what it weighed in air, as its bulk of the fluid weighs. Hence, all bodies of equal bulks, which would ſink in fluids, loſe equal weights when ſuſpended therein. And unequal bodies loſe in proportion to their bulks.

The *hydroſtatic balance.*

The *hydroſtatic balance* differs very little from a common balance that is nicely made: only it has a hook at the bottom of each ſcale, on which ſmall weights may be hung by horſe-hairs, or by ſilk threads. So that a body, ſuſpended by the hair or thread, may be immerſed in water without wetting the ſcale from which it hangs.

How to find the ſpecific gravity of any body.

If the body thus ſuſpended under the ſcale, at one end of the balance, be firſt counterpoiſed in air by weights in the oppoſite ſcale, and then immerſed in water, the equilibrium will be immediately deſtroyed. Then, if as much weight be put into the ſcale from which the body hangs, as will reſtore the equilibrium (without altering the weights in the oppoſite ſcale) that weight, which reſtores the equilibrium, will be equal to the weight of a quantity of water as big as the immerſed body. And if the weight of the body in air be divided by what it loſes in water, the quotient will ſhew how much that body is heavier than its bulk of water. Thus, if a guinea ſuſpended in air, be counterbalanced by 129 grains in the oppoſite ſcale of the balance; and then, upon its being immerſed in water, it becomes ſo much lighter, as to require 7¼ grains put into the ſcale over it, to reſtore the equilibrium, it ſhews that a quantity of water, of equal bulk with the guinea, weighs 7¼ grains, or 7.25; by which divide 129 (the weight of the guinea in air) and the quotient will be 17.793; which ſhews that the guinea is 17.793 times as heavy as its bulk of water. And thus, any piece of gold may be tried, by weighing it firſt in air, and then in water; and if upon dividing the weight in air by the loſs in water, the quotient comes out to be 17.793, the gold is good; if the quotient be 18, or between 18 and 19, the gold is very fine; but if it be leſs than 17, the gold is too much allayed, by being mixed with ſome other metal.

If ſilver be tried in this manner and found to be 11 times as heavy as water, it is very fine; if it be 10½ times as heavy, it is ſtandard; but if it be of any leſs weight compared with water, it is mixed with ſome lighter metal, ſuch as tin.

By this method, the ſpecific gravities of all bodies that will ſink in water, may be found. But as to thoſe which are lighter than water, as moſt ſorts of wood are, the following method may be taken, to ſhew how much lighter they are than their reſpective bulks of water.

Let an upright ſtud be fixed into a thick flat piece of braſs, and in this ſtud let a ſmall lever, whoſe arms are equally long, turn upon a fine pin as an axis. Let the thread which hangs from the ſcale of the balance be tied to one end of the lever, and a thread from the body to be weighed, tied to the other end. This done, put the braſs and lever into a veſſel; then pour water into the veſſel, and the body will riſe and float upon it, and draw down the end of the balance from which it hangs: then, put as much weight in the oppoſite ſcale as will raiſe that end of the balance, ſo as to pull the body down into the water by means of the lever; and this weight in the ſcale will ſhew how much the body is lighter than its bulk of water.

There are ſome things which cannot be weighed in this manner, ſuch as quickſilver, fragments of diamonds, &c. becauſe they cannot be ſuſpended in threads; and muſt therefore be put into a glaſs bucket, hanging by a thread from the hook of one ſcale, and counterpoiſed by weights put into the oppoſite ſcale. Thus, ſuppoſe you want to know the ſpecific gravity of quickſilver, with reſpect to that of water; let the empty bucket be firſt counterpoiſed in air, and then the quickſilver put into it and weighed. Write down the weight of the bucket, and alſo of the quickſilver; which done, empty the bucket, and let it be immerſed in water as it hangs by the thread, and counterpoiſed therein by weights in the oppoſite ſcale: then, pour the quickſilver into the bucket in the water, which will cauſe it to preponderate; and put as much weight into the oppoſite ſcale as will reſtore the balance to an equipoiſe; and this weight will be the weight of a quantity of water equal in bulk to the quickſilver. Laſtly, divide the weight of the quickſilver in air, by the weight of its bulk of water, and the quotient will ſhew how much the quickſilver is heavier than its bulk of water.

If a piece of braſs, glaſs, lead, or ſilver, be immerſed and ſuſpended in different ſorts of fluids, the different loſſes of weight therein will ſhew how much it is heavier than its bulk of the fluid; the fluid being lighteſt in which the immerſed body loſes leaſt of its aerial weight. A ſolid bubble of glaſs is generally uſed for finding the ſpecific gravities of fluids.

Hence we have an eaſy method of finding the ſpecific gravities both of ſolids and fluids, with regard to their ſpecific bulks of common pump water, which is generally made a ſtandard for comparing all others by.

In conſtructing tables of ſpecific gravities with accuracy, the gravity of water muſt be repreſented by unity or 1.000, where three cyphers are added, to give room for expreſſing the ratios of other gravities in decimal parts, as in the following table.

N. B. Although guinea gold has been generally reckoned 17.798 times as heavy as its bulk of water, yet, by many repeated trials, I cannot ſay that I have found it to be more than 17.200 (or $17\frac{2}{10}$) as heavy.

A Table of the ſpecific gravities of ſeveral ſolid and fluid bodies.

A cubic inch of	Troy weight.			Avoirdup.		Compa-rative weight.
	oz.	pw.	gr.	oz.	drams.	
Very fine gold -	10	7	3.83	1	5.80	19.637
Standard gold -	9	19	6.44	10	14.90	18.888
Guinea gold -	9	7	17.18	10	4.76	17.793
Moidore gold -	9	0	19.84	9	14.71	17.140
Quickſilver - -	7	7	11.61	8	1.45	14.019
Lead - -	5	19	17.55	6	9.08	11.325
Fine ſilver - -	5	16	23.23	6	6.66	11.087
Standard ſilver -	5	11	3.36	6	1.54	10.535
Copper - -	4	13	7.04	5	1.89	8.843
Plate-braſs -	4	4	9.60	4	10.09	8.000
Steel - - -	4	2	20.12	4	8.70	7.852
Iron - - -	4	0	15 20	4	6.77	7.645
Block-tin - -	3	17	5.68	4	3.79	7.321
Speltar - -	3	14	12.86	4	1.42	7.065
Lead ore - -	3	11	17.76	3	14.96	6.800
Glaſs of antimony	2	15	16.89	3	0.89	5.280
German antimony	2	2	4.80	2	5.04	4.000
Copper ore - -	2	1	11.83	2	4.43	3.775
Diamond - -	1	15	20.88	1	15.48	3.400
Clear glaſs - -	1	13	5.58	1	13.16	3.150
Lapis lazuli - -	1	12	5.27	1	12.27	3.054
Welch aſbeſtos -	1	10	17.57	1	10.97	2.913
White marble -	1	8	13.41	1	9.06	2.707
Black ditto -	1	8	12.65	1	9.02	2.704
Rock cryſtal -	1	8	1.00	1	8.61	2.658
Green glaſs - -	1	7	15.38	1	8.26	2.620
Cornelian ſtone -	1	7	1.21	1	7.73	2.568
Flint - - -	1	6	19.63	1	7.53	2.542
Hard paving ſtone	1	5	22.87	1	6.77	2.460
Live ſulphur -	1	1	2.40	1	2.52	2.000
Nitre - - -	1	0	1.08	1	1.59	1.900
Alabaſter - -	0	19	18.74	1	1.35	1.875
Dry ivory - -	0	19	6.09	1	0.89	1.825
Brimſtone - -	0	18	23.76	1	0.66	1.800
Alum - - -	0	17	21.92	0	15.72	1.714

The Table concluded.

A cubic inch of	Troy weight.			Avoirdup.		Comparative weight.
	oz.	pw.	gr.	oz.	drams.	
Ebony - -	0	11	18.82	0	10.34	1.117
Human blood -	0	11	2.89	0	9.76	1.054
Amber - -	0	10	20.79	0	9.54	1.030
Cow's milk -	0	10	20.79	0	9.54	1.030
Sea water - -	0	10	20.79	0	9.54	1.030
Pump water -	0	10	13.30	0	9.26	1.000
Spring water -	0	10	12.94	0	9.25	0.999
Diſtilled water -	0	10	11.42	0	9.20	0.993
Red wine - -	0	10	11.42	0	9.20	0.993
Oil of amber -	0	10	7.63	0	9.06	0.978
Proof ſpirits -	0	9	19.73	0	8.62	0.931
Dry oak - -	0	9	18.00	0	8.56	0.925
Olive oil - -	0	9	15.17	0	8.45	0.913
Pure ſpirits - -	0	9	3.27	0	8.02	0.866
Spirit of turpentine	0	9	2.76	0	7.99	0.864
Oil of turpentine	0	8	8.53	0	7.33	0.772
Dry crabtree -	0	8	1.69	0	7.08	0.765
Saſſafras wood -	0	5	2.04	0	4.46	0.482
Cork - -	0	2	12.77	0	2.21	0.240

Take away the decimal points from the numbers in the right-hand column, or (which is the ſame) multiply them by 1000, and they will ſhew how many avoirdupoiſe ounces are contained in a cubic foot of each body.

How to find out the quantity of adulteration in metals.

The uſe of the table of ſpecific gravities will beſt appear by an example. Suppoſe a body to be compounded of gold and ſilver, and it is required to find the quantity of each metal in the compound.

Firſt find the ſpecific gravity of the compound, by weighing it in air and in water, and dividing its aerial weight by what it loſes thereof in water, the quotient will ſhew its ſpecific gravity, or how many times it is heavier than its bulk of water. Then, ſubtract the ſpecific gravity of ſilver (found in the table) from that of the compound, and the ſpecific gravity of the compound from that of gold; the firſt remaineder ſhews the bulk of gold, and the latter the bulk of ſilver, in the whole compound: and if theſe remainders be multiplied by the reſpective ſpecific gravities, the products will ſhew the proportion of weights of each metal in the body. Example.

Suppoſe the ſpecific gravity of the compounded body be 13; that of ſtandard ſilver, (by the table) is 10.5, and that of gold 19.63: therefore 10.5 from 13, remains 2.5, the proportional bulk of the gold; and 13 from 19.63, remains 6.63 the proportional bulk of ſilver in the compound. Then, the firſt remainder 2.5, multiplied by 19.63, the ſpecific gravity of gold, produces 49.075 for the proportional weight of gold; and the laſt remainder 6.63 multiplied by 10.5, the ſpecific gravity of ſilver produces 69.615 for the proportional weight of ſilver in the whole body. So that for every 49.07 ounces or pounds of gold, there are 69.6 pounds or ounces of ſilver in the body.

Hence it is eaſy to know whether any ſuſpected metal be genuine,, or allayed, or counterfeit; by finding how much it is heavier than its bulk of water, and comparing the ſame with the table: if they agree, the metal is good; if they differ, it is allayed or counterfeited.

How to try ſpirituous liquors.

A cubical inch of good brandy, rum, or other proof ſpirits, weighs 235.7 grains: therefore, if a true inch cube of any metal weighs 235.7 grains leſs in ſpirits than in air, it ſhews the ſpirits are proof. If it loſes leſs of its aerial weight in ſpirits, they are above proof; if it loſes more, they are under. For, the better the ſpirits are, they are the lighter; and the worſe, the heavier. All bodies expand with heat and contract with cold, but ſome more and ſome leſs than others. And therefore the ſpecific gravities of bodies are not preciſely the ſame in ſummer as in winter. It has been found, that a cubic inch of good brandy is ten grains heavier in winter than in ſummer; as much ſpirit of nitre, 20 grains; vinegar 6 grains, and ſpring-water 3. Hence it is moſt profitable to buy ſpirits in winter, and ſell them in ſummer, ſince they are always bought and ſold by meaſure. It has been found, that 32 gallons of ſpirits in winter will make 33 in ſummer.

The expanſion of all fluids is proportionable to the degree of heat; that is, with a double or triple heat a fluid will expand two or three times as much.

The *thermometer.*

Upon theſe principles depends the conſtruction of the thermometer, in which the globe or bulb, and part of the tube, are filled with a fluid, which, when joined to the barometer, is ſpirits of wine tinged, that it may be more eaſily ſeen in the tube. But when thermometers are made by themſelves, quickſilver is generally uſed.

In the thermometer, a ſcale is fitted to the tube, to ſhew the expanſion of the quickſilver, and conſequently the degree of heat. And, as *Fahrenheit*'s ſcale is moſt in eſteem at preſent, I ſhall explain the conſtruction and graduation of thermometers according to that ſcale.

Firſt, Let the globe or bulb, and part of the tube, be filled with a fluid; then immerſe the bulb in water juſt freezing, or ſnow juſt thawing;

ing; and even with that part in the ſcale where the fluid then ſtands in the tube, place the number 32, to denote the freezing point: then put the bulb under your arm-pit, when your body is of a moderate degree of heat, ſo that it may acquire the ſame degree of heat with your ſkin; and when the fluid has riſen as far as it can by that heat, there place the number 97: then divide the ſpace between theſe numbers into 65 equal parts, and continue thoſe diviſions both above 97 and below 32, and number them accordingly.

This may be done in any part of the world; for it is found that the freezing point is always the ſame in all places, and the heat of the human body differs but very little; ſo that the thermometers made in this manner will agree with one another; and the heat of ſeveral bodies will be ſhewn by them, and expreſſed by the numbers upon the ſcale, thus.

Air, in ſevere cold weather, in our climate, from 15 to 25. Air in winter, from 26 to 42. Air in ſpring and autumn, from 43 to 53. Air at midſummer, from 65 to 68. Extreme heat of the ſummer ſun, from 86 to 100. Butter juſt melting, 95. Alcohol boils with 174 or 175. Brandy with 190. Water 212. Oil of turpentine 550. Tin melts with 408, and lead with 540. Milk freezes about 30, vinegar 38, and blood 27.

A body ſpecifically lighter than a fluid will ſwim upon its ſurface, in ſuch a manner, that a quantity of the fluid equal in bulk with the immerſed part of the body, will be as heavy as the whole body. Hence, the lighter a fluid is, the deeper a body will ſink in it; upon which depends the conſtruction of the *hydrometer* or water-poiſe.

How the weight of a ſhip may be eſtimated.

From this we can eaſily find the weight of a ſhip, or any other body that floats in water. For, if we multiply the number of cubic feet which are under the ſurface, by 62.5, the number of pounds in one cubic foot of freſh water; or by 64.4, the number of pounds in a cubic foot of ſalt water; the product will be the weight of the ſhip, and all that is in it. For, ſince it is the weight of the ſhip that diſplaces the water, it muſt continue to ſink until it has removed as much water as is equal to it in weight; and therefore the part immerſed muſt be equal in bulk to ſuch a portion of the water as is equal to the weight of the whole ſhip.

To prove this by experiment, let a ball of ſome light wood, ſuch as fir or pear-tree, be put into water contained in a glaſs veſſel; and let the veſſel be put into a ſcale at one end of a balance, and counterpoiſed by weights in the oppoſite ſcale: then, marking the height of the water in the veſſel, take out the ball; and fill up the veſſel with water to the ſame height that it ſtood at when the ball was in it; and the ſame weight will counterpoiſe it as before.

From the veſſel's being filled up to the ſame height at which the water ſtood when the ball was in it, it is evident that the quantity poured in is equal in magnitude to the immerſed part of the ball; and from the ſame weight counterpoiſing, it is plain that the water poured in, is equal in weight to the whole ball.

In troy weight, 24 grains make a pennyweight, 20 pennyweights make an ounce, and 12 ounces a pound. In avoirdupoiſe weight, 16 drams make an ounce, and 16 ounces a pound. The troy pound contains 5760 grains, and the avoirdupoiſe pound 7000; and hence, the avoirdupoiſe dram weighs 27.34375 grains, and the avoirdupoiſe ounce 437.5.

Becauſe it is often of uſe to know how much any given quantity of goods in troy weight do make in avoirdupoiſe weight, and the reverſe; we ſhall here annex two tables for converting theſe weights into one another. Thoſe from page 135 to page 146 are near enough for common hydraulic purpoſes; but the two following are better, where accuracy is required in comparing the weights with one another: and I find, by trial, that 175 troy ounces are preciſely equal to 192 avoirdupoiſe ounces, and 175 troy pounds are equal to 144 avoirdupoiſe. And although there are ſeveral leſſer integral numbers, which come very near to agree together, yet I have found none leſs than the above to agree exactly. Indeed 41 troy ounces are ſo nearly equal to 45 avoirdupoiſe ounces, that the latter contains only $7\frac{1}{2}$ grains more than the former: and 45 troy pounds weigh only $7\frac{3}{10}$ drams more than 37 avoirdupoiſe.

I have lately made a ſcale for comparing theſe weights with one another, and ſhewing the weight of pump-water, proof ſpirits, pure ſpirits, and guinea gold, taken in cubic inches, to any quantity leſs than a pound, both in troy and avoirdupoiſe; only by ſliding one ſide of a ſquare along the ſcale, and the other ſide croſſing it.

A Table for reducing Troy weight into Avoirdupoiſe weight.

Troy weight.		Avoirdupoiſe.			Troy weight.		Avoir.
		lb.	oz.	drams.			Drams.
Pounds—	4000	3291	6	13.68	Penny wt.	19	16.67
	3000	2528	9	2.26		18	15.79
	2000	1645	11	6.84		17	14.92
	1000	822	13	11.42		16	14.04
	900	740	9	2.28		15	13.16
	800	658	4	9.14		14	12.29
	700	576	0	0.00		13	11.41
	600	493	11	6.85		12	10.53
	500	411	6	13.71		11	9.65
	400	329	2	4.57		10	8.78
	300	246	13	11.42		9	7.90
	200	164	9	2.28		8	7.02
	100	82	4	9.15		7	6.14
	90	74	0	13.62		6	5.27
	80	65	13	4.11		5	4.39
	70	57	9	9.60		4	3.51
	60	49	5	15.08		3	2.63
	50	41	2	4.57		2	1.75
	40	32	14	10.05		1	0.88
	30	24	10	15.54	Grains —	23	.84
	20	16	7	5.03		22	.80
	10	8	3	10.52		21	.77
	9	7	6	7.86		20	.73
	8	6	9	5.21		19	.69
	7	5	12	2.56		18	.66
	6	4	14	15.90		17	.62
	5	4	1	13.25		16	.58
	4	3	4	10.60		15	.55
	3	2	7	7.95		14	.51
	2	1	10	5.30		13	.47
	1	0	13	2.65		12	.44
Ounces —	11		12	1.09		11	.40
	10		10	15.54		10	.36
	9		9	13.99		9	.33
	8		8	12.43		8	.29
	7		7	10.88		7	.26
	6		6	9.32		6	.22
	5		5	7.77		5	.18
	4		4	6.22		4	.15
	3		3	4.66		3	.11
	2		2	3.11		2	.07
	1		1	1.55		1	.04

A Table for reducing Avoirdupoiſe weight into Troy Weight.

Avoirdupoiſe Weight.	Troy weight.				Avoird. weight.	Troy Weight.			
	lb.	oz.	pw.	gr.		lb.	oz.	pw.	gr.
Pounds 6000	7291	8	0	0	Ounces 15	1	1	13	10.50
5000	6076	4	13	8	14	1	0	15	5
4000	4861	1	6	16	13		11	16	23.50
3000	3645	10	0	0	12		10	18	18
2000	2430	6	13	8	11		10	0	12.50
1000	1215	3	6	16	10		9	2	7
900	1093	9	0	0	9		8	4	1.50
800	972	2	13	8	8		7	5	20
700	850	8	6	16	7		6	7	14.50
600	729	2	0	0	6		5	9	9
500	607	7	13	8	5		4	11	3.50
400	486	1	6	16	4		3	12	22
300	364	7	0	0	3		2	14	16.50
200	243	0	13	8	2		1	16	11
100	121	6	6	16	1		0	18	5.50
90	109	4	10	0	Drams 15			17	2.10
80	97	2	13	8	14			15	22.76
70	85	0	16	16	13			14	19.42
60	72	11	0	0	12			13	15.08
50	60	9	3	8	11			12	12.74
40	48	7	6	16	10			11	9.40
30	36	5	10	0	9			10	6.06
20	24	3	13	8	8			9	2.72
10	12	1	16	16	7			8	23.38
9	10	11	5	0	6			7	20.04
8	9	8	13	8	5			6	16.70
7	8	6	1	16	4			5	13.36
6	7	3	10	0	3			3	10.02
5	6	0	18	8	2			2	6.68
4	4	10	6	16	1			1	3.34
3	3	7	15	0	3/4			0	20.51
2	2	5	3	8	1/2				13.67
1	1	2	11	16	1/4				6.83

The two following examples will be ſufficient to explain theſe two tables, and ſhew their agreement.

Ex. I. *In* 6835 *Troy pounds* 6 *ounces* 9 *pennyweights* 6 *grains, Qu. How much Avoirdupoiſe weight?* (See page 165.)

		Avoirdupoiſe.		
		lb.	oz.	drams.
Pounds troy—	4000	3291	6	13.68
	2000	1645	11	6.84
	800	658	4	9.14
	20	16	7	5.03
	10	8	3	10.52
	5	4	1	13.25
oz.	6		6	9.32
pw.	9			7.90
gr.	6			.22
Anſwer.		5624	10	11.90

Ex. II. *In* 5624 *pounds* 10 *ounces* 12 *drams Avoirdupoiſe, Qu. How much Troy weight?* (See page 166.)

		Troy.			
		lb.	oz.	pw.	gr.
Pounds avoird.	5000	6076	4	13	8
	600	729	2	0	0
	20	24	3	13	8
	4	4	10	6	16
oz.	10		9	2	7
dr.	12			13	15,08
Anſwer.		6835	6	9	6.08

LECT. VI.

Of Pneumatics.

THIS ſcience treats of the nature, weight, preſſure, and ſpring of the air, and the effects ariſing therefrom.

The properties of air.

The air is that thin tranſparent fluid body in which we live and breathe. It encompaſſes the whole earth to a conſiderable height; and, together with the clouds and vapours that float in it, it is called the atmoſphere. The air is juſtly reckoned among the number of fluids, becauſe it has all the properties by which a fluid is diſtinguiſhed. For, it yields to the leaſt force impreſſed, its parts are eaſily moved among one another, it preſſes according to its perpendicular height, and its preſſure is every way equal.

That the air is a fluid, conſiſting of ſuch particles as have no coheſion betwixt them, but eaſily glide over one another, and yield to the ſlighteſt impreſſion, appears from that eaſe and freedom with which animals breathe in it, and move through it without any difficulty or ſenſible reſiſtance.

But it differs from all other fluids in the four following particulars. 1. It can be compreſſed into a much leſs ſpace than what it naturally poſſeſſeth, which no other fluid can. 2. It cannot be congealed or fixed, as other fluids may. 3. It is of a different denſity in every part, upward from the earth's ſurface, decreaſing in its weight, bulk for bulk, the higher it riſes; and therefore muſt alſo decreaſe in denſity. 4. It is of an elaſtic or ſpringy nature, and the force of its ſpring is equal to its weight.

That air is a body, is evident from its excluding all other bodies out of the ſpace it poſſeſſes: for, if a glaſs jar be plunged with its mouth downward into a veſſel of water, there will but very little water get into the jar, becauſe the air of which it is full keeps the water out.

As air is a body, it muſt needs have gravity or weight: and that it is weighty, is demonſtrated by experiment. For, let the air be taken out of a veſſel by means of the air-pump, then, having weighed the veſſel, let in the air again, and upon weighing it when re-filled with air, it will be found conſiderably heavier. Thus, a bottle that holds a wine quart, being emptied of air and weighed, is found to be about 16 grains lighter than when the air is let into it again; which ſhews that a quart of air weighs 16 grains. But a quart of water weighs 14621 grains; this divided by 16, quotes 914 in round numbers; which ſhews, that water is 914 times as heavy as air near the ſurface of the earth.

As the air riſes above the earth's ſurface, it grows rarer, and conſequently lighter, bulk for bulk. For, becauſe it is of an elaſtic or ſpringy nature, and its lowermoſt parts are preſſed with the weight of all that is above them, it is plain that the air muſt be more denſe or compact at the earth's ſurface, than at any height above it; and gradually rarer the higher up. For, the denſity of the air is always as the force that compreſſeth it; and therefore, the air towards the upper parts of the atmoſphere being leſs preſſed than that which is near the earth, it will expand itſelf, and thereby become thinner than at the earth's ſurface.

Dr. *Cotes* has demonſtrated, that if altitudes in the air be taken in arithmetical proportion, the rarity of the air will be in geometrical proportion. For inſtance,

At the altitude of	Miles above the ſurface of the earth, the air is	times thinner and lighter than at the earth's ſurface.
	7	4
	14	16
	21	64
	28	256
	35	1024
	42	4096
	49	16384
	56	65536
	63	262144
	70	1048576
	77	4194304
	84	16777216
	91	67108864
	98	268435456
	105	1073741824
	112	4294967296
	119	17179869184
	126	68719476736
	133	274877906944
	140	1099511627776

And hence it is eaſy to prove by calculation, that a cubic inch of ſuch air as we breathe, would be ſo much rarefied at the altitude of 500 miles, that it would fill a hollow ſphere equal in diameter to the orbit of Saturn.

The weight or preſſure of the air is exactly determined by the following experiment.

The *Torricellian* experiment.

Take a glaſs tube about three feet long, and open at one end; fill it with quickſilver, and putting your finger upon the open end, turn that end downward, and immerſe it into a ſmall veſſel of quickſilver, without letting in any air: then take away your finger; and the quickſilver will remain ſuſpended in the tube 29½ inches above its ſurface in the veſſel; ſometimes more, and at other times leſs, as the weight of the air is varied by winds and other cauſes. That the quickſilver is kept up in the tube by the preſſure of the atmoſphere upon that in the baſon, is evident; for, if the baſon and tube be put under a glaſs, and the air be then taken out of the glaſs, all the quickſilver in the tube will fall down into the baſon; and if the air be let in again, the quickſilver will riſe to the ſame height as before. Therefore the air's preſſure on the ſurface of the earth, is equal to the weight of 29½ inches depth of quickſilver all over the earth's ſurface, at a mean rate.

A ſquare column of quickſilver, 29½ inches high, and one inch thick, weighs juſt 15 pounds, which is equal to the preſſure of air upon every ſquare inch of the earth's ſurface; and 144 times as much, or 2160 pounds, upon every ſquare foot; becauſe a ſquare foot contains 144 ſquare inches. At this rate, a middle-ſized man, whoſe ſurface may be about 14 ſquare feet, ſuſtains a preſſure of 30240 pounds, when the air is of a mean gravity: a preſſure which would be inſupportable, and even fatal to us, were it not equal on every part, and counterbalanced by the ſpring of the air within us, which is diffuſed through the whole body; and re-acts with an equal force againſt the outward preſſure.

Now, ſince the earth's ſurface contains (in round numbers) 200,000,000 ſquare miles, and every ſquare mile 27,878,400 ſquare feet, there muſt be 5,575,680,000,000,000 ſquare feet on the earth's ſurface; which multiplied by 2160 pounds (the preſſure on each ſquare foot) gives 12,043,468,800,000,000,000 pounds for the preſſure or weight of the whole atmoſphere.

When the end of a pipe is immerſed in water, and the air is taken out of the pipe, the water will riſe in it to the height of 33 feet above the ſurface of the water in which it is immerſed; but will go no higher: for it is found, that a common pump will draw water no higher than 33 feet above the ſurface of the well: and unleſs the bucket goes within that diſtance from the well, the water will never get above it. Now, as it is the preſſure of the atmoſphere, on the ſurface of the water in the well, that cauſes the water to aſcend in the pump, and follow the piſton or bucket, when the air above it is lifted up; it is evident, that a column of water 33 feet high, is equal in weight to a column of quickſilver of the ſame diameter, 29½ inches high; and to as thick a column of air, reaching from the earth's ſurface to the top of the atmoſphere.

The *barometer.* In ſerene calm weather, the air has weight enough to ſupport a column of quickſilver 31 inches high; but in tempeſtuous ſtormy weather, not above 28 inches. The quickſilver, thus ſupported in a glaſs tube, is found to be a nice counterbalance to the weight or preſſure of the air, and to ſhew its alterations at different times. And being now generally uſed to denote the changes in the weight of the air, and of the weather conſequent upon them, it is called the *barometer*, or weather-glaſs.

The preſſure of the air being equal on all ſides of a body expoſed to it, the ſofteſt bodies ſuſtain this preſſure without ſuffering any change in their figure; and ſo do the moſt brittle bodies without being broke.

The air is rarefied, or made to ſwell with heat; and of this property, *wind* is a neceſſary conſequence. For, when any part of the air is heated by the ſun, or otherwiſe, it will ſwell, and thereby affect the adjacent air: and ſo, by various degrees of heat in different places, there will ariſe various winds.

The cauſe of *winds*.

When the air is much heated, it will aſcend towards the upper part of the atmoſphere, and the adjacent air will ruſh in to ſupply its place; and therefore, there will be a ſtream or current of air from all parts towards the place where the heat is. And hence we ſee the reaſon why the air ruſhes with ſuch force into a glaſs-houſe, or towards any place where a great fire is made. And alſo, why ſmoke is carried up a chimney, and why the air ruſhes in at the key-hole of the door, or any ſmall chink, when there is a fire in the room. So we may take it in general, that the air will preſs towards that part of the world where it is moſt heated.

Upon this principle, we can eaſily account for the *trade-winds*, which blow conſtantly from eaſt to weſt about the equator. For, when the ſun ſhines perpendicularly on any part of the earth, it will heat the air very much in that part, which air will therefore riſe upward, and when the ſun withdraws, the adjacent air will ruſh in to fill its place; and conſequently will cauſe a ſtream or current of air from all parts towards that which is moſt heated by the ſun. But as the ſun, with reſpect to the earth, moves from eaſt to weſt, the common courſe of the air will be that way too; continually preſſing after the ſun: and therefore,

The *trade-winds*.

therefore, at the equator, where the ſun ſhines ſtrongly, there will be a continual wind from the eaſt; but, on the north ſide, it will incline a little to the north, and on the ſouth-ſide, to the ſouth.

This general courſe of the wind about the equator, is changed in ſeveral places, and upon ſeveral accounts; as, 1. By exhalations that riſe out of the earth at certain times, and from certain places; in earthquakes, and from volcanos. 2. By the falling of great quantities of rain, cauſing thereby a ſudden condenſation or contraction of the air. 3. By burning ſands, that often retain the ſolar heat to a degree incredible to thoſe who have not felt it, cauſing a more than ordinary rarefaction of the air contiguous to them. 4. By high mountains, which alter the direction of the winds in ſtriking againſt them. 5. By the declination of the ſun towards the north or ſouth, heating the air on the north or ſouth ſide of the equator.

The *monſoons.* To theſe and ſuch like cauſes is owing, 1. The irregularity and uncertainty of winds in climates diſtant from the equator, as in moſt parts of *Europe.* 2. Thoſe periodical winds, called *monſoons*, which in the *Indian* ſeas blow half a year one way, and the other half another. 3. Thoſe winds which, on the coaſt of *Guiney*, and on the weſtern coaſts of *America*, blow always from weſt to eaſt. 4. The ſea-breezes, which, in hot countries, blow generally from ſea to land, in the day-time; and the land-breezes, which blow in the night; and, in ſhort, all thoſe ſtorms, hurricanes, whirlwinds, and irregularities, which happen at different times and places.

The *vivifying ſpirit in air.*

All common air is impregnated with a certain kind of *vivifying ſpirit* or quality, which is neceſſary to continue the lives of animals: and this, in a gallon of air, is ſufficient for one man during the ſpace of a minute, and not much longer.

This ſpirit in air is deſtroyed by paſſing through the lungs of animals: and hence it is, that an animal dies ſoon, after being put under a veſſel which admits no freſh air to come to it. This ſpirit is alſo in the air which is in water; for fiſh die when they are excluded from freſh air, as in a pond that is cloſely frozen over. And the little eggs of inſects, ſtopped up in a glaſs, do not produce their young, though aſſiſted by a kindly warmth. The ſeeds alſo of plants mixed with good earth, and incloſed in a glaſs, will not grow.

This enlivening quality in air, is alſo deſtroyed by the air's paſſing through fire; particularly charcoal fire, or the flame of ſulphur. Hence, ſmoking chimneys muſt be very unwholeſome, eſpecially if the rooms they are in be ſmall and cloſe.

Air is alſo vitiated, by remaining cloſely pent up in any place for a conſiderable time; or perhaps, by being mixed with malignant ſteams and particles flowing from the neighbouring bodies: or laſtly, by the corruption of the vivifying ſpirit; as in the holds of ſhips, in oil-ciſterns, or wine-cellars, which have been ſhut for a conſiderable time. The air in any of them is ſometimes ſo much vitiated, as to be immediate death to any animal that comes into it.

Damps.

Air that has loſt its vivifying ſpirit, is called *damp*, not only becauſe it is filled with humid or moiſt vapours, but becauſe it deadens fire, extinguiſhes flame, and deſtroys life. The dreadful effects of damps are ſufficiently known to ſuch as work in mines.

If part of the vivifying ſpirit of air in any country begins to putrefy, the inhabitants of that country will be ſubject to an epidemical diſeaſe, which will continue until the putrefaction is over. And as the putrefying ſpirit occaſions the diſeaſe, ſo if the diſeaſed body contributes towards the putrefying of the air, then the diſeaſe will not only be epidemical, but peſtilential and contagious.

The atmoſphere is the common receptacle of all the effluvia or vapours ariſing from different bodies; of the ſteams and ſmoke of things burnt or melted; the fogs or vapours proceeding from damp watery places; and of the effluvia from ſulphureous, nitrous, acid, and alkaline bodies. In ſhort, whatever may be called volatile, riſes in the air to greater or leſs heights, according to its ſpecific gravity.

Fermentations. When the effluvia, which ariſe from acid and alkaline bodies, meet each other in the air, there will be a ſtrong conflict or *fermentation* between them; which will ſometimes be ſo great, as to produce a fire; then if the effluvia be combuſtible, the fire will run from one part to another, juſt as the inflammable matter happens to lie.

Any one may be convinced of this, by mixing an acid and an alkaline fluid together, as the ſpirit of nitre and oil of cloves; upon the doing of which, a ſudden ferment, with a fine flame, will ariſe; and if the ingredients be very pure and ſtrong, there will be a ſudden exploſion.

Thunder and lightning. Whoever conſiders the effects of fermentation, cannot be at a loſs to account for the dreadful effects of *thunder* and *lightning:* for the effluvia of ſulphureous and nitrous bodies, and others that may riſe into the atmoſphere, will ferment with each other, and take fire very often of themſelves; ſometimes by the aſſiſtance of the ſun's heat.

If the inflammable matter be thin and light, it will riſe to the upper part of the atmoſphere, where it will flaſh without doing any harm: but if it be denſe, it will lie near the ſurface of the earth, where taking fire, it will explode with a ſurpriſing force; and by its heat rarefy and drive away the air, kill men and cattle, ſplit trees, walls, rocks, &c. and be accompanied with terrible claps of thunder.

The heat of lightning appears to be quite different from that of other fires; for it has been known to run through wood, leather, cloth, &c. without hurting them, while it has broken and melted iron, ſteel, ſilver, gold, and other hard bodies. Thus it has melted or burnt aſunder a ſword, without hurting the ſcabbard; and money in a man's pocket, without hurting his cloaths: the reaſon of this ſeems to be, that the particles of *that* fire are ſo fine, as to paſs through ſoft looſe bodies without diſſolving them; whilſt they ſpend their whole force upon the hard ones.

It is remarkable, that knives and forks which have been ſtruck with lightning have a very ſtrong magnetical virtue for ſeveral years after; and I have heard that lightning ſtriking upon the mariner's compaſs, will ſometimes turn it round; and often make it ſtand the contrary way; or with the north-pole towards the ſouth.

Much of the ſame kind with lightning, are thoſe exploſions, called *fulminating* or *fire-damps*, which *Fire-damps.*

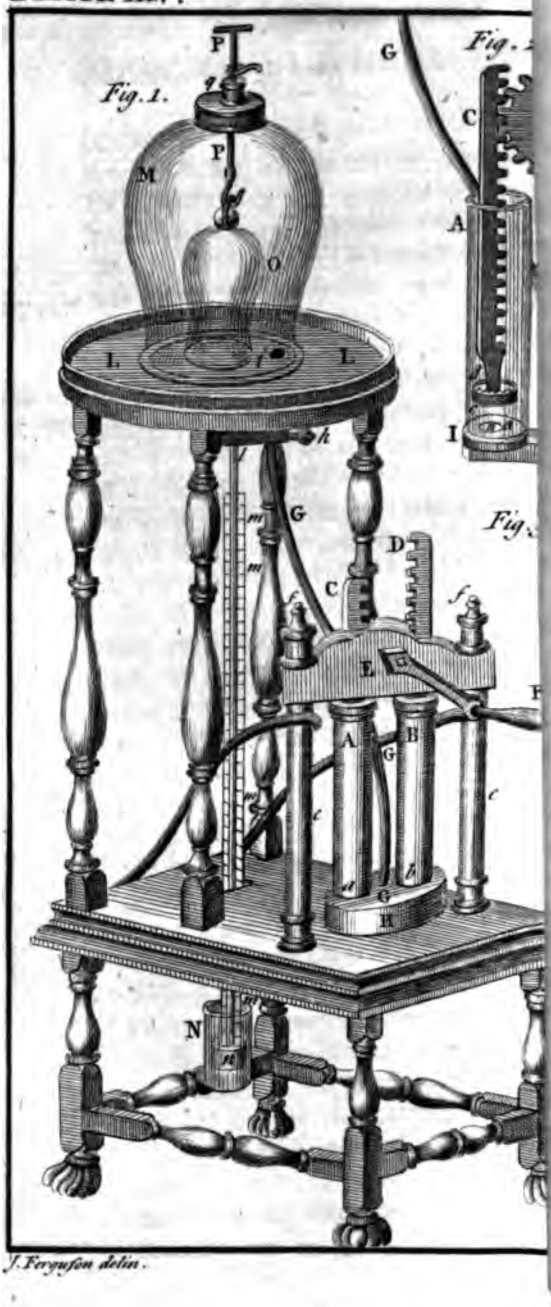
which ſometimes happen in mines; and are occaſioned by ſulphureous and nitrous, or rather oleaginous particles, riſing from the mine, and mixing with the air, where they will take fire by the lights which the workmen are obliged to make uſe of. The fire being kindled will run from one part of the mine to another, like a train of gunpowder, as the combuſtible matter happens to lie. And as the elaſticity of the air is increaſed by heat, *that* in the mine will conſequently ſwell very much, and ſo, for want of room, will explode with a greater or leſs degree of force, according to the denſity of the combuſtible vapours. It is ſometimes ſo ſtrong, as to blow up the mine; and at other times ſo weak, that when it has taken fire at the flame of a candle, it is eaſily blown out.

Air that will take fire at the flame of a candle may be produced thus. Having exhauſted a receiver of the air-pump, let the air run into it through the flame of the oil of turpentine; then remove the cover of the receiver, and holding a candle to that air, it will take fire, and burn quicker or ſlower, according to the denſity of the oleaginous vapour.

Earthquakes. When ſuch combuſtible matter, as is abovementioned, kindles in the bowels of the earth, where there is little or no vent, it produces *earthquakes*, and violent ſtorms or hurricanes of wind when it breaks forth into the air.

An artificial earthquake may be made thus. Take 10 or 15 pounds of ſulphur, and as much of the filings of iron, and knead them with common water into the conſiſtency of a paſte: this being buried in the ground, will, in 8 or 10 hours time, buſt out in flames, and cauſe

PLATE XIV.



J. Ferguson delin.

the earth to tremble all around to a conſiderable diſtance.

From this experiment we have a very natural account of the fires of mount *Ætna*, *Veſuvius*, and other volcanos; they being probably ſet or fire at firſt by the mixture of ſuch metalline and ſulphureous particles.

Plate XIV. Fig. 1.

The *air pump* being conſtructed the ſame way as the water-pump, whoever underſtands the one, will be at no loſs to underſtand the other.

The *air-pump.*

Having put a wet leather on the plate *L L* of the air-pump, place the glaſs receiver *M* upon the leather, ſo that the hole *i* in the plate may be within the glaſs. Then, turning the handle *F* backward and forward, the air will be pumped out of the receiver; which will then be held down to the plate by the preſſure of the external air, or atmoſphere. For, as the handle *F* (Fig. 2.) is turned backwards, it raiſes the piſton *d e* in the barrel *B K*, by means of the wheel *E* and rack *D d:* and, as the piſton is leathered ſo tight as to fit the barrel exactly, no air can get between the piſton and barrel; and therefore, all the air above *d* in the barrel is lifted up towards *B*, and a vacuum is made in the barrel from *b* to *e*; upon which, part of the air in the receiver *M* (Fig. 1.) by its ſpring, ruſhes through the hole *i*, in the braſs plate *L L*, along the pipe *G C G* (which communicates with both barrels by the hollow trunk *IHK* (Fig. 2.) and puſhing up the valve *b*, enters into the vacant place *b e* of the barrel *B K*. For, wherever the reſiſtance or preſſure is taken off, the air will run to that place, if it can find a paſſage.—Then, if the handle *F* be turned forward, the piſton *d e* will be depreſſed in the barrel; and, as the air which had got into the barrel cannot be puſhed back through the valve *b*, it will aſcend through a hole in the piſton, and eſcape through a valve at *d*; and be hindered by that valve from returning into the barrel, when the piſton is again raiſed. At the next raiſing of the piſton, a vacuum is again made in the ſame manner as before, between *b* and *e*; upon which, more of the air that was left in the receiver *M*, gets out thence by its ſpring, and runs into the barrel *B K*, through the valve *B*. The ſame thing is to be underſtood with regard to the other barrel *A I*; and as the handle *F* is turned backwards and forwards, it alternately raiſes and depreſſes the piſtons in their barrels; always raiſing one whilſt it depreſſes the other. And, as there is a vacuum made in each barrel when its piſton is raiſed, the particles of air in the receiver *M* puſh out another by their ſpring or elaſticity, through the hole *i*, and pipe *G G* into the barrels; until at laſt the air in the receiver comes to be ſo much dilated, and its ſpring ſo far weakened, that it can no longer get through the valves; and then no more can be taken out. Hence, there is no ſuch thing as making a perfect vacuum in the receiver; for the quantity of air taken out at any one ſtroke, will always be as the denſity thereof in the receiver: and therefore it is impoſſible to take it all out, becauſe, ſuppoſing the receiver and barrels of equal capacity, there will be always as much left as was taken out at the laſt turn of the handle.

There is a cock *k* below the pump-plate, which being turned, lets the air into the receiver again; and then the receiver becomes looſe, and may be taken off the plate. The barrels are fixed to the frame *E e e* by two ſcrew-nuts *f f*, which preſs down the top piece *E* upon the barrels: and the hollow trunk *H* (in Fig. 2.) is covered by a box, as *G H* in Fig. 1.

There is a glaſs tube *l m m m n* open at both ends, and about 34 inches long; the upper end communicating with the hole in the pump-plate, and the lower end immerſed in quickſilver at *n* in the veſſel *N*. To this tube is fitted a wooden ruler *m m*, called the *gage*, which is divided into inches and parts of an inch, from the bottom at *n* (where it is even with the ſurface of the quickſilver) and continued up to the top, a little below *l*, to 30 or 31 inches.

As the air is pumped out of the receiver *M*, it is likewiſe pumped out of the glaſs tube *l m n*, becauſe that tube opens into the receiver through the pump-plate; and as the tube is gradually emptied of air, the quckſilver in the veſſel *N* is forced up into the tube by the preſſure of the atmoſphere. And if the receiver could be perfectly exhauſted of air, the quickſilver would ſtand as high in the tube as it does at that time in the barometer: for it is ſupported by the ſame power or weight of the atmoſphere in both.

The quantity of air exhauſted out of the receiver on each turn of the handle, is always proportionable to the aſcent of the quickſilver on that turn; and the quantity of air remaining in the receiver, is proportionable to the defect of the height of the quickſilver in the gage, from what it is at that time in the barometer.

I ſhall now give an account of the experiments made with the air-pump in my lectures; ſhewing the reſiſtance, weight, and elaſticity of the air.

I. *To ſhew the reſiſtance of the air.*

1. There is a little machine, conſiſting of two Fig. 3. mills, *a* and *b*, which are of equal weights, independent of each other, and turn equally free on their axes in the frame. Each mill has four thin arms or ſails, fixed into the axis: thoſe of the mill *a* have their planes at right angles to its axis, and thoſe of *b* have their planes parallel to it. Therefore, as the mill *a* turns round in common air, it is but little reſiſted thereby, becauſe it's ſails cut the air with their thin edges: but the mill *b* is much reſiſted, becauſe the broad ſides of it's ſails move againſt the air when it turns round. In each axle is a pin near the middle of the frame, which goes quite through the axle, and ſtands out a little on each ſide of it: upon theſe pins, the ſlider *d* may be made to bear, and ſo hinder the mills from going, when the ſtrong ſpring *c* is ſet on bend againſt the oppoſite ends of the pins.

Having ſet this machine upon the pump-plate *L L* (Fig. 1.) draw up the ſlider *d* to the pins on one ſide, and ſet the ſpring *c* at bend upon the oppoſite ends of the pins: then puſh down the ſlider *d*, and the ſpring acting equally ſtrong upon each mill, will ſet them both a going with equal forces and velocities: but the mill *a* will run much longer than the mill *b*, becauſe the air makes much leſs reſiſtance againſt the edges of its ſails, than againſt the ſides of the ſails of *b*.

Draw up the ſlider again, and ſet the ſpring upon the pins as before; then cover the machine with the receiver *M* upon the pump-Fig. 1. plate, and having exhauſted the receiver of air, puſh down the wire *P P* (through the coller of leathers in the neck *q)* upon the ſlider; which will diſengage it from the pins, and allow the mills to turn round by the impulſe of the ſpring: and as there is no air in the receiver to make any ſenſible reſiſtance againſt them, they will both move a conſiderable time longer than they did in the open air; and the moment that one ſtops, the other will do ſo too.—This ſhews that air reſiſts bodies in motion, and that equal bodies meet with different degrees of reſiſtance, according as they preſent greater or leſs ſurfaces to the air, in the planes of their motions.

2. Take off the receiver *M*, and the mills; and having put the guinea *a* and feather *b* upon the braſs flap *c*, turn up the flap, and ſhut it into the notch *d*. Then, putting a wet leather over the top of the tall receiver *AB*(it being open both at top and battom) cover it with the plate *C*, from which the guinea and feather tongs *e d* will then hang within the receiver. This done, pump the air out of the receiver; and then draw up the wire *f* a little, which by a ſquare piece on its lower end will open the tongs *e d*; and the flap falling down as at *c*, the guinea and feather will deſcend with equal velocities in the receiver; and both will fall upon the pump-plate at the ſame inſtant. *N. B.* In this experiment, the obſervers ought not to look at the top, but at the bottom of the receiver; in order to ſee the guinea and feather fall upon the plate: otherwiſe, on account of the quickneſs of their motion, they will eſcape the ſight of the beholders. Fig. 4.

II. *To ſhew the weight of the air.*

1. Having fitted a braſs cap, with a valve tied over it, to the mouth of a thin bottle or *Florence* flaſk, whoſe contents are exactly known, ſcrew the neck of this cap into the hole *i* of the pump-plate: then, having exhauſted the air out of the flaſk, and taken it off from the pump, let it be ſuſpended at one end of a balance, and nicely counterpoiſed by weights in the ſcale at the other end: this done, raiſe up the valve with a pin, and the air will ruſh into the flaſk with an audible noiſe: during which time, the flaſk will deſcend, and pull down that end of the beam. When the noiſe is over, put as many grains into the ſcale at the other end as will reſtore the equilibrium; and they will ſhew exactly the weight of the quantity of air which has got into the flaſk, and filled it. If the flaſk holds an exact quart, it will be found, that 16 grains will reſtore the equipoiſe of the balance, when the quickſilver ſtands at 29½ inches in the barometer: which ſhews, that when the air is at a mean rate of denſity, a quart of it weighs 16 grains: it weighs more when the quickſilver ſtands higher; and leſs when it ſtands lower.

2. Place the ſmall receiver *O* (Fig. 1.) over the hole *i* in the pump-plate, and upon exhauſting the air, the receiver will be fixed down to the plate by the preſſure of the air on its outſide, which is left to act alone, without any air in the receiver to act againſt it: and this preſſure will be equal to as many times 15 pounds, as there are ſquare inches in that part of the plate which the receiver covers; which will hold down the receiver ſo faſt, that it cannot be got off, until

the air be let into it by turning the cock *k*; and then it becomes looſe.

3. Set the little glaſs *A B* (which is open at both ends) over the hole *i* upon the pump-plate *L L*, and put your hand cloſe upon the top of it at *B*: then, upon exhauſting the air out of the glaſs, you will find your hand preſſed down with a great weight upon it: ſo that you can hardly releaſe it, until the air be re-admitted into the glaſs by turning the cock *k*; which air, by acting as ſtrongly upward againſt the hand as the external air acted in preſſing it downward, will releaſe the hand from its confinement. Fig. 5.

4. Having tied a piece of wet bladder *b* over the open top of the glaſs *A* (which is alſo open at bottom) ſet it to dry, and then the bladder will be tight like a drum. Then place the open end *A* upon the pump-plate, over the hole *i*, and begin to exhauſt the air out of the glaſs. As the air is exhauſting, its ſpring in the glaſs will be weakened, and give way to the preſſure of the outward air on the bladder, which, as it is preſſed down, will put on a ſpherical concave figure, which will grow deeper and deeper, until the ſtrength of the bladder be overcome by the weight of the air; and then it will break with a report as loud as that of a gun.—If a flat piece of glaſs be laid upon the open top of this receiver, and joined to it by a flat ring of wet leather between them; upon pumping the air out of the receiver, the preſſure of the outward air upon the flat glaſs will break it all to pieces. Fig. 6.

5. Immerſe the neck *c d* of the hollow glaſs ball *e b* in water, contained in the phial *a a*; then ſet it upon the pump-plate, and cover it and the hole *i* with the cloſe receiver *A*; and then begin Fig. 7.

to pump out the air. As the air goes out of the receiver by its ſpring, it will alſo by the ſame means go out of the hollow ball *e b*, through the neck *d c*, and riſe up in bubbles to the ſurface of the water in the phial; from whence it will make its way, with the reſt of the air in the receiver, through the air-pipe *GG* and valves *a* and *b*, into the open air. When it has done bubbling in the phial, the ball is ſufficiently exhauſted; and then, upon turning the cock *k*, the air will get into the receiver, and preſs ſo upon the ſurface of the water in the phial, as to force the water up into the ball in a jet, through the neck *c d*; and will fill the ball almoſt full of water. The reaſon why the ball is not quite filled, is becauſe all the air could not be taken out of it; and the ſmall quantity that was left in, and had expanded itſelf ſo as to fill the whole ball, is now condenſed into the ſame ſtate as the outward air, and remains in a ſmall bubble at the top of the ball; and ſo keeps the water from filling that part of the ball.

Fig. 8. 6. Pour ſome quickſilver into the jar *D*, and ſet it on the pump-plate near the hole *i*; then ſet on the tall open receiver *AB*, ſo as to be over the jar and hole; and cover the receiver with the braſs plate *C*. Screw the open glaſs tube *fg* (which has a braſs top on it at *h*) into the ſyringe *H*, and putting the tube through a hole in the middle of the plate, ſo as to immerſe the lower end of the tube *e* in the quickſilver at *D*, ſcrew the end *h* of the ſyringe into the plate. This done, draw up the piſton in the ſyringe by the ring *I*, which will make a vacuum in the ſyringe, below the piſton; and as the upper end of the tube opens into the ſyringe, the air will be dilated in the tube, becauſe part of it, by its ſpring, gets up into the ſyringe; and the ſpring of the un-dilated air in the receiver acting upon the ſurface of the quickſilver in the jar, will force part of it up into the tube: for the quickſilver will follow the piſton in the ſyringe, in the ſame way, and for the ſame reaſon, that water follows the piſton of a common pump when it is raiſed in the pump-barrel; and this, according to ſome, is done by ſuction. But to refute that erroneous notion, let the air be pumped out of the receiver *AB*, and then all the quickſilver in the tube will fall down by its own weight into the jar; and cannot be again raiſed one hair's breadth in the tube by working the ſyringe: which ſhews that ſuction had no hand in raiſing the quickſilver; and, to prove that it is done by preſſure, let the air into the receiver by the cock *k* (Fig. 1.) and its action upon the ſurface of the quickſilver in the jar will raiſe it up into the tube, although the piſton of the ſyringe continues motionleſs.—If the tube be about 32 or 33 inches high, the quickſilver will riſe in it very near as high as it ſtands at that time in the barometer. And, if the ſyringe has a ſmall hole, as *m*, near the top of it, and the piſton be drawn up above that hole, the air will ruſh through the hole into the ſyringe and tube, and the quickſilver will immediately fall down into the jar. If this part of the apparatus be air-tight, the quickſilver may be pumped up into the tube to the ſame height that it ſtands in the barometer; but it will go no higher, becauſe then the weight of the column in the tube is the ſame as the weight of a column of air of the ſame thickneſs with the quickſilver, and reaching from the earth to the top of the atmoſphere.

Fig. 9. 7. Having placed the jar *A*, with ſome quickſilver in it, on the pump-plate, as in the laſt experiment, cover it with the receiver *B*; then puſh the open end of the glaſs tube *d e* through the collar of leathers in the braſs neck *C* (which it fits ſo as to be air-tight) almoſt down to the quickſilver in the jar. Then exhauſt the air out of the receiver, and it will alſo come out of the tube, becauſe the tube is cloſe at top. When the gauge *m m* ſhews that the receiver is well exhauſted, puſh down the tube, ſo as to immerſe its lower end into the quickſilver in the jar. Now, although the tube be exhauſted of air, none of the quickſilver will riſe into it, becauſe there is no air left in the receiver to preſs upon its ſurface in the jar. But let the air into the receiver by the cock *k*, and the quickſilver will immediately riſe in the tube; and ſtand as high in it, as it was pumped up in the laſt experiment.

Both theſe experiments ſhew, that the quickſilver is ſupported in the barometer by the preſſure of the air on its ſurface in the box, in which the open end of the tube is placed. And that the more denſe and heavy the air is, the higher does the quickſilver riſe; and, on the contrary, the thinner and lighter the air is, the more will the quickſilver fall. For if the handle *F* be turned ever ſo little, it takes ſome air out of the receiver, by raiſing one or other of the piſtons in its barrel; and conſequently, that which remains in the receiver is ſo much the rarer, and has ſo much the leſs ſpring and weight; and thereupon, the quickſilver falls a little in the tube: but upon turning the cock, and re-admitting the air into the receiver, it becomes as weighty as before, and the quickſilver riſes again to the ſame height.—Thus we ſee the reaſon why the quickſilver in the barometer falls before rain or ſnow, and riſes before fair weather; for, in the former caſe, the air is too thin and light to bear up the vapours, and in the latter, too denſe and heavy to let them fall.

N. B. In all mercurial experiments with the air-pump, a ſhort pipe muſt be ſcrewed into the hole *i*, ſo as to riſe about an inch above the plate, to prevent the quickſilver from getting into the air-pipe and barrels, in caſe any of it ſhould be accidentally ſpilt over the jar: for if it once gets into the pipes or barrels, it ſpoils them, by looſening the ſolder, and corroding the braſs.

8. Take the tube out of the receiver, and put one end of a bit of dry hazel branch, about an inch long, tight into the hole, and the other end tight into a hole quite through the bottom of a ſmall wooden cup: then pour ſome quickſilver into the cup, and exhauſt the receiver of air, and the preſſure of the outward air, on the ſurface of the quickſilver, will force it through the pores of the hazel, from whence it will deſcend in a beautiful ſhower into a glaſs cup placed under the receiver to catch it.

9. Put a wire through the collar of leathers in the top of the receiver, and fix a bit of dry wood on the end of the wire within the receiver; then exhauſt the air, and puſh the wire down, ſo as to immerſe the wood into a jar of quickſilver on the pump-plate; this done, let in the air, and upon taking the wood out of the jar, and ſplitting it, its pores will be found full of quickſilver, which the force of the air, upon being let into the receiver, drove into the wood.

10. Join the two braſs hemiſpherical cups *A* and *B* together, with a wet leather between them, having Fig. 10.

ing a hole in the middle of it; then ſcrew the end *D* of the pipe *C D* into the plate of the pump at *i*, and turn the cock *E*, ſo as the pipe may be open all the way into the cavity of the hemiſpheres: then exhauſt the air out of them, and turn the cock a quarter round, which will ſhut the pipe *C D*, and keep out the air. This done, unſcrew the pipe at *D* from the pump; and ſcrew the piece *F b* upon it at *D*; and let two ſtrong men try to pull the hemiſpheres aſunder by the rings *g* and *b*, which they will find hard to do: for if the diameter of the hemiſpheres be four inches, they will be preſſed together by the external air with a force equal to 190 pounds. And to ſhew that it is the preſſure of the air that keeps them together, hang them by either of the rings upon the hook *P* of the wire in the receiver *M* (Fig. 1.) and upon exhauſting the air out of the receiver, they will fall aſunder of themſelves.

11. Place a ſmall receiver *O* (Fig. 1.) near the hole *i* on the pump-plate, and cover both it and the hole with the receiver *M*; and turn the wire ſo by the top *P*, that its hook may take hold of the little receiver by a ring at its top, allowing that receiver to ſtand with its own weight on the plate. Then, upon working the pump, the air will come out of both receivers; but the large one *M* will be forcibly held down to the pump by the preſſure of the external air; whilſt the ſmall one *O*, having no air to preſs upon it, will continue looſe, and may be drawn up and let down at pleaſure, by the wire *P P*. But, upon letting it quite down to the plate, and admitting the air into the receiver *M*, by the cock *k*, the air will preſs ſo ſtrongly upon the ſmall receiver *O*, as to fix it down to the plate; and at

the ſame time, by counterbalancing the outward preſſure on the large receiver *M*, it will become looſe. This experiment evidently ſhews, that the receivers are held down by preſſure, and not by ſuction, for the internal receiver continued looſe whilſt the operator was pumping, and the external one was held down; but the former became faſt immediately by letting in the air upon it.

12. Screw the end *A* of the braſs pipe *ABF* into the hole of the pump-plate, and turn the cock *e* until the pipe be open; then put a wet leather upon the plate *cd*, which is fixed on the pipe, and cover it with the tall receiver *GH*, which is cloſe at top: then exhauſt the air out of the receiver, and turn the cock *e* to keep it out; which done, unſcrew the pipe from the pump, and ſet its end *A* into a baſon of water, and turn the cock *e* to open the pipe; on which, as there is no air in the receiver, the preſſure of the atmoſphere on the water in the baſon will drive the water forcibly through the pipe, and make it play up in a jet to the top of the receiver. Fig. 11.

13. Set the ſquare phial *A* (Fig. 14.) upon the pump-plate, and having covered it with the wire cage *B*, put a cloſe receiver over it, and exhauſt the air out of the receiver; in doing of which, the air will alſo make its way out of the phial through a ſmall hole in its neck under the valve *b*. When the air is exhauſted, turn the cock below the plate, to re-admit the air into the receiver; and as it cannot get into the phial again, becauſe of the valve, the phial will be broke into ſome thouſands of pieces by the preſſure of the air upon it. Had the phial been of a round form, it would have ſuſtained this

preſſure like an arch, without breaking; but as its ſides are flat, it cannot.

To ſhew the elaſticity or ſpring of the air.

14. Tie up a very ſmall quantity of air in a bladder, and put it under a receiver; then exhauſt the air out of the receiver; and the ſmall quantity which is confined in the bladder (having nothing to act againſt it) will expand itſelf ſo by the force of its ſpring, as to fill the bladder as full as it could be blown of common air. But upon letting the air into the receiver again, it will overpower the air in the bladder, and preſs its ſides almoſt cloſe together.

15. If the bladder ſo tied up be put into a wooden box, and have 20 or 30 pound weight of lead put upon it in the box, and the box be covered with a cloſe receiver; upon exhauſting the air out of the receiver, that air which is confined in the bladder will expand itſelf ſo, as to raiſe up all the lead by the force of its ſpring.

Fig. 7. 16. Take the glaſs ball mentioned in the fifth experiment, which was left full of water all but a ſmall bubble of air at top, and having ſet it with its neck downward into the empty phial *aa*, and covered it with a cloſe receiver, exhauſt the air out of the receiver, and the ſmall bubble of air in the top of the ball will expand itſelf, ſo as to force all the water out of the ball into the phial.

Fig. 11. 17. Screw the pipe *AB* into the pump-plate, place the tall receiver *GH* upon the plate *cd*, as in the twelfth experiment, and exhauſt the air out of the receiver; then, turn the cock *e* to keep out the air, unſcrew the pipe from the pump, and ſcrew it into the mouth of the copper veſſel *C C* (Fig. 15.) the veſſel having firſt been about half filled with water. Then open the cock *e* (Fig. 11.) and the ſpring of the air which is confined in the copper veſſel will force the water up through the pipe *A B* in a jet into the exhauſted receiver, as ſtrongly as it did by its preſſure on the ſurface of the water in a baſon, in the twelfth experiment.

18. If a fowl, a cat, rat, mouſe, or bird, be put under a receiver, and the air be exhauſted, the animal will be at firſt oppreſſed as with a great weight, then grow convulſed, and at laſt expire in all the agonies of a moſt bitter and cruel death. But as this experiment is too ſhocking to every ſpectator who has the leaſt degree of humanity, we ſubſtitute a machine called the *lungs-glaſs* in place of the animal.

19. If a butterfly be ſuſpended in a receiver, by a fine thread tied to one of its horns, it will fly about in the receiver, as long as the receiver continues full of air; but if the air be exhauſted, though the animal will not die, and will continue to flutter its wings, it cannot remove itſelf from the place where it hangs in the middle of the receiver, until the air be let in again, and then the animal will fly about as before.

20. Pour ſome quickſilver into the ſmall bottle *A*, and ſcrew the braſs collar *c* of the tube *B C* into the braſs neck *b* of the bottle, and the lower end of the tube will be immerſed into the quickſilver, ſo that the air above the quickſilver in the bottle will be confined there, becauſe it cannot get out about the joinings, nor can it be drawn out through the quickſilver into the tube. This tube is alſo open at top, and is to be covered with the receiver *G* and large tube *E F*, which tube is fixed by braſs collars to the receiver, and is cloſe at Fig. 12.

at the top. This preparation being made, exhauſt the air both out of the receiver and its tube; and the air will by the ſame means be exhauſted out of the inner tube *BC*, through its open top at *C*; and as the receiver and tubes are exhauſting, the air that is confined in the glaſs bottle *A* will preſs ſo by its ſpring upon the ſurface of the quickſilver, as to force it up in the inner tube as high as it was raiſed in the ninth experiment by the preſſure of the atmoſphere: which demonſtrates that the ſpring of the air is equivalent to its weight.

Fig. 13. 21. Screw the end *C* of the pipe *C D* into the hole of the pump-plate, and turn all the three cocks *d*, *G*, and *H*, ſo as to open the communications between all the three pipes *E*, *F*, *D C*, and the hollow trunk *A B*. Then, cover the plates *g* and *h* with wet leathers, which have holes in their middle where the pipes open into the plates; and place the cloſe receiver *I* upon the plate *g*: this done, ſhut the pipe *F* by turning the cock *H*, and exhauſt the air out of the receiver *I*. Then, turn the cock *d* to ſhut out the air, unſcrew the machine from the pump, and having ſcrewed it to the wooden foot *L*, put the receiver *K* upon the plate *h*; this receiver will continue looſe on the plate as long as it keeps full of air; which it will do until the cock *H* be turned to open the communication between the pipes *F* and *E*, through the trunk *AB*; and then the air in the receiver *K*, having nothing to act againſt its ſpring, will run from *K* into *I*, until it be ſo divided between theſe receivers, as to be of equal denſity in both; and then they will be held down with equal forces to their plates by the preſſure of the atmoſphere; though each receiver will then be kept down but with one half of preſſure upon it, that the receiver *I* had, when it was exhauſted of air; becauſe it has now one half of the common air in it which filled the receiver *K* when it was ſet upon the plate; and therefore, a force equal to half the force of the ſpring of common air, will act within the receivers againſt the whole preſſure of the common air upon their outſides. This is called transferring the air out of one veſſel into another.

22. Put a cork into the ſquare phial *A*, and fix it in with wax or cement; put the phial upon the pump-plate with the wire cage *B* over it, and cover the cage with a cloſe receiver. Then, exhauſt the air out of the receiver, and the air that was corked up in the phial will break the phial outwards by the force of its ſpring, becauſe there is no air left on the outſide of the phial to act againſt the air within it. Fig. 14.

23. Put a ſhrivelled apple under a cloſe receiver, and exhauſt the air; then the ſpring of the air within the apple will plump it out, ſo as to cauſe all the wrinkles diſappear; but upon letting the air into the receiver again, to preſs upon the apple, it will inſtantly return to its former decayed and ſhrivelled ſtate.

24. Take a freſh egg, and cut off a little of the ſhell and film from its ſmalleſt end, then put the egg under a receiver, and pump out the air; upon which, all the contents in the egg will be forced out into the receiver, by the expanſion of a ſmall bubble of air contained in the great end, between the ſhell and film.

25. Put ſome warm beer into a glaſs, and having ſet it on the pump, cover it with a cloſe receiver, and then exhauſt the air. Whilſt this is doing, and thereby the preſſure more and more

taken off from the beer in the glaſs, the air therein will expand itſelf, and riſe up in innumerable bubbles to the ſurface of the beer; and from thence it will be taken away with the other air in the receiver. When the receiver is nearly exhauſted, the air in the beer, which could not diſentangle itſelf quick enough to get off with the reſt, will now expand itſelf ſo, as to cauſe the beer to have all the appearance of boiling; and the greateſt part of it will go over the glaſs.

26. Put ſome warm water into a glaſs, and put a bit of dry wainſcot or other wood into the water. Then, cover the glaſs with a cloſe receiver, and exhauſt the air; upon which, the air in the wood having liberty to expand itſelf, will come out plentifully, and make all the water to bubble about the wood, eſpecially about the ends, becauſe the pores lie lengthwiſe. A cubic inch of dry wainſcot has ſo much air in it, that it will continue bubbling for near half an hour together.

Miſcellaneous Experiments.

27. Screw the ſyringe *H* (Fig. 8.) to a piece of lead that weighs one pound at leaſt; and, holding the lead in one hand, pull up the piſton in the ſyringe with the other; then, quitting hold of the lead, the air will puſh it upward, and drive back the ſyringe upon the piſton. The reaſon of this is, that the drawing up of the piſton makes a vacuum in the ſyringe, and the air, which preſſes every way equally, having nothing to reſiſt its preſſure upward, the lead is thereby preſſed upward, contrary to its natural tendency by gravity. If the ſyringe, ſo loaded,

be hung in a receiver, and the air be exhauſted, the ſyringe and lead will deſcend upon the piſton-rod by their natural gravity; and, upon admitting the air into the receiver, they will be drove upward again, until the piſton be at the very bottom of the ſyringe.

28. Let a large piece of cork be ſuſpended by a thread at one end of a balance, and counterpoiſed by a leaden weight, ſuſpended in the ſame manner, at the other. Let this balance be hung to the inſide of the top of a large receiver; which being ſet on the pump, and the air exhauſted, the cork will preponderate, and ſhew itſelf to be heavier than the lead; but upon letting in the air again, the equilibrium will be reſtored. The reaſon of this is, that ſince the air is a fluid, and all bodies loſe as much of their abſolute weight in it, as is equal to the weight of their bulk of the fluid, the cork being the larger body, loſes more of its real weight than the lead does; and therefore muſt in fact be heavier, to balance it under the diſadvantage of loſing ſome of its weight: which diſadvantage being taken off by removing the air, the bodies then gravitate according to their real quantities of matter, and the cork, which balanced the lead in air, ſhews itſelf to be heavier when *in vacuo*.

29. Set a lighted candle upon the pump, and cover it with a tall receiver. If the receiver holds a gallon, the candle will burn a minute; and then, after having gradually decayed from the firſt inſtant, it will go out: which ſhews, that a conſtant ſupply of freſh air is neceſſary to feed flame; and ſo it alſo is for animal life. For a bird kept under a cloſe receiver will ſoon die, although no air be pumped out; and it is

ſound that, in the diving-bell, a gallon of air is ſufficient only for one minute for a man to breathe in.

The moment when the candle goes out, the ſmoke will be ſeen to aſcend to the top of the receiver, and there it will form a ſort of cloud: but upon exhauſting the air, the ſmoke will fall down to the bottom of the receiver, and leave it as clear at the top as it was before it was ſet upon the pump. This ſhews, that ſmoke does not aſcend on account of its being poſitively light, but becauſe it is lighter than air; and its falling to the bottom when the air is taken away, ſhews, that it is not deſtitute of weight. So moſt ſorts of wood aſcend or ſwim in water; and yet there are none who doubt of the wood's having gravity or weight.

30. Set a receiver, which is open at top, upon the air-pump, and cover it with a braſs plate, and wet leather; and having exhauſted it of air, let the air in again at top through an iron pipe, making it paſs through a charcoal flame at the end of the pipe; and when the receiver is full of that air, lift up the cover, and let down a mouſe or bird into the receiver, and the burnt air will immediately kill it. If a candle be let down into that air, it will go out directly; but, by letting it down gently, it will purify the air ſo far as it goes; and ſo, by letting it down more and more, the flame will drive out the bad air, and good air will get in.

31. Set a bell upon a cuſhion on the pump-plate, and cover it with a receiver; then ſhake the pump to make the clapper ſtrike againſt the bell, and the ſound will be very well heard: but, exhauſt the receiver of air, and then, if the clapper be made to ſtrike ever ſo hard againſt

the bell, it will make no ſound at all; which ſhews, that air is abſolutely neceſſary for the propagation of ſound.

32. Let a candle be placed on one ſide of a receiver, and viewed through the receiver at ſome diſtance; then, as ſoon as the air begins to be exhauſted, the receiver will be filled with vapours which riſe from the wet leather, by the ſpring of the air in it; and the light of the candle being refracted through that medium of vapours, will have the appearance of circles of various colours, of a faint reſemblance to thoſe in the rain-bow.

The air-pump was invented by *Otho Guerick* of *Magdeburg*, but was much improved by Mr. *Boyle*, to whom we are indebted for our greateſt part of the knowledge of the wonderful properties of the air, demonſtrated in the above experiments.

The elaſtic air which is contained in many bodies, and is kept in them by the weight of the atmoſphere, may be got out of them either by boiling, or by the air-pump, as ſhewn in the 25th experiment: but the fixed air, which is by much the greater quantity, cannot be got out but by diſtillation, fermentation, or putrefaction.

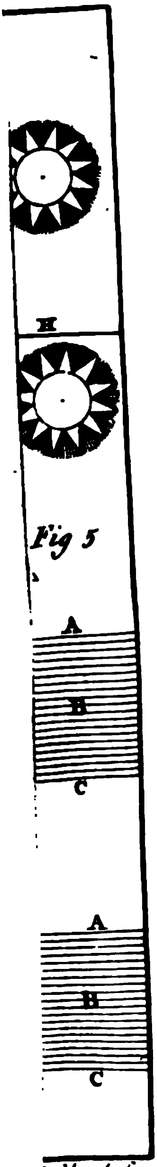
If fixed air did not come out of bodies with difficulty, and ſpend ſome time in extricating itſelf from them, it would tear them to pieces. Trees would be rent by the change of air from a fixt, to an elaſtic ſtate, and animals would be burſt in pieces by the exploſion of air in their food.

Dr. *Hales* found by experiment, that the air in apples is ſo much condenſed, that if it were let out into the common air, it would fill a ſpace

48 times as great as the bulk of the apples themſelves; ſo that its preſſure outwards was equal to 11776 lb. and, in a cubic inch of oak, to 19860 lb. againſt its ſides. So that if the air was let looſe at once in theſe ſubſtances, they would tear every thing to pieces about them with a force ſuperior to that of gunpowder. Hence, in eating apples, it is well that they part with the air by degrees, as they are chewed, and ferment in the ſtomach, otherwiſe an apple would be immediate death to him who eats it.

The mixing of ſome ſubſtances with others will releaſe the air from them, all of a ſudden, which may be attended with very great danger. Of this we have a remarkable inſtance in an experiment made by Dr. *Slare*; who having put half a dram of oil of carraway-ſeeds into one glaſs, and a dram of compound ſpirit of nitre in another, covered them both on the air-pump with a receiver ſix inches wide, and eight inches deep, and then exhauſted the air, and continued pumping until all that could poſſibly be got both out of the receiver, and out of the two fluids, was extricated: then, by a particular contrivance from the top of the receiver, he mixed the fluids together; upon which they produced ſuch a prodigious quantity of air, as inſtantly blew up the receiver, although it was preſſed down by the atmoſphere with upwards of 400 pound weight.

N. B. In the 28th Experiment, the cork muſt be covered all over with a piece of thin wet bladder glued to it, and not uſed until it be thoroughly dry.



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LECT. VIII.

Of Optics.

LIGHT conſiſts of an inconceivably great number of particles flowing from a luminous body in all manner of directions; and theſe particles are ſo ſmall, as to ſurpaſs all human comprehenſion.

That the number of particles of light is inconceivably great, appears from the light of a candle; which, if there be no obſtacle in the way to obſtruct the paſſage of its rays, will fill all the ſpace within two miles of the candle every way, with luminous particles, before it has loſt the leaſt ſenſible part of its ſubſtance.

A ray of light is a continued ſtream of theſe particles, flowing from any viſible body in a ſtraight line: and that the particles themſelves are incomprehenſibly ſmall, is manifeſt from the following experiment. Make a ſmall pin-hole in a piece of black paper, and hold the paper upright on a table facing a row of candles ſtanding by one another; then place a ſheet of paſteboard at a little diſtance behind the paper, and ſome of the rays which flow from all the candles through the hole in the paper, will form as many ſpecks of light on the paſteboard, as there are candles on the table before the plate: each ſpeck being as diſtinct and clear, as if there was only one ſpeck from one ſingle candle: which ſhews that the particles of light are exceedingly ſmall, otherwiſe they could not paſs through the hole from ſo many different candles without confuſion.—Dr. *Niewentyt* has computed, that there flows more than 6,000,000,000,000 times as

The amazing ſmallneſs of the particles of light.

many particles of light from a candle in one ſecond of time, as there are grains of ſand in the whole earth, ſuppoſing each cubic inch of it to contain 1,000,000.

Plate XV. Fig. 1.

Reflected light.

Theſe particles, by falling directly upon our eyes, excite in our minds the idea of light. And when they fall upon bodies, and are thereby reflected to our eyes, they excite in us the ideas of theſe bodies. And as every point of a viſible body reflects the rays of light in all manner of directions, every point will be viſible in every part to which the light is reflected from it. Thus the object *A C B* is viſible to an eye in any part where the rays *A a*, *A b*, *A c*, *A d*, *A e*, *B a*, *B b*, *B c*, *B d*, *B e*, and *C a*, *C b*, *C c*, *C d*, *C e*, come. Here we have ſhewn the rays as if they were only reflected from the ends *A* and *B*, and from the middle point *C* of the object; every other point being ſuppoſed to reflect rays in the ſame manner. So that wherever a ſpectator is placed with regard to the body, every point of that part of the ſurface which is towards him will be viſible, when no intervening object ſtops the paſſage of the light.

As no object can be ſeen through the bore of a bended pipe, it is evident that the rays of light move in ſtraight lines, whilſt there is nothing to refract or turn them out of their rectilineal courſe.

While the rays of light continue in any * medium of an uniform denſity, they are ſtraight; but when they paſs obliquely out of one medium into another, which is either more denſe or more rare, they are refracted towards the denſer medium: and this refraction is more or leſs, as the rays fall more or leſs obliquely on the refracting ſurface which divides the mediums.

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* Any thing through which the rays of light can paſs, is called a medium; as air, water, glaſs, diamond, or even a vacuum.

To prove this by experiment, ſet the empty veſſel *ABCD* into any place where the ſun ſhines obliquely, and obſerve the part where the ſhadow of the edge *B C* falls on the bottom of the veſſel at *E*; then fill the veſſel with water, and the ſhadow will reach no farther than *e*; which ſhews, that the ray *a B E*, which came ſtraight in the open air, juſt over the edge of the veſſel at *B* to its bottom at *E*, is refracted by falling obliquely on the ſurface of the water at *B*; and inſtead of going on in the rectilineal direction *a B E*, it is bent downward in the water from *B* to *e*; the whole bend being at the ſurface of the water: and ſo of all the other rays *a b c*. Fig. 2. Refracted light.

If a ſtick be laid over the veſſel, and the ſun's rays be reflected from a glaſs perpendicularly into the veſſel, the ſhadow of the ſtick will fall upon the ſame part of the bottom, whether the veſſel be empty or full, which ſhews, that the rays of light are not refracted when they fall perpendicularly on the ſurface of any medium.

The rays of light are as much refracted by paſſing out of water into air, as by paſſing out of air into water. Thus, if a ray of light flows from the point *e*, under water, in the direction *e B*; when it comes to the ſurface of the water at *B*, it will not go on thence in the rectilineal courſe *B d*, but will be refracted into the line *B a*. Therefore,

To an eye at *e* looking through a plane glaſs in the bottom of the empty veſſel, the point *a* cannot be ſeen, becauſe the ſide *B c* of the veſſel interpoſes; and the point *d* will juſt be ſeen over the edge of the veſſel at *B*. But if the veſſel be filled with water, the point *a* will be ſeen from *e*; and will appear as at *d*, elevated in the direction of the ray *e B* *.

The days are made longer by the refraction of the ſun's rays.

The time of ſun-riſing or ſetting, ſuppoſing its rays ſuffered no refraction, is eaſily found by calculation. But obſervation proves that the ſun riſes ſooner, and ſets later every day than the calculated time; the reaſon of which is plain, from what was ſaid immediately above. For, though the ſun's rays do not come part of the way to us through water, yet they do through the air or atmoſphere, which being a groſſer medium than the free ſpace between the ſun and the top of the atmoſphere, the rays, by entering obliquely into the atmoſphere, are there refracted, and thence bent down to the earth. And although there are many places of the earth to which the ſun is vertical at noon, and conſequently his rays can ſuffer no refraction at that time, becauſe they come perpendicularly through the atmoſphere: yet there is no place to which the ſun's rays do not fall obliquely on the top of the atmoſphere, at his riſing and ſetting; and conſequently, no clear day in which the ſun will not be viſible before he riſes in the horizon, and after he ſets in it: and the longer or ſhorter, as the atmoſphere is more or leſs replete with vapours. For, let *A B C* be part of the earth's ſurface, *D E F* the atmoſphere that covers it, Fig. 3.

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* Hence a piece of money lying at *e*, in the bottom of an empty veſſel, cannot be ſeen by an eye at *a*, becauſe the edge of the veſſel intervenes; but let the veſſel be filled with water, and the ray *e a* being then refracted at *B*, will ſtrike the eye at *a*, and ſo render the money viſible, which will appear as if it were raiſed up to *f* in the line *a B f*.

and *E B G H* the ſenſible horizon of an obſerver at *B*. As every point of the ſun's ſurface ſends out rays of light in all manner of directions, ſome of his rays will conſtantly fall upon, and enlighten, ſome half of the atmoſphere; and therefore, when the ſun is at *I*, below the horizon *H*, thoſe rays which go on in the free ſpace *I k K* preſerve a rectilineal courſe until they fall upon the top of the atmoſphere; and thoſe which fall ſo about *K*, are refracted at their entrance into the atmoſphere, and bent down in the line *K m B*, to the obſerver's place at *B*: and therefore, to him, the ſun will appear at *L*, in the direction of the ray *B m K*, above the horizon *B G H*, when he is really below it at *I*.

The angle contained between a ray of light, and a perpendicular to the refracting ſurface, is called *the angle of incidence*; and the angle contained between the ſame perpendicular, and the ſame ray after refraction, is called *the angle of refraction*. Thus, let *L B M* be the refracting ſurface of a medium (ſuppoſe water) and *A B C* a perpendicular to that ſurface; let *D B* be a ray of light, going out of air into water at *B*, and therein refracted in the line *B H*; the angle *A B D*, is the angle of incidence, of which *D F* is the ſine; and the angle *K B H* is the angle of refraction, whoſe ſine is *K I*.

Angle of incidence.

Fig. 4.

Angle of refraction.

When the refracting medium is water, the ſine of the angle of incidence is to the ſine of the angle of refraction, as 4 to 3; which is confirmed by the following experiment, taken from Doctor SMITH's Optics.

Deſcribe the circle *D A E C* on a plane ſquare board, and croſs it at right angles with the ſtraight lines *A B C*, and *L B M*; then, from the interſection, *A*, with any opening of the compaſſes, ſet off the equal arcs *A D* and *A E*, and draw the right line *D F E:* then, taking *F a* which is three quarters of the length *F E*, from the point *a*, draw *a I* parallel to *A B K*, and join *K I*, parallel to *B M:* ſo *K I* will be equal to three quarters of *F E* or of *D F*. This done, fix the board upright upon the leaden pedeſtal *O*, and ſtick three pins perpendicularly into the board, at the points *D*, *B*, and *I:* then ſet the board upright into the veſſel *V U T*, and fill up the veſſel with water to the line *L B M*. When the water has ſettled, look along the line *D B*, ſo as you may ſee the head of the pin *B* over the head of the pin *D*; and the pin *I* will appear in the ſame right line produced to *G*, for its head will be ſeen juſt over the head of the pin at *B:* which ſhews that the ray *I B*, coming from the pin at *I*, is ſo refracted at *B*, as to proceed from thence in the line *B D* to the eye of the obſerver; the ſame as it would do from any point *G* in the right line *D B G*, if there were no water in the veſſel: and alſo ſhews that *K I*, the ſine of refraction in water, is to *D F*, the ſine of incidence in air, as 3 to 4[*]

Hence, if *D B H* were a crooked ſtick put obliquely into the water, it would appear a ſtraight one, as *D B G*. Therefore, as the line *B H* appears at *B G*, ſo the line *B G* will appear at *B g*; and conſequently, a ſtraight ſtick *D B G* put obliquely into water, will ſeem bent at the ſurface of the water in *B*, and crooked, as *D B g*.

When a ray of light paſſes out of air into glaſs, the ſine of incidence is to the ſine of refraction, as 3 to 2; and when out of air into a diamond, as 5 to 2.

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[*] This is ſtrictly true of the red rays only, for the other coloured rays are differently refracted; but the difference is ſo ſmall, that it need not be conſidered in this place.

Glaſs may be ground into eight different shapes at leaſt, for optical purpoſes, viz. Fig. 5.

1. A *plane glaſs*, which is flat on both ſides, and of equal thickneſs in all its parts, as *A*.

2. A *plano convex*, which is flat on one ſide, and convex on the other, as *B*. Lenſes.

3. A *double convex*, which is convex on both ſides, as *C*.

4. A *plano-concave* which is flat on one ſide, and concave on the other, as *D*.

5. A *double-concave*, which is concave on both ſides, as *E*.

6. A *meniſcus*, which is concave on one ſide, and convex on the other, as *F*.

7. A *flat plano-convex*, whoſe convex ſide is ground into ſeveral little flat ſurfaces, as *G*.

8. A *priſm*, which has three flat ſides, and when viewed endwiſe, appears like an equilateral triangle, as *H*.

Glaſſes ground into any of the ſhapes *B*, *C*, *D*, *E*, *F*, are generally called *lenſes*.

A right line *L I K*, going perpendicularly through the middle of a lens, is called *the axis of the lens*.

A ray of light *G h*, falling perpendicularly on a plane glaſs *E F*, will paſs through the glaſs in the ſame direction *h i*, and go out of it into the air in the ſame right courſe *i H*. Fig. 6.

A ray of light *A B*, falling obliquely on a plane glaſs, will go out of the glaſs in the ſame direction, but not in the ſame right line; for in touching the glaſs, it will be refracted in the line *B C*, and in leaving the glaſs, it will be refracted in the line *C D*.

Fig. 6. A ray of light *C D*, falling obliquely on the middle of a convex glaſs, will go forward in the ſame direction *D E*, as if it had fallen with the ſame degree of obliquity on a plane glaſs; and will go out of the glaſs in the ſame direction with which it entered: for it will be equally refracted at the points *D* and *E*, as if it had paſſed through a plane ſurface. But the rays *C G* and *C I* will be ſo refracted, as to meet again at the point *F*. Therefore, all the rays which flow from the point *C*, ſo as to go through the glaſs, will meet again at *F*; and if they go farther onward, as to *L*, they croſs at *F*, and go forward on the oppoſite ſides of the middle ray *C D E F*, to what they were in approaching it in the directions *H F* and *K F*.

Fig. 8. The properties of different lenſes.

When parallel rays, as *A B C*, fall directly upon a plano-convex glaſs *D E*, and paſs through it, they will be ſo refracted, as to unite in a point *f* behind it: and this point is called the *principal focus*: the diſtance of which, from the middle of the glaſs, is called the *focal diſtance*; which is equal to twice the radius of the ſphere of the glaſs's convexity. And,

Fig. 9. When parallel rays, as *A B C*, fall directly upon a glaſs *D E*, which is equally convex on both ſides, and paſs through it; they will be ſo refracted, as to meet in a point or principal focus *f*, whoſe diſtance is equal to the radius or ſemidiameter of the ſphere of the glaſs's convexity. But if a glaſs be more convex on one ſide than on the other, the rule for finding the focal diſtance is this; as the ſum of the ſemidiameters of both convexities is to the ſemidiameter of either, ſo is double the ſemidiameter of the other to the diſtance of the focus. Or, divide the

PLATE XVI.

Fig. 1.

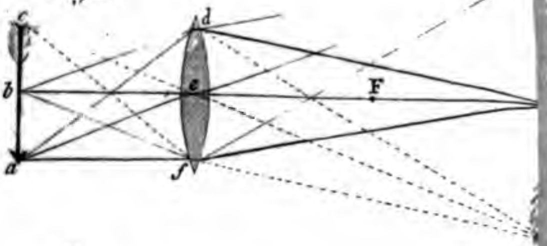
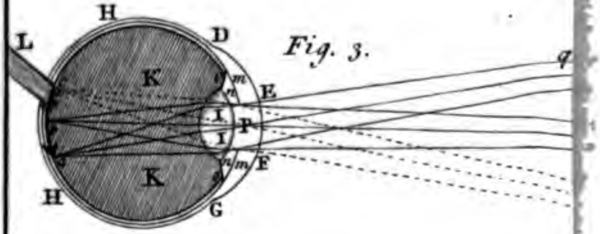


Fig. 3.



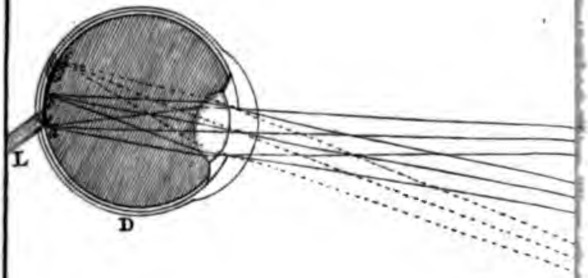
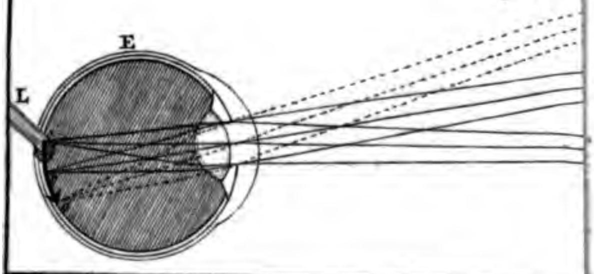


Fig. 4.



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The double product of the radii by their ſum, and the quotient will be the diſtance ſought.

Since all thoſe rays of the ſun which paſs through a convex glaſs are collected together in its focus, the force of all their heat is collected into that part; and is in proportion to the common heat of the ſun, as the area of the glaſs is to the area of the focus. Hence we ſee the reaſon why a convex glaſs cauſes the ſun's rays to burn after paſſing through it.

All theſe rays croſs the middle ray in the focus *f*, and then diverge from it, to the contrary ſides, in the ſame manner *F f G*, as they converged in the ſpace *D f E* in coming to it.

If another glaſs *F G*, of the ſame convexity as *D E*, be placed in the rays at the ſame diſtance from the focus, it will refract them ſo, as that after going out of it, they will be all parallel, as *a b c*; and go on in the ſame manner as they came to the firſt glaſs *D E*, through the ſpace *A B C*; but on the contrary ſides of the middle ray *B f b*: for the ray *A D f* will go on from *f* in the direction *f G a*, and the ray *C E f* in the direction *f F c*; and ſo of the reſt.

The rays diverge from any radiant point, as from a principal focus: therefore, if a candle be placed at *f*, in the focus of the convex glaſs *F G*, the diverging rays in the ſpace *F f G* will be ſo refracted by the glaſs, as, that after going out of it, they will become parallel, as ſhewn in the ſpace *c b a*.

If the candle be placed nearer the glaſs than its focal diſtance, the rays will diverge after paſſing through the glaſs, more or leſs, as the candle is more or leſs diſtant from the focus.

If the candle be placed farther from the glaſs than its focal diſtance, the rays will converge after paſſing through the glaſs, and meet in a point which will be more or leſs diſtant from the glaſs, as the candle is nearer to, or farther from its focus; and where the rays meet, they will form an inverted image of the flame of the candle; which may be ſeen on a paper placed in the meeting of the rays.

Plate XVI. Fig. 1. Hence, if any object *A B C* be placed beyond the focus *F* of the convex glaſs *d e f*, ſome of the rays which flow from every point of the object, on the ſide next the glaſs, will fall upon it, and after paſſing through it, they will be converged into as many points on the oppoſite ſide of the glaſs, where the image of every point will be formed: and conſequently, the image of the whole object, which will be inverted. Thus, the rays *A d*, *A e*, *A f*, flowing from the point *A*, will converge in the ſpace *d a f*, and by meeting at *a*, will there form the image of the point *A*. The rays *B d*, *B e*, *B f*, flowing from the point *B*, will be united at *b* by the refraction of the glaſs, and will there form the image of the point *B*. And the rays *C d*, *C e*, *C f*, flowing from the point *C*, will be united at *c*, where they will form the image of the point *C*. And ſo of all the other intermediate points between *A* and *C*. The rays which flow from every particular point of the object, and are united again by the glaſs, are called ***pencils of rays***.

If the object *A B C* be brought nearer to the glaſs, the picture *a b c* will be removed to a greater diſtance. For then, more rays flowing from every ſingle point, will fall more diverging upon the glaſs; and therefore cannot be ſo ſoon collected into the correſponding points behind it. Conſequently, if the diſtance of the object

A B C

A B C be equal to the diſtance *e B* of the focus of the glaſs, the rays of each pencil will be ſo refracted by paſſing through the glaſs, that they will go out of it parallel to each other; as *d I*, *e H*, *f b*, from the point *C*; *d G*, *e K*, *f D*, from the point *B*; and *d K*, *e E*, *f L*, from the point *A*: and therefore, there will be no picture formed behind the glaſs.

Plate XVI. Fig. 2.

If the focal diſtance of the glaſs, and the diſtance of the object from the glaſs, be known, the diſtance of the picture from the glaſs may be found by this rule, viz. multiply the diſtance of the focus by the diſtance of the object, and divide the product by their difference; the quotient will be the diſtance of the picture.

Fig. 1.

The picture will be as much bigger or leſs than the object, as its diſtance from the glaſs is greater or leſs than the diſtance of the object. For, as *B e* is to *e b*, ſo is *AC* to *c a*. So that if *ABC* be the object, *c b a* will be the picture; or, if *c b a* be the object, *A B C* will be the picture.

The manner of viſion.

Having deſcribed how the rays of light, flowing from objects and paſſing through convex glaſſes, are collected into points, and form the images of the objects; it will be eaſy to underſtand how the rays are affected by paſſing through the humours of the eye, and are thereby collected into innumerable points on the bottom of the eye, and thereon form the images of the objects which they flow from. For, the different humours of the eye, and particularly the chryſtalline humour, are to be conſidered as a convex glaſs; and the rays in paſſing through them to be affected in the ſame manner as in paſſing through a convex glaſs.

PlateXVI. Fig. 3.

The eye deſcribed.

The eye is nearly globular. It conſiſts of three coats and three humours. The part *D H H G* of the outer coat, is called the *ſclerotica*, the reſt *D E F G* the *cornea*. Next within this coat is that called the *choroides*, which ſerves as it were for a lining to the other, and joins with the *iris m n*, *m n*. The *iris* is compoſed of two ſets of muſcular fibres; the one of a circular form, which contracts the hole in the middle called the *pupil*, when the light would otherwiſe be too ſtrong for the eye; and the other of radial fibres, tending every where from the circumference of the iris towards the middle of the pupil; which fibres, by their contraction, dilate and enlarge the pupil when the light is weak, in order to let in the more of its rays. The third coat is only a fine expanſion of the optic nerve *L*, which ſpreads like net-work all over the inſide of the choroides, and is therefore called the *retina*; upon which are painted (as it were) the images of all viſible objects, by the rays of light which either flow or are reflected from them.

Under the *cornea* is a fine tranſparent fluid like water, which is therefore called the *aqueous humour*. It gives a protuberant figure to the cornea, fills the two cavities *m m* and *n n*, which communicate by the pupil *P*, and has the ſame limpidity, ſpecific gravity, and refractive power as water. At the back of this lies the *chryſtalline humour I I*, which is ſhaped like a double convex glaſs; and is a little more convex on the back than the fore-part. It converges the rays, which paſs through it from every viſible object to its focus at the bottom of the eye. This humour is tranſparent like chryſtal, is much of the conſiſtence of hard jelly, and exceeds the ſpecific gravity of water in the proportion of 11 to 10. It is incloſed in a fine tranſparent membrane; from which proceed radial fibres *o o*, called the *ligamentum ciliare*, all around its edge; and join to the circumference of the iris. Theſe fibres have a power of contracting and dilating occaſionally, by which means they alter the ſhape or convexity of the chryſtalline humour, and alſo ſhift it a little backward or forward in the eye, ſo as to adapt its focal diſtance at the bottom of the eye to the different diſtances of objects; without which proviſion, we could only ſee thoſe objects diſtinctly, that were all at one diſtance from the eye.

At the back of the chryſtalline, lies the *vitreous humour K K*, which is tranſparent like glaſs, and is largeſt of all in quantity, filling the whole orb of the eye, and giving it a globular ſhape. It is much of a conſiſtence with the white of an egg, and very little exceeds the ſpecific gravity and refractive power of water.

As every point of an object *A B C* ſends out rays in all directions, ſome rays, from every point on the ſide next the eye, will fall upon the cornea between *E* and *F*; and by paſſing on through the humours and pupil of the eye, they will be converged to as many points on the retina or bottom of the eye, and will thereon form a diſtinct inverted picture *c b a* of the object. Thus, the pencil of rays *q r s* that flows from the point *A* of the object, will be converged to the point *a* on the retina; thoſe from the point *B* will be converged to the point *b*; thoſe from the point *C* will be converged to the point *c*; and ſo of all the intermediate points: by which means the whole image *a b c* is formed, and the object made viſible; although it muſt be owned, that the method by which this ſenſation is carried from the eye by the optic nerve to the common ſenſory in the brain, and there diſcerned, is above the reach of our comprehenſion.

But that viſion is effected in this manner, may be demonſtrated experimentally. Take a bullock's eye whilſt it is freſh, and having cut off the three coats from the back part, quite to the vitreous humour, put a piece of white paper over that part, and hold the eye towards any bright object, and you will ſee an inverted picture of the object upon the paper.

Seeing the image is inverted, many have wondered why the object appears upright. But we are to conſider, 1. That *inverted* is only a relative term: and 2. That there is a very great difference between the real object and the means or image by which we perceive it. When all the parts of a diſtant proſpect are painted upon the retina, they are all right with reſpect to one another, as well as the parts of the proſpect itſelf; and we can only judge of an object's being inverted, when it is turned reverſe to its natural poſition, with reſpect to other objects which we ſee and compare it with.—If we lay hold of an upright ſtick in the dark, we can tell which is the upper or lower part of it, by moving our hand upward or downward; and know very well that we cannot feel the upper end by moving our hand downward. Juſt ſo we find by experience, that upon directing our eyes towards a tall object, we cannot ſee its top by turning our eyes downward, nor its foot by turning our eyes upward; but muſt trace the object the ſame way by the eye to ſee it from head to foot, as we do by the hand to feel it;

J. Mynde ſc.

and as the judgment is informed by the motion of the hand in one caſe, ſo it is alſo by the motion of the eye in the other.

Fig. 4. In Fig. 4. is exhibited the manner of ſeeing the ſame object *A B C*, by both the eyes *D* and *E* at once.

When any part of the image *c b a* falls upon the optic nerve *L*, the correſponding part of the object becomes inviſible. On which account nature has wiſely placed the optic nerve of each eye, not in the middle of the bottom of the eye, but towards the ſide next the noſe; ſo that whatever part of the image falls upon the optic nerve of one eye, may not fall upon the optic nerve of the other. Thus the point *a* of the image *c b a* falls upon the optic nerve of the eye *D*, but not of the eye *E*; and the point *c* falls upon the optic nerve of the eye *E*, but not of the eye *D*: and therefore to both eyes taken together, the whole object *A B C* is viſible.

Plate XVII. Fig. 1. The nearer that any object is to the eye, the larger is the angle under which it it ſeen, and the magnitude under which it appears. Thus to the eye *D*, the object *A B C* is ſeen under the angle *A P C*; and its image *c b a* is very large upon the retina: but to the eye *E*, at a double diſtance, the ſame object is ſeen under the angle *A p C*, which is equal only to half the angle *A P C*, as is evident by the figure. The image *c b a* is likewiſe twice as large in the eye *D*, as the other image *c b a* is in the eye *E*. In both theſe repreſentations, a part of the image falls on the optic nerve, and the object in the correſponding part is inviſible.

As the ſenſe of ſeeing is allowed to be occaſioned by the impulſe of the rays from the viſible object upon the retina of the eye, and forming

the image of the object thereon, and that the retina is only the expanſion of the optic nerve all over the choroides; it ſhould ſeem ſurpriſing that the part of the image which falls on the optic nerve ſhould render the like part of the object inviſible; eſpecially as that nerve is allowed to be the inſtrument by which the impulſe and image are conveyed to the common ſenſory in the brain. But this difficulty vaniſhes, when we conſider that there is an artery within the trunk of the optic nerve, which entirely obſcures the image in that part, and conveys no ſenſation to the brain.

That the part of the image which falls upon the middle of the optic nerve is loſt, and conſequently the correſponding part of the object is rendered inviſible, is plain by experiment. For, if Fig. 2. a perſon fixes three patches, *A*, *B*, *C*, horizontally, upon a white wall, at the height of the eye, and the diſtance of about a foot from each other, and places himſelf before them, ſhutting the right eye, and directing the left towards the patch *C*, he will ſee the patches *A* and *C*, but the middle patch *B* will diſappear. Or, if he ſhuts his left eye, and directs the right towards *A*, he will ſee both *A* and *C*, but *B* will diſappear; and if he directs his eye towards *B*, he will ſee both *B* and *A*, but not *C*. For whatever patch is directly oppoſite to the optic nerve *N*, vaniſhes. This requires a little practice, after which he will find it eaſy to direct his eye, ſo as to loſe the ſight of whichever patch he pleaſes.

We are not commonly ſenſible of this diſappearance, becauſe the motions of the eye are ſo quick and inſtantaneous, that we no ſooner loſe the ſight of any part of an object, than we recover it again; much the ſame as in the twinkling of our eyes, for at each twinkling we are blinded; but it is ſo ſoon over, that we are ſcarce ever ſenſible of it.

Fig. 4. Why ſome eyes require ſpectacles.

Some eyes require the aſſiſtance of convex glaſſes to make them ſee objects diſtinctly, and others of concave. If either the cornea *a b c* or chryſtalline humour *e*, or both of them, be too flat, as in the eye *A*, their focus will not be on the retina, as at *d*, where it ought to be, in order to render viſion diſtinct; but beyond the eye, as at *f*. Conſequently thoſe rays which flow from the object *C*, and paſs through the humours of the eye, are not converged enough to unite at *d*; and therefore the obſerver can have but a very indiſtinct view of the object. This is remedied by placing a convex glaſs *g h* before the eye, which makes the rays converge ſooner, and imprints the image duly on the retina at *d*.

If either the cornea, or chryſtalline humour, or both of them, be too convex, as in the eye *f*, the rays that enter in from the object *C*, will be converged to a focus in the vitreous humour, as at *f*; and by diverging from thence to the retina, will form a very confuſed image thereon: and ſo, of courſe, the obſerver will have as confuſed a view of the object, as if his eye had been too flat. This inconvenience is remedied by placing a concave glaſs *g h* before the eye; which glaſs, by cauſing the rays to diverge between it and the eye, lengthens the focal diſtance ſo, that if the glaſs be properly choſen, the rays will unite at the retina, and form a diſtinct picture of the object upon it.

Such eyes as have their humours of a due convexity, cannot ſee any object diſtinctly at a leſs diſtance than ſix inches; and there are numberleſs objects too ſmall to be ſeen at that diſtance, becauſe they cannot appear under any ſenſible angle. The method of viewing ſuch minute objects is by a *microſcope*, of which there are three ſorts, viz. the *ſingle*, the *double*, and the *ſolar*.

Fig. 5. The *ſingle microſcope.*

The *ſingle microſcope*, is only a ſmall convex glaſs, as *c d*, having the object *a b* placed in its focus, and the eye at the ſame diſtance on the other ſide; ſo that the rays of each pencil, flowing from every point of the object on the ſide next the glaſs, may go on parallel in the ſpace between the eye and the glaſs; and then, by entering the eye at *C*, they will be converged to as many different points on the retina, and form a large inverted picture *A B* upon it, as in the figure.

To find how much this glaſs magnifies, divide the leaſt diſtance (which is about ſix inches) at which an object can be ſeen diſtinctly with the bare eye, by the focal diſtance of the glaſs; and the quotient will ſhew how much the glaſs magnifies the diameter of the object.

Fig. 6. The *double microſcope.*

The *double* or *compound microſcope*, conſiſts of an object-glaſs *c d*, and an eye-glaſs *e f*. The ſmall object *a b* is placed at a little greater diſtance from the glaſs *c d* than its principal focus, ſo that the pencils of rays flowing from the different points of the object, and paſſing through the glaſs, may be made to converge and unite in as many points between *g* and *b*, where the image of the object will be formed: which image is viewed by the eye through the eye-glaſs *e f*. For the eye-glaſs being ſo placed, that the image *g b* may be in its focus, and the eye much about the ſame diſtance on the other ſide, the rays of each pencil will be parallel, after going out of the eye-glaſs, as at *e* and *f*, till they come to the eye at *k*, where they will begin to converge by the refractive power of the humours; and after having croſſed each other in the pupil, and paſſed through the chryſtalline and vitreous humours they will be collected into points on the retina, and form the large inverted image *A B* thereon.

The magnifying power of this microſcope is as follows. Suppoſe the image *g h* to be ſix times the diſtance of the object *a b* from the object-glaſs *c d*; then will the image be ſix times the length of the object: but ſince the image could not be ſeen diſtinctly by the bare eye at a leſs diſtance than ſix inches, if it be viewed by an eye-glaſs *e f*, of one inch focus, it will thereby be brought ſix times nearer the eye; and conſequently viewed under an angle ſix times as large as before; ſo that it will be again magnified ſix times; that is, ſix times by the object-glaſs, and ſix times by the eye-glaſs, which multiplied into one another, makes 36 times; and ſo much is the object magnified in diameter more than what it appears to the bare eye; and conſequently 36 times 36, or 1296 times in ſurface.

But becauſe the extent or field of view is very ſmall in this microſcope, there are generally two eye-glaſſes placed ſometimes cloſe together, and ſometimes an inch aſunder; by which means, although the object appears leſs magnified, yet the viſible area is much enlarged by the interpoſition of a ſecond eye-glaſs; and conſequently a much pleaſanter view is obtained.

Fig. 7. The *ſolar microſcope.*

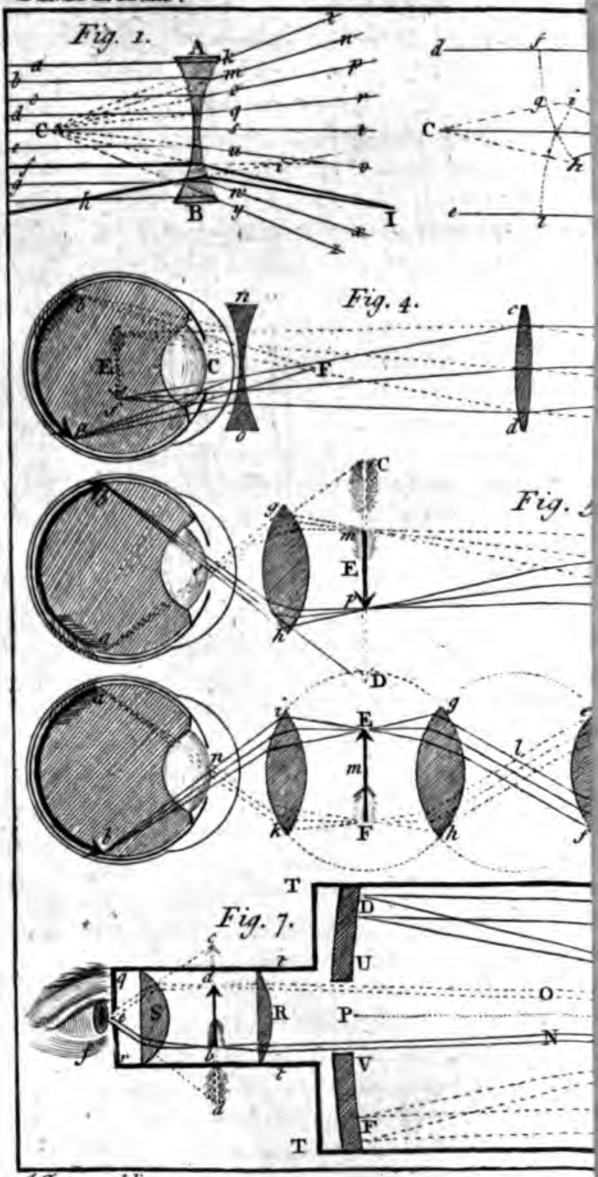
The *ſolar microſcope*, invented by Dr. *Lieberkhun*, is conſtructed in the following manner. Having procured a very dark room, let a round hole be made in the window-ſhutter, about three inches diameter, through which the ſun may caſt a cylinder of rays *A A* into the room. In this hole, place the end of a tube, containing two convex glaſſes and an object, viz. 1. A convex glaſs *a a*, of about two inches diameter, and three inches focal diſtance, is to be placed in that end of the tube which is put into the hole. 2. The object *b b*, being put between two glaſſes (which muſt be concave to hold it at liberty) is placed about two inches and a half from the glaſs *a a*. 3. A little more than a quarter of an inch from the object is placed the ſmall convex glaſs *c c*, whoſe focal diſtance is a quarter of an inch.

The tube may be ſo placed, when the ſun is low, that his rays *A A* may enter directly into it: but when he is high, his rays *B B* muſt be reflected into the tube by the plane mirrour or looking-glaſs *C C*.

Things being thus prepared, the rays that enter the tube will be conveyed by the glaſs *a a* towards the object *b b*, by which means it will be ſtrongly illuminated; and the rays *d* which flow from it, through the magnifying glaſs *c c*, will make a large inverted picture of the object at *D D*, which, being received on a white paper, will repreſent the object magnified in length, in proportion of the diſtance of the picture from the glaſs *c c*, to the diſtance of the object from the ſame glaſs. Thus, ſuppoſe the diſtance of the object from the glaſs to be $\frac{3}{10}$ parts of an inch, and the diſtance of the diſtinct picture to be 12 feet or 144 inches, in which there are 1440 tenths of an inch; and this number divided by 3 tenths, gives 480; which is the number of times the picture is longer or broader than the object; and the length multiplied by the breadth, ſhews how much the whole ſurface is magnified.

Before

PLATE XVIII.



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Before we enter upon the deſcription of teleſcopes, it will be proper to ſhew how the rays of light are affected by paſſing through concave glaſſes, and alſo by falling upon concave mirrours. Teleſcopes.

When parallel rays, as *a b c d e f g h*, paſs directly through a glaſs *A B*, which is equally concave on both ſides, they will diverge after paſſing through the glaſs, as if they had come from a radiant point *C*, in the center of the glaſs's concavity; which point is called the negative or virtual focus of the glaſs. Thus the ray *a*, after paſſing through the glaſs *A B*, will go on in the direction *k l*, as if it had proceeded from the point *C*, and no glaſs been in the way. The ray *b* will go on in the direction *m n*; the ray *c* in the direction *o p*, &c.—The ray *C*, that falls directly upon the middle of the glaſs, ſuffers no refraction in paſſing through it; but goes on in the ſame rectilineal direction, as if no glaſs had been in its way. Plate. XVIII. Fig. 1.

If the glaſs had been concave only on one ſide, and the other ſide quite plane, the rays would have diverged, after paſſing through it, as if they had come from a radiant point at double the diſtance of *C* from the glaſs; that is, as if the radiant had been at the diſtance of a whole diameter of the glaſs's concavity.

If rays come more converging to ſuch a glaſs, than parallel rays diverge after paſſing through it, they will continue to converge after paſſing through it; but will not meet ſo ſoon as if no glaſs had been in the way; and will incline towards the ſame ſide to which they would have diverged, if they had come parallel to the glaſs. Thus the rays *f* and *h*, going in a converging ſtate towards the edge of the glaſs at *B*, and converging more in their way to it than the parallel rays diverge after paſſing through it, they will go on converging after they paſs through it, though in a leſs degree than they did before, and will meet at *I*: but if no glaſs had been in their way, they would have met at *i*.

Fig. 2. When the parallel rays, as *d f a*, *C m b*, *e l c*, fall upon a concave mirror *A B* (which is not tranſparent, but has only the ſurface *A b B* of a clear poliſh) they will be reflected back from that mirror, and meet in a point *m*, at half the diſtance of the ſurface of the mirror from *C*, the center of its concavity: for they will be reflected at as great an angle from the perpendicular to the ſurface of the mirror, as they fell upon it, with regard to that perpendicular; but on the other ſide thereof. Thus, let *C* be the center of concavity of the mirror *A b B*, and let the parallel rays *d f a*, *C m b*, and *e l c*, fall upon it at the points *a b*, and *c*. Draw the lines *C i a*, *C m b*, and *C b c*, from the center *C* to theſe points; and all theſe lines will be perpendicular to the ſurface of the mirror, becauſe they proceed thereto like ſo many *radii* or ſpokes from its center. Make the angle *C a b* equal to the angle *d a C*, and draw the line *a m b*, which will be the direction of the ray *d f a*, after it is reflected from the point *a* of the mirror: ſo that the angle of incidence *d a C*, is equal to the angle of reflection *C a b*; the rays making equal angles with the perpendicular *C i a* on its oppoſite ſides.

Draw alſo the perpendicular *C b c* to the point *c*, where the ray *e l c* touches the mirror; and, having made the angle *C c i*, equal to the angle *C c e*, draw the line *c m i*, which will be the courſe of the ray *e l c*, after it is reflected from the mirror.

The ray *C m b* paſſes through the center of concavity of the mirror, and falls upon it, at *b*, the perpendicular to it; and is therefore reflected back from it in the ſame line *b m C*.

All theſe reflected rays meet in the point *m*; and in that point the image of the body which emits the parallel rays *d a*, *C b*, and *e c*, will be formed: which point is diſtant from the mirror equal to half the radius *b m C* of its concavity.

The rays which proceed from any celeſtial object may be eſteemed parallel at the earth; and therefore, the images of that object will be formed at *m*, when the reflecting ſurface of the concave mirror is turned directly towards the object. Hence, the focus *m* of parallel rays is not in the center of the mirror's concavity, but half way between the mirror and that center.

The rays which proceed from any remote terreſtrial object, are nearly parallel at the mirror; not ſtrictly ſo, but come diverging to it, in ſeparate pencils, or, as it were, bundles of rays, from each point of the ſide of the object next the mirror: and therefore they will not be converged to a point, at the diſtance of half the radius of the mirror's concavity from its reflecting ſurface; but into ſeparate points at a little greater diſtance from the mirror. And the nearer the object is to the mirror, the farther theſe points will be from it; and an inverted image of the object will be formed in them, which will ſeem to hang pendent in the air; and will be ſeen by an eye placed beyond it (with regard to the mirror) in all reſpects like the object, and as diſtinct as the object itſelf.

Fig. 3. Let *A c B* be the reflecting ſurface of a mirror, whoſe center of concavity is at *C*; and let the upright object *D E* be placed beyond the center *C*, and ſend out a conical pencil of diverging rays from its upper extremity *D*, to every point of the concave ſurface of the mirror *A c B*. But to avoid confuſion, we only draw three rays of that pencil, as *D A*, *D c*, *D B*.

From the center of concavity *C*, draw the three right lines *C A*, *C c*, *C B*, touching the mirror in the ſame points where the foreſaid rays touch it; and all theſe lines will be perpendicular to the ſurface of the mirror. Make the angle *C A d* equal to the angle *D A C*, and draw the right line *A d* for the courſe of the reflected ray *D A*: make the angle *C c d* equal to the angle *D c C*, and draw the right line *c d* for the courſe of the reflected ray *D d*: make alſo the angle *C B d* equal to the angle *D B C*, and draw the right line *B d* for the courſe of the reflected ray *D B*. All theſe reflected rays will meet in the point *d*, where they will form the extremity *d* of the inverted image *e d*, ſimilar to the extremity *D* of the upright object *D E*.

If the pencils of rays *E f*, *E g*, *E h* be alſo continued to the mirror, and their angles of reflection from it be made equal to their angles of incidence upon it, as in the former pencil from *D*, they will all meet at the point *e* by reflection, and form the extremity *e* of the image *e d*, ſimilar to the extremity *E* of the object *D E*.

And as each intermediate point af the object, between *D* and *E*, ſends out a pencil of rays in like manner to every part of the mirror, the rays of each pencil will be reflected back from it, and meet in all the intermediate points between the extremities *e* and *d* of the image; and ſo the whole image will be formed, not at *i*, half the diſtance of the mirror from its center of concavity *C*; but at a greater diſtance, between *i* and the object *D E*; and the image will be inverted with reſpect to the object.

This being well underſtood, the reader will eaſily ſee how the image is formed by the large concave mirror of the reflecting teleſcope, when he comes to the deſcription of that inſtrument.

When the object is more remote from the mirror than its center of concavity *C*, the image will be leſs than the object, and between the object and mirror: when the object is nearer than the center of concavity, the image will be more remote and bigger than the object: thus, if *D E* be the object, *e d* will be its image; for, as the object recedes from the mirror, the image approaches nearer to it; and as the object approaches nearer to the mirror, the image recedes farther from it; on account of the leſſer or greater divergency of the pencils of rays which proceed from the object; for, the leſs they diverge, the ſooner they are converged to points by reflection; and the more they diverge, the farther they muſt be reflected before they meet.

If the radius of the mirror's concavity and the diſtance of the object from it be known, the diſtance of the image from the mirror is found by this rule: divide the product of the diſtance and radius by double the diſtance made leſs by the radius, and the quotient is the diſtance required.

If the object be in the center of the mirror's concavity, the image and object will be coincident, and equal in bulk.

If a man places himſelf directly before a large concave mirror, but farther from it than its center of concavity, he will ſee an inverted image of himſelf in the air, between him and the mirror, of a leſs ſize than himſelf. And if he holds out his hand towards the mirror, the hand of the image will come out towards his hand, and coincide with it, of an equal bulk, when his hand is in the center of concavity; and he will imagine he may ſhake hands with his image. If he reaches his hand farther, the hand of the image will paſs by his hand, and come between his hand and his body: and if he moves his hand towards either ſide, the hand of the image will move towards the other; ſo that whatever way the object moves, the image will move the contrary way.

All the while a by-ſtander will ſee nothing of the image, becauſe none of the reflected rays that form it enter his eyes.

If a fire be made in a large room, and a ſmooth mahogany table be placed at a good diſtance near the wall, before a large concave mirror, ſo placed, that the light of the fire may be reflected from the mirror to its focus upon the table; if a perſon ſtands by the table, he will ſee nothing upon it but a longiſh beam of light: but if he ſtands at a diſtance towards the fire, not directly between the fire and mirror, he will ſee an image of the fire upon the table, large and erect. And if another perſon, who knows nothing of this matter beforehand, ſhould chance to come into the room, and ſhould look from the fire towards the table, he

Plate XVIII.

he would be ſtartled at the appearance; for the table would ſeem to be on fire, and by being near the wainſcot, to endanger the whole houſe. In this experiment, there ſhould be no light in the room but what proceeds from the fire; and the mirror ought to be at leaſt fifteen inches in diameter.

If the fire be darkened by a ſcreen, and a large candle be placed at the back of the ſcreen; a perſon ſtanding by the candle will ſee the appearance of a fine large ſtar, or rather planet, upon the table, as bright as Venus or Jupiter. And if a ſmall wax taper (whoſe flame is much leſs than the flame of the candle) be placed near the candle, a ſatellite to the planet will appear on the table: and if the taper be moved round the candle, the ſatellite will go round the planet.

For theſe two pleaſing experiments, I am indebted to the late reverend Dr. LONG, *Lowndes*'s profeſſor of aſtronomy at Cambridge, who favoured me with the ſight of them, and many more of his curious inventions.

The refracting teleſcope.

In a *refracting teleſcope*, the glaſs which is neareſt the object in viewing it, is called the *object-glaſs*; and that which is neareſt the eye, is called *the eye-glaſs*. The object-glaſs muſt be convex, but the eye-glaſs may be either convex or concave: and generally, in looking through a teleſcope, the eye is in the focus of the eye-glaſs; though that is not very material: for the diſtance of the eye, as to diſtinct viſion, is indifferent, provided the rays of the pencils fall upon it parallel: only, the nearer the eye is to the end of the teleſcope, the larger is the ſcope or area of the field of view.

Fig. 4.

Let *c d* be a convex-glaſs fixed in a long tube, and have its focus at *E*. Then, a pencil of rays *g b i*, flowing from the upper extremity *A* of the remote object *A B*, will be ſo refracted by paſſing through the glaſs, as to converge and meet in the point *f*; whilſt the pencil of rays *k l m* flowing from the lower extremity *B*, of the ſame object *A B*, and paſſing through the glaſs, will converge and meet in the point *e*: and the images of the points *A* and *B*, will be formed in the points *f* and *e*. And as all the intermediate points of the object, between *A* and *B*, ſend out pencils of rays in the ſame manner, a ſufficient number of theſe pencils will paſs through the object-glaſs *c d*, and converge to as many intermediate points between *e* and *f*; and ſo will form the whole inverted image *e E f*, of the diſtinct object. But becauſe this image is ſmall, a concave glaſs *n o* is ſo placed in the end of the tube next the eye, that its virtual focus may be at *F*. And as the rays of the pencils paſs converging through the concave glaſs, but converge leſs after paſſing through it than before, they go on further, as to *b* and *a*, before they meet; and the pencils themſelves being made to diverge by paſſing through the concave glaſs, they enter the eye, and form the large picture *a b* upon the retina, whereon it is magnified under the angle *b F a*.

But this teleſcope has one inconveniency which renders it unfit for moſt purpoſes, which is, that the pencils of rays being made to diverge by paſſing through the concave glaſs *n o*, very few of them can enter the pupil of the eye; and therefore the field of view is but very ſmall, as is evident by the figure. For none of the pencils which flow either from the top or bottom of the object *A B* can enter the pupil of the eye at *C*, but are all ſtopt by falling upon the iris above and below the pupil: and therefore, only the middle part of the object can be ſeen when the teleſcope lies directly towards it, by means of thoſe rays which proceed from the middle of the object. So that to ſee the whole of it, the teleſcope muſt be moved upwards and downwards, unleſs the object be very remote; and then it is never ſeen diſtinctly.

Fig. 5. This inconvenience is remedied by ſubſtituting a convex eye-glaſs, as *g h*, in place of the concave one; and fixing it ſo in the tube, that its focus may be coincident with the focus of the object-glaſs *c d*, as at *E*. For then, the rays of the pencils flowing from the object *A B*, and paſſing through the object-glaſs *c d*, will meet in its focus, and form the inverted image *m E p*: and as the image is formed in the focus of the eye-glaſs *g h*, the rays of each pencil will be parallel, after paſſing through that glaſs; but the pencils themſelves will croſs in its focus, on the other ſide, as at *e*: and the pupil of the eye being in this focus, the image will be viewed through the glaſs, under the angle *g e h*; and being at *E*, it will appear magnified, ſo as to fill the whole ſpace *C m e p D*.

But, as this teleſcope inverts the image with reſpect to the object, it gives an unpleaſant view of terreſtrial objects; and is only fit for viewing the heavenly bodies, in which we regard not their poſition, becauſe their being inverted does not appear, on account of their being round. But whatever way the object ſeems to move, this teleſcope muſt be moved the contrary way, in order to keep ſight of it; for, ſince the object is inverted, its motion will be ſo too.

The magnifying power of this teleſcope is, as the focal diſtance of the object-glaſs to the focal diſtance of the eye-glaſs. Therefore, if the former be divided by the latter, the quotient will expreſs the magnifying power.

When we ſpeak of magnifying by a teleſcope or microſcope, it is only meant with regard to the diameter, not to the area or ſolidity of the object. But as the inſtrument magnifies the vertical diameter, as much as it does the horizontal, it is eaſy to find how much the whole viſible area or ſurface is magnified: for, if the diameters be multiplied into one another, the product will expreſs the magnification of the whole viſible area. Thus, ſuppoſe the focal diſtance of the object-glaſs be ten times as great as the focal diſtance of the eye-glaſs; then, the object will be magnified ten times, both in length and breadth: and 10 multiplied by 10, produces 100; which ſhews, that the area of the object will appear 100 times as big when ſeen through ſuch a teleſcope, as it does to the bare eye.

Hence it appears, that if the focal diſtance of the eye-glaſs, were equal to the focal diſtance of the object-glaſs, the magnifying power of the teleſcope would be nothing.

This teleſcope may be made to magnify in any given degree, provided it be of a ſufficient length. For, the greater the focal diſtance of the object-glaſs, the leſs may be the focal diſtance of the eye-glaſs; though not directly in proportion. Thus, an object-glaſs, of 10 feet focal diſtance, will admit of an eye glaſs whoſe focal diſtance is little more than 2½ inches; which will magnify near 48 times: but an object-glaſs, of 100 feet focus, will require an eye-glaſs ſomewhat more than 6 inches; and will therefore magnify almoſt 200 times.

A teleſcope for viewing terreſtrial objects, ſhould be ſo conſtructed, as to ſhew them in their natural poſture. And this is done by one object-glaſs Fig. 6. *c d*, and three eye-glaſſes *e f*, *g h*, *i k*, ſo placed, that the diſtance between any two, which are neareſt to each other, may be equal to the ſum of their focal diſtances; as in the figure, where the focus of the glaſſes *c d* and *e f* meet at *F*, thoſe of the glaſſes *e f* and *g h*, meet at *l*, and of *g h* and *i k*, at *m*; the eye being at *n*, in or near the focus of the eye-glaſs *i k*, on the other ſide. Then, it is plain, that theſe pencils of rays, which flow from the object *A B*, and paſs through the object-glaſs *c d*, will meet and form an inverted image *C F D* in the focus of that glaſs; and the image being alſo in the focus of the glaſs *e f*, the rays of the pencils will become parallel, after paſſing through that glaſs, and croſs at *l*, in the focus of the glaſs *e f*; from whence they paſs on to the next glaſs *g h*, and by going through it they are converged to points in its other focus, where they form an erect image *E m F*, of the object *A B*: and as this image is alſo in the focus of the eye-glaſs *i k*, and the eye on the oppoſite ſide of the ſame glaſs; the image is viewed through the eye-glaſs in this teleſcope, in the ſame manner as through the eye-glaſs in the former one; only in a contrary poſition, that is, in the ſame poſition with the object.

The three glaſſes next the eye, have all their focal diſtances equal: and the magnifying power of this teleſcope is found the ſame way as that of the laſt above; viz. by dividing the focal diſtance of the object-glaſs *c d*, by the focal diſtance of the eye-glaſs *i k*, or *g h*, or *e f*, ſince all theſe three are equal.

When the rays of light are ſeparated by refraction, they become coloured, and if they be united again, they will be a perfect white. But

Why the object appears coloured when ſeen through a teleſcope.

thoſe rays which paſs through a convex glaſs, near its edges are more unequally refracted than thoſe which are nearer the middle of the glaſs. And when the rays of any pencil are unequally refracted by the glaſs, they do not all meet again in one and the ſame point, but in ſeparate points; which makes the image indiſtinct, and coloured, about its edges. The remedy is, to have a plate with a ſmall round hole in its middle, fixed in the tube at *m*, parallel to the glaſſes. For, the wandering rays about the edges of the glaſſes will be ſtopt, by the plate, from coming to the eye: and none admitted but thoſe which come through the middle of the glaſs, or at leaſt at a good diſtance from its edges, and paſs through the hole in the middle of the plate. But this circumſcribes the image, and leſſens the field of view, which would be much larger if the plate could be diſpenſed with.

The reflecting teleſcope,

The great inconvenience attending the management of long teleſcopes of this kind, has brought them much into diſuſe ever ſince the *reflecting teleſcope* was invented. For one of this ſort, ſix feet in length, magnifies as much as one of the other an hundred. It was invented by Sir *Iſaac Newton*, but has received conſiderable improvements ſince his time; and is now generally conſtructed in the following manner, which was firſt propoſed by Dr. *Gregory*.

Fig. 7.

At the bottom of the great tube *T T T T* is placed the large concave mirror *D U V F*, whoſe principal focus is at *m*; and in its middle is a round hole *P*, oppoſite to which is placed the ſmall mirror *L*, concave toward the great one; and ſo fixed to a ſtrong wire *M*, that it may be moved farther from the great mirror, or nearer to it, by means of a long ſcrew on the outſide of the tube, keeping its axis ſtill in the ſame line *P m n* with that of the great one.—Now, ſince in viewing a very remote object, we can ſcarce ſee a point of it but what is at leaſt as broad as the great mirror, we may conſider the rays of each pencil, which flow from every point of the object, to be parallel to each other, and to cover the whole reflecting ſurface *D U V F.* But to avoid confuſion in the figure, we ſhall only draw two rays of a pencil flowing from each extremity of the object into the great tube, and trace their progreſs, through all their reflections and refractions, to the eye *f*, at the end of the ſmall tube *t t*, which is joined to the great one.

Let us then ſuppoſe the object *A B* to be at ſuch a diſtance, that the rays *C* may flow from its lower extremity *B*, and the rays *E* from its upper extremity *A*. Then the rays *C* falling parallel upon the great mirror at *D*, will be thence reflected, converging in the direction *D G*; and by croſſing at *I* in the principal focus of the mirror, they will form the upper extremity *I* of the inverted image *I K*, ſimilar to the lower extremity *B* of the object *A B*: and paſſing on to the concave mirror *L* (whoſe focus is at *n*) they will fall upon it at *g*, and be thence reflected converging, in the direction *g N*, becauſe *g m* is longer than *g n*; and paſſing through the hole *P* in the large mirror, they would meet ſomewhere about *r*, and form the lower extremity *d* of the erect image *a d*, ſimilar to the lower extremity *B* of the object *A B*. But by paſſing through the plano-convex-glaſs *R* in their way, they form that extremity of the image at *b*. In like manner, the rays *E*, which come from the top of the object *A B*, and fall parallel upon the great mirror at *F*, are thence reflected converging to its focus, where they form the lower extremity *K* of the inverted image *I K*, ſimilar to the upper extremity *A* of the object *A B*; and thence paſſing on to the ſmall mirror *L*, and falling upon it at *b*, they are thence reflected in the converging ſtate *b O*; and going on through the hole *P* of the great mirror, they will meet ſomewhere about *q*, and form there the upper extremity *a* of the erect image *a d*, ſimilar to the upper extremity *A* of the object *A B*: but by paſſing through the convex glaſs *R* in their way, they meet and croſs ſooner, as at *a*, where that point of the erect image is formed.—The like being underſtood of all thoſe rays which flow from the intermediate points of the object, between *A* and *B*, and enter the tube *T T*; all the intermediate points of the image between *a* and *b* will be formed: and the rays paſſing on from the image through the eye-glaſs *S*, and through a ſmall hole *e* in the end of the leſſer tube *t t*, they enter the eye *f*, which ſees the image *a d* (by means of the eye-glaſs) under the large angle *c e d*, and magnified in length, under that angle from *c* to *d*.

In the beſt reflecting teleſcopes, the focus of the ſmall mirror is never coincident with the focus *m* of the great one, where the firſt image *I K* is formed, but a little beyond it (with reſpect to the eye) as at *n*: the conſequence of which is, that the rays of the pencils will not be parallel after reflection from the ſmall mirror, but converge ſo as to meet in points about *q*, *e*, *r*; where they will form a larger upright image than *a d*, if the glaſs *R* was not in their way: and this image might be viewed by means of a ſingle eye-glaſs properly placed between the image and the eye: but then the field of view would be leſs, and conſequently not ſo pleaſant; for which reaſon, the glaſs *R* is ſtill retained, to enlarge the ſcope or area of the field.

To find the magnifying power of this teleſcope, multiply the focal diſtance of the great mirror by the diſtance of the ſmall mirror from the image next the eye, and multiply the focal diſtance of the ſmall mirror by the focal diſtance of the eye-glaſs: then, divide the product of the former multiplication by the product of the latter, and the quotient will expreſs the magnifying power.

I ſhall here ſet down the dimenſions of one of Mr. *Short*'s reflecting teleſcopes, as deſcribed in Dr. *Smith*'s Optics.

The focal diſtance of the great mirror 9.6 inches, its breadth 2.3; the focal diſtance of the ſmall mirror 1.5, its breadth 0.6: the breadth of the hole in the great mirror 0.5; the diſtance between the ſmall mirror and the next eye-glaſs 14.2; the diſtance between the two eye-glaſſes 2.4; the focal diſtance of the eye-glaſs next the metals 3.8; and the focal diſtance of the eye-glaſs next the eye 1.1.

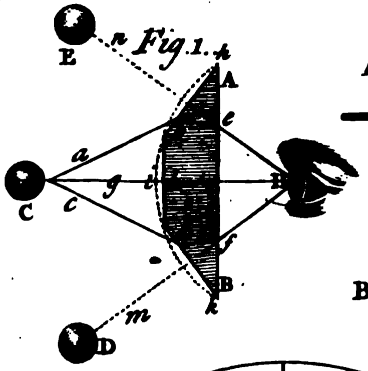
One great advantage of the reflecting teleſcope is, that it will admit of an eye-glaſs of a much ſhorter focal diſtance than a refracting teleſcope will; and, conſequently, it will magnify ſo much the more: for the rays are not coloured by reflection from a concave mirror, if it be ground to a true figure, as they are by paſſing through a convex-glaſs, let it be ground ever ſo true.

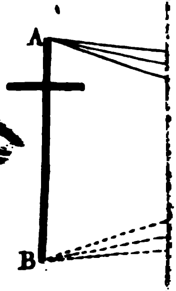
The adjuſting ſcrew on the outſide of the great tube fits this teleſcope to all ſorts of eyes, by bringing the ſmall mirror either nearer to the eye, or removing it farther: by which means, the rays are made to diverge a little for ſhort-ſighted eyes, or to converge for thoſe of a long ſight.

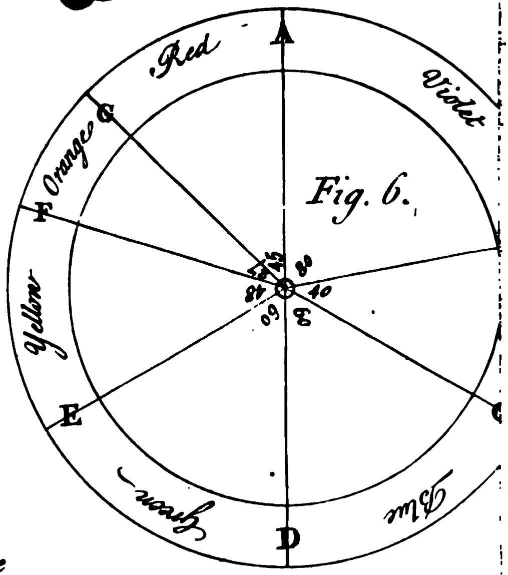
The nearer an object is to the teleſcope, the more its pencils of rays will diverge before they fall upon the great mirror, and therefore they will be the longer of meeting in points after reflection; ſo that the firſt image *I K* will be formed at a greater diſtance from the large mirror, when the object is near the teleſcope, than when it is very remote. But as this image muſt be formed farther from the ſmall mirror than its principal focus *n*, this mirror muſt be always ſet at a greater diſtance from the large one, in viewing near objects, than in viewing remote ones. And this is done by turning the ſcrew on the outſide of the tube, until the ſmall mirror be ſo adjuſted, that the object (or rather its image) appears perfect.

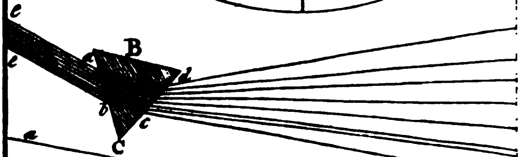
In looking through any teleſcope towards an object, we never ſee the object itſelf, but only that image of it which is formed next the eye in the teleſcope. For, if a man holds his finger or a ſtick between his bare eye and an object, it will hide part (if not the whole) of the object from his view. But if he ties a ſtick acroſs the mouth of a teleſcope, before the object-glaſs, it will hide no part of the imaginary object he ſaw through the teleſcope before, unleſs it covers the whole mouth of the tube: for, all the effect will be, to make the object appear dimmer, becauſe it intercepts part of the rays. Whereas, if he puts only a piece of wire acroſs the inſide of the tube, between the eye-glaſs and his eye, it will hide part of the object which he thinks he ſees: which proves that he ſees not the real object, but its image. This is alſo confirmed by means of the ſmall

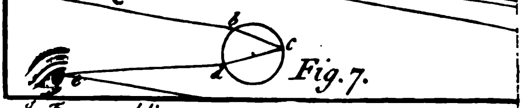
PLATE XIX.











J. Ferguson delin.

ſmall mirror *L*, in the reflecting teleſcope, which is made of opake metal, and ſtands directly between the eye and the object towards which the teleſcope is turned; and will hide the whole object from the eye at *e*, if the two glaſſes *R* and *S* are taken out of the tube.

Plate XIX. Fig. 1. *The multiplying-glaſs.*

The multiplying glaſs is made by grinding down the round ſide *b i k* of a convex glaſs *A B* into ſeveral flat ſurfaces, as *b b*, *b l d*, *d k*. An object *C* will not appear magnified, when ſeen through this glaſs, by the eye at *H*; but it will appear multiplied into as many different objects as the glaſs contains plane ſurfaces. For, ſince rays will flow from the object *C* to all parts of the glaſs, and each plane ſurface will refract theſe rays to the eye, the ſame object will appear to the eye, in the direction of the rays which enter it through each ſurface. Thus, a ray *g i H*, falling perpendicularly on the middle ſurface will go through the glaſs to the eye without ſuffering any refraction; and will therefore ſhew the object in its true place at *C*: whilſt a ray *a b* flowing from the ſame object, and falling obliquely on the plane ſurface *b b*, will be refracted in the direction *b e*, by paſſing through the glaſs; and upon leaving it, will go on to the eye in the direction *e H*; which will cauſe the ſame object *C* to appear alſo at *E*, in the direction of the ray *H e*, produced in the right line *H e n*. And the ray *c d*, flowing from the object *C*, and falling obliquely on the plane ſurface *d k*, will be refracted (by paſſing through the glaſs and leaving it at *f*) to the eye at *H*; which will cauſe the ſame object to appear at *D*, in the direction *H f m*.— If the glaſs be turned round the line *g l H*, as an axis, the object *C* will keep its place, becauſe the ſurface *b l d* is not removed; but all the other objects will ſeem to go round *C*, becauſ the oblique planes, on which the rays *a b*, *c d* fall, will go round by the turning of the glaſs.

Fig. 2. The *camera obſcura.*

The *camera obſcura* is made by a convex glaſs *C D*, placed in a hole of a window-ſhutter. Then, if the room be darkened ſo as no light can enter but what comes through the glaſs, the pictures of all the objects (as fields, trees, buildings, men, cattle, &c.) on the outſide, will be ſhewn in an inverted order, on a white paper placed at *G H* in the focus of the glaſs; and will afford a moſt beautiful and perfect piece of perſpective or landſcape of whatever is before the glaſs; eſpecially if the ſun ſhines upon the objects.

If the convex glaſs *C D* be placed in a tube in the ſide of a ſquare box, within which is the plane mirror *E F*, reclining backwards in an angle of 45 degrees from the perpendicular *k q*, the pencils of rays flowing from the outward objects, and paſſing through the convex glaſs to the plane mirror, will be reflected upwards from it, and meet in points, as *I* and *K* (at the ſame diſtance that they would have meet at *H* and *G*, if the mirror had not been in the way) and will form the aforeſaid images on an oiled paper ſtretched horizontally in the direction *I K*; on which paper, the out-lines of the images may be eaſily drawn with a black lead pencil; and then copied on a clean ſheet, and coloured by art, as the objects themſelves are by nature.—In this machine, it is uſual to place a plane glaſs, unpoliſhed, in the horizontal ſituation *I K*, which glaſs receives the images of the outward objects; and their outlines may be traced upon it by a black-lead pencil.

N. B. The tube in which the convex glaſs *C D* is fixed, muſt be made to draw out, or puſh in, ſo as to adjuſt the diſtance of that glaſs from the plane mirror, in proportion to the diſtance of the outward objects; which the operator does, until he ſees their images diſtinctly painted on the horizontal glaſs at *I K*.

The forming a horizontal image, as *I K*, of an upright object *A B*, depends upon the angles of incidence of the rays upon the plane mirror *E F*, being equal to their angles of reflection from it. For, if a perpendicular be ſuppoſed to be drawn to the ſurface of the plane mirror at *e*, where the ray *A a C e* falls upon it, that ray will be reflected upwards in an equal angle with the other ſide of the perpendicular, in the line *e d I*. Again, if a perpendicular be drawn to the mirror from the point *f*, where the ray *A b f* falls upon it, that ray will be reflected in an equal angle from the other ſide of the perpendicular, in the line *f b I*. And if a perpendicular be drawn from the point *g*, where the ray *A c g* falls upon the mirror, that ray will be reflected in an equal angle from the other ſide of the perpendicular, in the line *g i I*. So that all the rays of the pencil *a b c*, flowing from the upper extremity of the object *A B*, and paſſing through the convex glaſs *C D*, to the plane mirror *E F*, will be reflected from the mirror and meet at *I*, where they will form the extremity *I* of the image *I K*, ſimilar to the extremity *A* of the object *A B*. The like is to be underſtood of the pencil *q r s*, flowing from the lower extremity of the object *A B*, and meeting at *K* (after reflection from the plane mirror) the rays form the extremity *K* of the image, ſimilar to the extremity *B* of the object: and ſo of all the pencils that flow from the intermediate points of the object to the mirror, through the convex glaſs.

The *opera-glaſs.*

If a convex glaſs, of a ſhort focal diſtance, be placed near the plane mirror, in the end of a ſhort tube, and a convex glaſs be placed in a hole in the ſide of the tube, ſo as the image may be formed between the laſt-mentioned convex glaſs, and the plane mirror, the image being viewed through this glaſs will appear magnified. —In this manner the *opera-glaſſes* are conſtructed; with which a gentleman may look at any lady at a diſtance in the company, and the lady know nothing of it.

The common *looking glaſs.*

The image of any object that is placed before a plane mirror, appears as big to the eye as the object itſelf; and is erect, diſtinct, and ſeemingly as far behind the mirror, as the object is before it: and that part of the mirror, which reflects the image of the object to the eye (the eye being ſuppoſed equally diſtant from the glaſs with the object) is juſt half as long and half as broad as the object itſelf. Let *A B* be an object placed before the reflecting ſurface *g b i* of the plane mirror *C D*; and let the eye be at *o*. Let *A b* be a ray of light flowing from the top *A* of the object, and falling upon the mirror at *b*: and *b m* be a perpendicular to the ſurface of the mirror at *b*, the ray *A b* will be reflected from the mirror to the eye at *o*, making an angle *m b o* equal to the angle *A b m*: then will the top of the image *E* appear to the eye in the direction of the reflected ray *o b* produced to *E*, where the right line *A p E*, from the top of the object, cuts the right line *o b E*, at *E*. Let *B i* be a ray of light proceeding from the foot of the object at *B* to the mirror at *i*, and *n i* a perpendicular to the mirror from the point *i*, where

Fig. 3.

where the ray *B i* falls upon it: this ray will be reflected in the line *i o*, making an angle *n i o*, equal to the angle *B i n*, with that perpendicular, and entering the eye at *o*: then will the foot *F* of the image appear in the direction of the reflected ray *o i*, produced to *F*, where the right line *B F* cuts the reflected ray produced to *F*. All the other rays that flow from the intermediate points of the object *A B*, and fall upon the mirror between *b* and *i* will be reflected to the eye at *o*; and all the intermediate points of the image *E F* will appear to the eye in the direction-line of theſe reflected rays produced. But all the rays that flow from the object, and fall upon the mirror above *b*, will be reflected back above the eye at *o*; and all the rays that flow from the object, and fall upon the mirror below *i*, will be reflected back below the eye at *o*: ſo that none of the rays that fall above *b*, or below *i*, can be reflected to the eye at *o*; and the diſtance between *b* and *i* is equal to half the length of the object *A B*.

A man will ſee his image in a plane looking-glaſs, that is but half his height. Fig. 4.

Hence it appears, that if a man ſee his whole image in a plane looking-glaſs, the part of the glaſs that reflects his image muſt be juſt half as long and half as broad as himſelf, let him ſtand at any diſtance from it whatever; and that his image muſt appear juſt as far behind the glaſs as he is before it. Thus, the man *A B* viewing himſelf in the plane mirror *C D*, which is juſt half as long as himſelf, ſees his whole image as at *E F*, behind the glaſs, exactly equal to his own ſize. For, a ray *A C* proceeding from his eye at *A*, and falling perpendicularly upon the ſurface of the glaſs at *C*, is reflected back to his eye in the ſame line *C A*; and the eye of his image will appear at *E*, in the ſame line produced to *E*, beyond the glaſs. And a ray *B D*, flowing from his foot, and falling obliquely on the glaſs at *D*, will be reflected as obliquely on the other ſide of the perpendicular *a b D*, in the direction *D A*; and the foot of his image will appear at *F*, in the direction of the reflected ray *A D*, produced to *F*, where it is cut by the right line *B G F*, drawn parallel to the right line *A C E*. Juſt the ſame as if the glaſs were taken away, and a real man ſtood at *F*, equal in ſize to the man ſtanding at *B*: for to his eye at *A*, the eye of the other man at *E* would be ſeen in the direction of the line *A C E*; and the foot of the man at *F* would be ſeen by the eye *A*, in the direction of the line *A D F*.

If the glaſs be brought nearer the man *A B*, as ſuppoſe to *c b*, he will ſee his image as at *C D G*: for the reflected ray *C A* (being perpendicular to the glaſs) will ſhew the eye of the image as at *C*; and the incident ray *B b*, being reflected in the line *b A*, will ſhew the foot of his image as at *G*; the angle of reflection *a b A* being always equal to the angle of incidence *B b a*: and ſo of all the intermediate rays from *A* to *B*. Hence, if the man *A B* advances towards the glaſs *C D*, his image will approach towards it; and if he recedes from the glaſs, his image will alſo recede from it.

Having already ſhewn, that the rays of light are refracted when they paſs obliquely through different mediums, we come now to prove that ſome rays are more refrangible than others; and that, as they are differently refracted, they excite in our minds the ideas of different colours. This will account for the colours ſeen about the edges of the images of thoſe objects which are viewed through ſome teleſcopes.

Let the ſun ſhine into a dark room through a ſmall hole, as at *e e*, in a window-ſhutter; and place a triangular priſm *BC* in the beam of rays *A*, in ſuch a manner, that the beam may fall obliquely on one of the ſides *a b C* of the priſm. Fig. 5.
The rays will ſuffer different refractions by paſſing through the priſm, ſo that inſtead of going all out of it on the ſide *d c C*, in one direction, they will go on from it in the different directions repreſented by the lines *f*, *g*, *h*, *i*, *k*, *l*, *m*, *n*; and falling upon the oppoſite ſide of the room, or on white paper placed as at *p q* to receive them, they will paint upon it a ſeries of moſt beautiful lively colours (not to be equalled by art) in this order, viz thoſe rays which are leaſt refracted by the priſm, and will therefore go on between the lines *n* and *m*, will be of a very bright intenſe red at *n*, degenerating from thence gradually into an orange colour, as they are nearer the line *m*: the next will be of a fine orange colour at *m*, and from thence degenerate into a yellow colour towards *l*: at *l* they will be of a fine yellow, which will incline towards a green, more and more, as they are nearer and nearer *k*: at *k* they will be a pure green, but from thence towards *i* they will incline gradually to a blue: at *i* they will be a perfect blue, inclining to an indigo colour from thence towards *h*: at *h* they will be quite the colour of indigo, which will gradually change towards a violet, the nearer they are to *g*: and at *g* they will be of a fine violet colour, which will incline gradually to a red as they come nearer to *f*, where the coloured image ends.

The priſm.

The colours of the light.

There is not an equal quantity of rays in each of theſe colours; for, if the oblong image *p q* be divided into 360 equal parts, the red ſpace

R will take up 45 of theſe parts: the orange *O*, 27; the yellow *Y*, 48; the green *G*, 60; th blue *B*, 60; the indigo *I*, 40; and the violet *V*, 80; all which ſpaces are as nearly proportioned i the figure as the ſmall ſpace *q p* would admit of.

If all theſe colours be blended together agai, they will make a pure white; as is proved thu Take away the paper on which the colours fell, and place a large convex glaſs *D* in the rays *f*, *g*, *b*, &c. which will refract them ſo, as to make them unite and croſs each other at *W*: where, if a white paper be placed to receive them, they will excite the idea of a ſtrong lively white. But if the paper be placed farther from the glaſs, as at *r s*, the different colours will appear again upon it, in an inverted order, occaſioned by the rays croſſing at *W*.

As white is a compoſition of all colours, ſo black is a privation of them all, and, therefore, properly no colour.

Fig. 6. Let two concentric circles be drawn on a ſmooth round board *A B C D E F G*, and the outermoſt of them divided into 360 equal parts or degrees: then, draw ſeven right lines, as ⊙ *A*, ⊙ *B*, &c. from the center to the outermoſt circle; making the lines ⊙ *A* and ⊙ *B* include 80 degrees of that circle; the lines ⊙ *B* and ⊙ *C* 40 degrees; ⊙ *C* and ⊙ *D* 60; ⊙ *D* and ⊙ *E* 60; ⊙ *E* and ⊙ *F* 48; ⊙ *F* and ⊙ *G* 27; ⊙ *G* and ⊙ *A* 45. Then, between theſe two circles, paint the ſpace *A G* red, inclining to orange near *G*; *G F* orange, inclining to yellow near *F*; *F E* yellow, inclining to green near *E*; *E D* green, inclining to blue near *D*; *D C* blue, inclining to indigo near *C*; *C B* indigo, inclining to violet near *B*; and *B A* violet, inclining to a ſoft red near *A*. This done, paint all that part of the board black

which lies within the inner circle; and putting an axis through the center of the board, let it be turned very ſwiftly round that axis, ſo as the rays proceeding from the above colours, may be all blended and mixed together in coming to the eye; and then, the whole coloured part will appear like a white ring, a little greyiſh; not perfectly white, becauſe no colours prepared by art are perfect.

All the priſmatic colours blended together, make a white.

Any of theſe colours, except red and violet, may be made by mixing together the two contiguous priſmatic colours. Thus, yellow is made by mixing together a due proportion of orange and green; and green may be made by a mixture of yellow and blue.

All bodies appear of that colour, whoſe rays they reflect moſt; as a body appears red when it reflects moſt of the red-making rays, and abſorbs the reſt.

Any two or more colours that are quite tranſparent by themſelves, become opake when put together. Thus, if water or ſpirits of wine be tinged red, and put in a phial, every object ſeen through it will appear red; becauſe it lets only the red rays paſs through it, and ſtops all the reſt. If water or ſpirits be tinged blue, and put in a phial, all objects ſeen through it will appear blue, becauſe it tranſmits only the blue rays, and ſtops all the reſt. But if theſe two phials are held cloſe together, ſo as both of them may be between the eye and object, the object will no more be ſeen through them than through a plate of metal; for whatever rays are tranſmitted through the fluid in the phial next the object, are ſtopped by that in the phial next the eye. In this experiment, the phials ought not to be round, but ſquare; becauſe nothing but the

Tranſparent colours become opake if put together.

light itſelf can be ſeen through a round tranſparent body, at any diſtance.

As the rays of light ſuffer different degrees of refraction by paſſing obliquely through a priſm, or through a convex glaſs, and are thereby ſeparated into all the ſeven original or primary colours; ſo they alſo ſuffer different degrees of refraction by paſſing through drops of falling rain; and then, being reflected towards the eye, from the ſides of theſe drops which are fartheſt from the eye, and again refracted by paſſing out of theſe drops into the air, in which refracted directions they come to the eye; they make all the colours to appear in the form of a fine arch in the heavens, which is called the *rain-bow*.

Fig. 7.

There are always two rain-bows ſeen together, the interior of which is formed by the rays *a b*, which falling upon the upper part *b*, of the drop *b c d*, are refracted into the line *b c* as they enter the drop, and are reflected from the back of it at *c*, in the line *c d*, and then, by paſſing out of the drop into air, they are again refracted at *d*; and from thence they paſs on to the eye at *e*: ſo that to form the interior bow, the rays ſuffer two refractions, as at *b* and *d*; and one reflection, as at *c*.

Fig. 8.

The exterior bow is formed by rays which ſuffer two reflections, and two refractions; which is the occaſion of its being leſs vivid than the interior, and alſo of its colours being inverted with reſpect to thoſe of the interior. For, when a ray *a b* falls upon the lower part of the drop *b c d e*, it is refracted into the direction *b c* by entering the drop; and paſſing on to the back of the drop at *c*, it is thence reflected in the line *c d*, in which direction it is impoſſible for it to enter the eye at *f*: but by being again reflected from the point *d* of the drop, it goes on in the drop to *e*, where it paſſes out of the drop into the air, and is there refracted downward to the eye, in the direction *ef*.

L E C T. VIII. AND IX.

The deſcription and uſe of the globes, and armillary ſphere.

The terreſtrial globe.

IF a map of the world be accurately delineated on a ſpherical ball, the ſurface thereof will repreſent the ſurface of the earth: for the higheſt hills are ſo inconſiderable with reſpect to the bulk of the earth, that they take off no more from its roundneſs, than grains of ſand do from the roundneſs of a common globe; for the diameter of the earth is 8000 miles in round numbers, and no known hill upon it is three miles in perpendicular height.

Proof of the earth's being globular.

That the earth is ſpherical, or round like a globe, appears, 1. From its caſting a round ſhadow upon the moon, whatever ſide be turned towards her when ſhe is eclipſed. 2. From its having been ſailed round by ſeveral perſons. 3. From our ſeeing the farther, the higher we ſtand. 4. From our ſeeing the maſts of a ſhip, whilſt the hull is hid by the convexity of the water.

And that it may be peopled on all ſides without any one's being in danger of falling away from it.

The attractive power of the earth draws all terreſtrial bodies towards its center; as is evident from the deſcent of bodies in lines perpendicular to the earth's ſurface, at the places whereon they fall; even when they are thrown off from the earth on oppoſite ſides, and conſequently, in oppoſite directions. So that the earth may be compared to a great magnet rolled in filings of ſteel, which attracts and keeps them equally faſt to its ſurface on all ſides. Hence, as all terreſtrial bodies are attracted toward the earth's center, they can be in no danger of falling from any ſide of the earth, more than from any other.

Up and *down*, what.

The heaven or ſky ſurrounds the whole earth: and when we ſpeak of *up* or *down*, we mean only with regard to ourſelves; for no point, either in the heaven, or on the ſurface of the earth, is *above* or *below*, but only with reſpect to ourſelves. And let us be upon what part of the earth we will, we ſtand with our feet towards its center, and our heads towards the ſky: and ſo we ſay, it is *up* towards the ſky, and *down* toward the center of the earth.

All objects in the heaven appear equally diſtant.

To an obſerver placed any where in the indefinite ſpace, where there is nothing to limit his view, all remote objects appear equally diſtant from him; and ſeem to be placed in a vaſt concave ſphere, of which his eye is the center. Every aſtronomer can demonſtrate, that the moon is much nearer to us than the ſun is; that ſome of the planets are ſometimes nearer to us, and ſometimes farther from us, than the ſun; that others of them never come ſo near us as the ſun always is; that the remoteſt planet in our ſyſtem, is beyond compariſon nearer to us than any of the fixed ſtars are; and that it is highly probable ſome ſtars are, in a manner, infinitely more diſtant from us than others; and yet all theſe celeſtial objects appear equally diſtant from us. Therefore, if we imagine a large hollow ſphere of glaſs to have as many bright ſtuds fixed to its inſide, as there are ſtars viſible in the heaven, and theſe

The face of the heaven and earth

represented in a machine.

ſtuds to be of different magnitudes, and placed at the ſame angular diſtances from each other as the ſtars are; the ſphere will be a true repreſentation of the ſtarry heaven, to an eye ſuppoſed to be in its center, and viewing it all around. And if a ſmall globe, with a map of the earth upon it, be placed on an axis in the center of this ſtarry ſphere, and the ſphere be made to turn round on this axis, it will repreſent the apparent motion of the heavens round the earth.

The equinoctial. The poles.

If a great circle be ſo drawn upon this ſphere, as to divide it into two equal parts, or hemiſpheres, and the plane of the circle be perpendicular to the axis of the ſphere, this circle will repreſent the *equinoctial*, which divides the heaven into two equal parts, called the *northern* and the *ſouthern hemiſpheres*; and every point of that circle will be equally diſtant from the *poles*, or ends of the axis in the ſphere. That pole which is in the middle of the northern hemiſphere, will be called the *north pole of the ſphere*, and that which is in the middle of the ſouthern hemiſphere, the *ſouth pole*.

The ecliptic.

If another great circle be drawn upon the ſphere, in ſuch a manner as to cut the equinoctial at an angle of 23½ degrees in two oppoſite points, it will repreſent the *ecliptic*, or circle of the ſun's apparent annual motion: one half of which is on the north ſide of the equinoctial, and the other half on the ſouth.

The ſun.

If a large ſtud be made to move eaſtward in this ecliptic, in ſuch a manner as to go quite round it, in the time that the ſphere is turned round weſtward 366 times upon its axis; this ſtud will repreſent the *ſun*, changing his place every day a 365th part of the ecliptic; and going round weſtward, the ſame way as the ſtars do; but with a motion ſo much ſlower than the motion of the ſtars, that they will make 366 revolutions about the axis of the ſphere, in the time that the ſun makes only 365. During one half of theſe revolutions, the ſun will be on the north ſide of the equinoctial; during the other half, on the ſouth: and at the end of each half, in the equinoctial.

The *earth.* If we ſuppoſe the terreſtrial globe in this machine to be about one inch in diameter, and the diameter of the ſtarry ſphere to be about five or ſix feet, a ſmall inſect on the globe would ſee only a very little portion of its ſurface; but it would ſee one half of the ſtarry ſphere; the convexity of the globe hiding the other half from its view. The apparent motion of the heavens. If the ſphere be turned weſtward round the globe, and the inſect could judge of the appearances which ariſe from that motion, it would ſee ſome ſtars riſing to its view in the eaſtern ſide of the ſphere, whilſt others were ſetting on the weſtern: but as all the ſtars are fixed to the ſphere, the ſame ſtars would always riſe in the ſame points of view on the eaſt ſide, and ſet in the ſame points of view on the weſt ſide. With the ſun it would be otherwiſe, becauſe the ſun is not fixed to any point of the ſphere, but moves ſlowly along an oblique circle in it. And if the inſect ſhould look towards the ſouth, and call that point of the globe, where the equinoctial in the ſphere ſeems to cut it on the left ſide, the *eaſt point*; and where it cuts the globe on the right ſide, the *weſt point*; the little animal would ſee the ſun riſe north of the eaſt, and ſet north of the weſt, for 182½ revolutions; after which, for as many more, the ſun would riſe ſouth of the eaſt, and ſet ſouth of the weſt,

weſt. And in the whole 365 revolutions, the ſun would riſe only twice in the eaſt point, and ſet twice in the weſt. All theſe appearances would be the ſame, if the ſtarry ſphere ſtood ſtill (the ſun only moving in the ecliptic) and the earthly globe were turned round the axis of the ſphere eaſtward. For, as the inſect would be carried round with the globe, he would be quite inſenſible of its motion; and the ſun and ſtars would appear to move weſtward.

We are but very ſmall beings when compared with our earthly globe, and *the globe itſelf* is but a dimenſionleſs point compared with the magnitude of the ſtarry heavens. Whether the earth be at reſt, and the heaven turns round it, or the heaven be at reſt, and the earth turns round, the appearance to us will be exactly the ſame. And becauſe the heaven is ſo immenſely large, in compariſon of the earth, we ſee one half of the heaven as well from the earth's ſurface, as we could do from its center, if the limits of our view are not entercepted by hills.

Circles of the ſphere.

We may imagine as many circles deſcribed upon the earth as we pleaſe; and we may imagine the plane of any circle deſcribed upon the earth to be continued, until it marks a circle in the concave ſphere of the heavens.

The horizon.

The *horizon* is either *ſenſible* or *rational.* The *ſenſible* horizon is that circle, which a man ſtanding upon a large plane, obſerves to terminate his view all around, where the heaven and earth ſeem to meet. The plane of our ſenſible horizon continued to the heaven, divides it into two hemiſperes; one viſible to us, the other hid by the convexity of the earth.

The plane of the *rational horizon*, is ſuppoſed parallel to the plane of the ſenſible; to paſs through the center of the earth, and to be continued to the heavens. And although the plane of the ſenſible horizon touches the earth in the place of the obſerver, yet *this* plane, and *that* of the rational horizon, will ſeem to coincide in the heaven, becauſe the whole earth is but a point compared to the ſphere of the heaven.

The earth being a ſpherical body, the horizon, or limit of our view, muſt change as we change our place.

Poles. The *poles of the earth*, are thoſe two points on its ſurface in which its axis terminates. The one is called the *north pole*, and the other the *ſouth pole*.

The *poles of the heaven*, are thoſe two points in which the earth's axis produced terminates in the heaven: ſo that the *north pole* of the heaven is directly over the north pole of the earth; and the *ſouth pole* of the heaven is directly over the ſouth pole of the earth.

Equator. The *equator* is a great circle upon the earth, every part of which is equally diſtant from either of the poles. It divides the earth into two equal parts, called the *northern* and *ſouthern hemiſpheres.* If we ſuppoſe the plane of this circle to be extended to the heaven, it will mark the *equinoctial* therein, and will divide the heaven into two equal parts, called the *northern* and *ſouthern* hemiſpheres of the heaven.

Meridian. The *meridian* of any place is a great circle paſſing through that place and the poles of the earth. We may imagine as many ſuch meridians as we pleaſe, becauſe any place that is ever ſo little to the eaſt or weſt of any other place, has a different meridian from that place; for no one circle can paſs through any two ſuch places and the poles of the earth.

The *meridian* of any place is divided by the poles, into two ſemicircles: that which paſſes through the place is called the *geographical*, or *upper meridian*; and that which paſſes through the oppoſite place, is called the *lower meridian*.

Noon and mid-night.

When the rotation of the earth brings the plane of the geographical meridian to the ſun, it is *noon* or *mid day* to that place; and when our lower meridian comes to the ſun, it is *mid-night*.

All places lying under the ſame geographical meridian, have their noon at the ſame time, and conſequently all the other hours. All thoſe places are ſaid to have the ſame *longitude*, becauſe no one of them lies either eaſtward or weſtward from any of the reſt.

Hour circles.

If we imagine 24 ſemicircles, one of which is the geographical meridian of a given place, to meet at the poles, and to divide the equator into 24 equal parts; each of theſe meridians will come round to the ſun in 24 hours, by the earth's equable motion round its axis in that time. And, as the equator contains 360 degrees, there will be 15 degrees contained between any two of theſe meridians which are neareſt to one another: for 24 times 15 is 360. And as the earth's motion is eaſtward, the ſun's apparent motion will be weſtward, at the rate of 15 degrees each hour. Therefore,

Longitude.

They whoſe geographical meridian is 15 degrees eaſtward from us, have noon, and every other hour, an hour ſooner than we have. They whoſe meridian is fifteen degrees weſtward from us, have noon, and every other hour, an hour later than we have: and ſo on in proportion, reckoning one hour for every fifteen degrees.

Ecliptic.

As the earth turns round its axis once in 24 hours, and ſhews itſelf all round to the ſun in that time; ſo it goes round the ſun once a year, in a great circle called the *ecliptic*, which croſſes the equinoctial in two oppoſite points, making an angle of 23½ degrees with the equinoctial on each ſide. So that one half of the ecliptic is in the northern hemiſphere, and the other in the ſouthern. It contains 360 equal parts, called degrees (as all other circles do, whether great or ſmall) and as the earth goes once round it every year, the ſun will appear to do the ſame, changing his place almoſt a degree, at a mean rate, every 24 hours. So that whatever place, or degree of the ecliptic, the earth is in at any time, the ſun will then appear in the oppoſite. And as one half of the ecliptic is on the north ſide of the equinoctial, and the other half on the ſouth; the ſun, as ſeen from the earth, will be half a year on the ſouth ſide of the equinoctial, and half a year on the north: and twice a year in the equinoctial itſelf.

Signs and *degrees.*

The ecliptic is divided by aſtronomers into 12 equal parts, called *ſigns*, each ſign into 30 *degrees*, and each degree into 60 *minutes*: but in uſing the globes, we ſeldom want the ſun's place nearer than half a degree of the truth.

The names and characters of the 12 ſigns are as follow; beginning at that point of the ecliptic where it croſſes the equinoctial to the northward, and reckoning eaſtward round to the ſame point again. And the days of the months on which the ſun now enters the ſigns, are ſet down below them.

Aries,

Aries	*Taurus*,	*Gemini*,	*Cancer*,
♈	♉	♊	♋
March	April	May	June
20	20	21	21
Leo,	*Virgo*,	*Libra*,	*Scorpio*,
♌	♍	♎	♏
July	Auguſt	September	October
23	23	23	23
Sagittarius,	*Capricornus*,	*Aquarius*,	*Piſces*,
♐	♑	♒	♓
November	December	January	February
22	21	20	18

By remembering on what day the ſun enters any particular ſign, we may eaſily find his place any day afterward, whilſt he is in that ſign, by reckoning a degree for each day; which will occaſion no error of conſequence in uſing the globes.

When the ſun is at the beginning of *Aries*, he is in the equinoctial; and from that time he declines northward every day, until he comes to the begnning of *Cancer*, which is 23½ degrees from the equinoctial: from thence he recedes ſouthward every day, for half a year; in the middle of which half, he croſſes the equinoctial at the beginning of *Libra*, and at the end of that half year, he is at his greateſt ſouth declination, in the beginning of *Capricorn*, which is alſo 23½ degrees from the equinoctial. Then, he returns northward from *Capricorn* every day, for half a year; in the middle of which half, he croſſes the equinoctial at the beginning of *Aries*; and at the end of it he arrives at *Cancer*.

The ſun's motion in the ecliptic is not perfectly equable, for he continues eight days longer in the northern half of the ecliptic, than in the ſouthern: ſo that the ſummer half year, in the northern hemiſphere, is eight days longer than the winter half year; and the contrary in the ſouthern hemiſphere.

Tropics. The *tropics* are leſſer circles in the heaven, parallel to the equinoctial; one on each ſide of it, touching the ecliptic in the points of its greateſt declination; ſo that each tropic is 23½ degrees from the equinoctial, one on the north ſide of it, and the other on the ſouth. The northern tropic touches the ecliptic at the beginning of *Cancer*, the ſouthern at the beginning of *Capricorn*; for which reaſon the former is called the *tropic of Cancer*, and the latter the *tropic of Capricorn*.

Polar circles. The *polar circles* in the heaven, are each 23½ degrees from the poles, all around. That which goes round the north pole, is called the *arctic circle*, from ἄρκτος, which ſignifies a *bear*; there being a collection or groupe of ſtars near the north pole, which goes by that name. The ſouth polar circle, is called the *antarctic circle*, from its being oppoſite to the arctic.

The ecliptic, tropics, and polar circles, are drawn upon the terreſtrial globe, as well as upon the celeſtial. But the ecliptic, being a great fixed circle in the heavens, cannot properly be ſaid to belong to the terreſtrial globe; and is laid down upon it only for the conveniency of ſolving ſome problems. So that, if this circle on the terreſtrial globe was properly divided into the months and days of the year, it would not only ſuit the globe better, but would alſo make the problems thereon much eaſier.

In order to form a true idea of the earth's motion round its axis every 24 hours, which is the cauſe of day and night; and of its motion in the ecliptic round the ſun every year, which is the cauſe of the different lengths of days and nights, and of the viciſſitude of ſeaſons; take the following method, which will be both eaſy and pleaſant.

An idea of the ſeaſons.

Let a ſmall terreſtrial globe, of about three inches diameter, be ſuſpended by a long thread of twiſted ſilk, fixt to its north pole: then having placed a lighted candle on a table, to repreſent the ſun, in the center of a hoop of a large caſk, which may repreſent the ecliptic, the hoop making an angle of 23½ degrees with the plane of the table; hang the globe within the hoop near to it; and if the table be level, the equator of the globe will be parallel to the table, and the plane of the hoop will cut the equator at an angle of 23½ degrees; ſo that one half of the equator will be above the hoop, and the other half below it: and the candle will enlighten one half of the globe, as the ſun enlightens one half of the earth, whilſt the other half is in the dark.

Things being thus prepared, twiſt the thread towards the left hand, that it may turn the globe the ſame way by untwiſting; that is, from weſt, by ſouth, to eaſt. As the globe turns round its axis or thread, the different places of its ſurface will go regularly through the light and dark; and have, as it were, an alternate return of day and night in each rotation. As the globe continues to turn round, and to ſhew itſelf all around to the candle, carry it ſlowly round the hoop by the thread, from weſt, by ſouth, to eaſt; which is the way that the earth moves round the ſun, once a year, in the ecliptic: and you will ſee, that whilſt the globe continues in the lower part of the hoop, the candle (being then north of the equator) will conſtantly ſhine round the north pole; and all the northern places which go through any part of the dark, will go through a leſs portion of it than they do of the light; and the more ſo, the farther they are from the equator: conſequently, their days are then longer than their nights. When the globe comes to a point in the hoop, mid-way between the higheſt and loweſt points, the candle will be directly over the equator, and will enlighten the globe juſt from pole to pole; and then every place on the globe will go through equal portions of light and darkneſs, as it runs round its axis; and conſequently, the day and night will be of equal length at all places upon it. As the globe advances thenceforward, towards the higheſt part of the hoop, the candle will be on the ſouth ſide of the equator, ſhining farther and farther round the ſouth pole, as the globe riſes higher and higher in the hoop; leaving the north pole as much in darkneſs, as the ſouth pole is then in the light, and making long days and ſhort nights on the ſouth ſide of the equator, and the contrary on the north ſide, whilſt the globe continues in the nothern or higher ſide of the hoop: and when it comes to the higheſt point, the days will be at the longeſt, and the nights at the ſhorteſt, in the ſouthern hemiſphere; and the reverſe in the northern. As the globe advances and deſcends in the hoop, the light will gradually recede from the ſouth pole, and approach towards the north pole, which will cauſe the northern days to lengthen, and the ſouthern days to ſhorten in

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the ſame proportion. When the globe comes to the middle point, between the higheſt and loweſt points of the hoop, the candle will be over the equator, enlightening the globe juſt from pole to pole, when every place of the earth (except the poles) will go through equal portions of light and darkneſs; and conſequently, the day and night will be then equal, all over the globe.

And thus, at a very ſmall expence, one may have a delightful and demonſtrative view of the cauſe of days and nights, with their gradual increaſe and decreaſe in length, through the whole year together, with the viciſſitudes of ſpring, ſummer, autumn, and winter, in each annual courſe of the earth round the ſun.

If the hoop be divided into 12 equal parts, and the ſigns be marked in order upon it, beginning with *Cancer* at the higheſt point of the hoop, and reckoning eaſtward (or contrary to the apparent motion of the ſun) you will ſee how the ſun appears to change his place every day in the ecliptic, as the globe advances eaſtward along the hoop, and turns round its own axis: and that when the earth is in a low ſign, as at *Capricorn*, the ſun muſt appear in a high ſign, as at *Cancer*, oppoſite to the earth's real place: and that whilſt the earth is in the ſouthern half of the ecliptic, the ſun appears in the northern half, and *vice verſâ*: that the farther any place is from the equator, between it and the polar circle, the greater is the difference between the longeſt and ſhorteſt day at that place; and that the poles have but one day and one night in the whole year.

Theſe things premiſed, we ſhall proceed to the deſcription and uſe of the terreſtrial globe, and explain the geographical terms as they occur in the problems.

The *ter-reſtrial globe* de-ſcribed.

This globe has the boundaries of land and water laid down upon it, the countries and kingdoms divided by dots, and coloured to diſtinguiſh them, the iſlands properly ſituated, the rivers and principal towns inſerted, as they have been aſcertained upon the earth by meaſurement and obſervation.

The equator, ecliptic, tropics, polar circles, and meridians, are laid down upon the globe in the manner already deſcribed. The ecliptic is divided into 12 ſigns, and each ſign into 30 degrees, which are generally ſubdivided into halves, and into quarters if the globe is large. Each tropic is 23½ degrees from the equator, and each polar circle 23½ degrees from its reſpective pole. Circles are drawn parallel to the equator, at every ten degrees diſtance from it on each ſide to the poles: theſe circles are called *parallels of latitude*. On large globes there are circles drawn perpendicularly through every tenth degree of the equator, interſecting each other at the poles: but on globes of or under a foot diameter, they are only drawn through every fifteenth degree of the equator: theſe circles are generally called *meridians*, ſometimes *circles of longitude*, and at other times *hour-circles*.

The globe is hung in a braſs ring, called the *braſen meridian*; and turns upon a wire in each pole ſunk half its thickneſs into one ſide of the meridian ring; by which means, *that* ſide of the ring divides the globe into two equal parts, called the *eaſtern* and *weſtern hemiſpheres*; as the equator divides it into two equal parts, called the *northern* and *ſouthern hemiſpheres*. This ring is divided into 360 equal parts or degrees, on the ſide wherein the axis of the globe turns. One half of theſe degrees are numbered, and reckoned, from the equator to the poles, where they end at 90: their uſe is to ſhew the latitudes of places. The degrees on the other half of the meridian ring, are numbered from the poles to the equator, where they end at 90: their uſe is to ſhew how to elevate either the north or ſouth pole above the horizon, according to the latitude of any given place, as it is north or ſouth of the equator.

The braſen meridian is let into two notches made in a broad flat ring, called the *wooden horizon*, the upper ſurface of which divides the globe into two equal parts, called the *upper* and *lower hemiſpheres.* One notch is in the north point of the horizon, and the other in the ſouth. On this horizon are ſeveral concentric circles, which contain the months and days of the year, the ſigns and degrees anſwering to the ſun's place for each month and day, and the 32 points of the compaſs.—The graduated ſide of the braſs meridian lies towards the eaſt ſide of the horizon, and ſhould be generally kept toward the perſon who works problems by the globes.

There is a ſmall *horary circle*, ſo fixed to the north part of the braſen meridian, that the wire in the north pole of the globe is in the center of that circle; and on the wire is an *index*, which goes over all the 24 hours of the circle, as the globe is turned round its axis. Sometimes there are two horary circles, one between each pole of the globe and the braſen meridian; which is the contrivance of the late ingenious Mr. *Joſeph Harris*, maſter of the aſſay-office in the Tower of London; and makes it very convenient for putting the poles of the globe through the horizon, and for elevating the pole to ſmall latitudes, and declinations of the ſun; which cannot be done where there is only one horary circle fixed to the outer edge of the braſen meridian.

There is a thin ſlip of braſs, called the *quadrant of altitude*, which is divided into 90 equal parts or degrees, anſwering exactly to ſo many degrees of the equator. It is occaſionally fixed to the uppermoſt point of the braſen meridian by a nut and ſcrew. The diviſions end at the nut, and the quadrant is turned round upon it.

As the globe has been ſeen by moſt people, and upon the figure of which, in a plate, neither the circles nor countries can be properly expreſſed, we judge it would ſignify very little to refer to a figure of it; and ſhall therefore only give ſome directions how to chooſe a globe, and then deſcribe its uſe.

Directions for chooſing of globes.

1. See that the papers be well and neatly paſted on the globes, which you may know, if the lines and circles thereon meet exactly, and continue all the way even and whole; the circles not breaking into ſeveral arches, nor the papers either coming ſhort, or lapping over one another.

2. See that the colours be tranſparent, and not laid too thick upon the globe to hide the names of places.

3. See that the globe hang evenly between the braſen meridian and the wooden horizon; not inclining either to one ſide or to the other.

4. See that the globe be as cloſe to the horizon and meridian as it conveniently may; otherwiſe, you will be too much puzzled to find

againſt what part of the globe any degree of the meridian or horizon is.

5. See that the equinoctial line be even with the horizon all around, with the north or ſouth pole is elevated 90 degrees above the horizon.

6. See that the equinoctial line cuts the horizon in the eaſt and weſt points, in all elevations of the pole from 0 to 90 degrees.

7. See that the degree of the braſen meridian marked with 0, be exactly over the equinoctial line of the globe.

8. See that there be exactly half of the braſen meridian above the horizon; which you may know, if you bring any of the decimal diviſions on the meridian to the north point of the horizon, and find their complement to 90 in the ſouth point.

9. See that when the quadrant of altitude is placed as far from the equator, on the braſen meridian, as the pole is elevated above the horizon, the beginning of the degrees of the quadrant reaches juſt to the plane ſurface of the horizon.

10. See that whilſt the index of the hour-circle (by the motion of the globe) paſſes from one hour to another, 15 degrees of the equator paſs under the graduated edge of the braſen meridian.

11. See that the wooden horizon be made ſubſtantial and ſtrong: it being generally obſerved, that in moſt globes, the horizon is the firſt part that fails, on account of its having been made too ſlight.

Directions for uſing them.

In uſing the globes, keep the eaſt ſide of the horizon towards you (unleſs your problem requires the turning of it) which ſide you may know by the word Eaſt upon the horizon; for then you have the graduated ſide of the meridian towards you, the quadrant of altitude before you, and the globle divided exactly into two equal parts, by the graduated ſide of the meridian.

In working ſome problems, it will be neceſſary to turn the whole globe and horizon about, that you may look on the weſt ſide thereof; which turning will be apt to jog the ball ſo, as to ſhift away that degree of the globe which was before ſet to the horizon or meridian: to avoid which inconvenience, you may thruſt in the feather-end of a quill between the ball of the globe and the braſen meridian; which, without hurting the ball, will keep it from turning in the meridian, whilſt you turn the weſt ſide of the horizon towards you.

PROBLEM I.

To find the [*] latitude and [†] longitude of any given place upon the globe.

Turn the globe on its axis, until the given place comes exactly under that graduated ſide of the braſen meridian, on which the degrees are numbered from the equator; and obſerve what degree of the meridian the place then lies under; which is its latitude, north or ſouth, as the place is north or ſouth of the equator.

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[*] The latitude of a place is its diſtance from the equator, and is north or ſouth, as the place is north or ſouth of the equator. Thoſe who live at the equator have no latitude, becauſe it is there that the latitude begins.

[†] The longitude of a place is the number of degrees (reckoned upon the equator) that the meridian of the ſaid place is diſtant from the meridian of any other place from which we reckon, either eaſtward or weſtward, for 180 degrees, or half round the globe. The Britiſh reckon the longitude from the meridian of London, and the French now reckon it from the meridian of Paris. The meridian of that place, from which the longitude is reckoned, is called

The globe remaining in this poſition, the degree of the equator, which is under the braſen meridian, is the longitude of the place, (from the meridian of *London* on the *Engliſh* globes) which is eaſt or weſt, as the place lies on the eaſt or weſt ſide of the firſt meridian of the globe.—All the *Atlantic Ocean*, and *America*, is on the weſt ſide of the meridian of *London*; and the greateſt part of *Europe*, and of *Africa*, together with all *Aſia*, is on the eaſt ſide of the meridian of *London*, which is reckoned the *firſt meridian* of the globe by the *Britiſh* geographers and aſtronomers.

PROBLEM II.

The longitude and latitude of a place being given, to find that place on the globe.

Look for the given longitude in the equator (counting it eaſtward or weſtward from the firſt meridian, as it is mentioned to be eaſt or weſt;) and bring the point of longtude in the equator to the braſen meridian, on that ſide which is above the ſouth point of the horizon: then count from the equator, on the braſen meridian, to the degree of the given latitude, towards the north or ſouth pole, according as the latitude is north or ſouth; and under that degree of latitude on the meridian, you will have the place required.

called the *firſt meridian*. The places upon this meridian have no longitude, becauſe it is there that the longitude begins.

PROBLEM III.

To find the difference of longitude, or difference of latitude, between any two given places.

Bring each of theſe places to the braſen meridian, and ſee what its latitude is: the leſſer latitude ſubtracted from the greater, if both places are on the ſame ſide of the equator, or both latitudes added together, if they are on different ſides of it, is the difference of latitude required. And the number of degrees contained between theſe places, reckoned on the equator, when they are brought ſeparately under the braſen meridian, is their difference of longitude; if it be leſs than 180: but if more, let it be ſubtracted from 360, and the remainder is the difference of longitude required. Or,

Having brought one of the places to the braſen meridian, and ſet the hour-index to XII, turn the globe until the other place comes to the braſen meridian, and the number of hours and parts of an hour, paſt over by the index, will give the longitude in time; which may be eaſily reduced to degrees, by allowing 15 degrees for every hour, and one degree for every four minutes.

N. B. When we ſpeak of bringing any place to the braſen meridian, it is the graduated ſide of the meridian that is meant.

PROBLEM IV.

Any place being given, to find all thoſe places that have the ſame longitude or latitude with it.

Bring the given place to the braſen meridian, then all thoſe places which lie under that ſide of the meridian, from pole to pole, have the ſame longitude with the given place. Turn the globe round its axis, and all thoſe places which paſs under the ſame degree of the meridian that the given place does, have the ſame latitude with that place.

Since all latitudes are reckoned from the equator, and all longitudes are reckoned from the firſt meridian, it is evident, that the point of the equator which is cut by the firſt meridian, has neither latitude nor longitude.—The greateſt latitude is 90 degrees, becauſe no place is more than 90 degrees from the equator. And the greateſt longitude is 180 degrees, becauſe no place is more than 180 degrees from the firſt meridian.

PROBLEM V.

To find the * antœci, ‡ periœci, *and* ‡ antipodes, *of any given place.*

Bring the given place to the braſen meridian, and having found its latitude, keep the globe in that ſituation, and count the ſame number of degrees of latitude from the equator towards the contrary pole, and where the reckoning ends, you have the *antæci* of the given place upon the globe. Thoſe who live at the equator have no *antæci*.

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* The *antœci* are thoſe people who live on the ſame meridian, and in equal latitudes, on different ſides of the equator. Being on the ſame meridian, they have the ſame hours; that is, when it is noon to the one, it is alſo noon to the other; and when it is mid-night to the one, it is alſo midnight to the other, &c. Being on different ſides of the equator

The globe remaining in the ſame poſition, ſet the hour-index to the upper XII. on the horary circle, and turn the globe until the index comes to the lower XII; then, the place which lies under the meridian, in the ſame latitude with the given place, is the *periæci* required. Thoſe who live at the poles have no *periæci*.

As the globe now ſtands (with the index at the lower XII.) the *antipodes* of the given place will be under the ſame point of the braſen meridian where its *antæci* ſtood before. Every place upon the globe has its *antipodes*.

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tor, they have different or oppoſite ſeaſons at the ſame time; the length of any day to the one is equal to the length of the night of that day to the other; and they have equal elevations of the different poles.

† The *periæci* are thoſe people who live on the ſame parallel of latitude, but on oppoſite meridians: ſo that though their latitude be the ſame, their longitude differs 180 degrees. By being in the ſame latitude, they have equal elevations of the ſame pole (for the elevation of the pole is always equal to the latitude of the place) the ſame length of days or nights, and the ſame ſeaſons. But being on oppoſite meridians, when it is noon to the one, it is mid-night to the other.

‡ The *antipodes* are thoſe who live diametrically oppoſite to one another upon the globe, ſtanding with feet towards feet, on oppoſite meridians and parallels. Being on oppoſite ſides of the equator, they have oppoſite ſeaſons, winter to one, when it is ſummer to the other; being equally diſtant from the equator, they have their contrary poles equally elevated above the horizon; being on oppoſite meridians, when it is noon to the one, it muſt be mid-night to the other; and as the ſun recedes from the one when he approaches to the other, the length of the day to one muſt be equal to the length of the night at the ſame time to the other.

PROBLEM VI.

To find the diſtance between any two places on the globe.

Lay the graduated edge of the quadrant of altitude over both the places, and count the number of degrees intercepted between them on the quadrant; then multiply theſe degrees by 60, and the product will give the diſtance in geographical miles: but to find the diſtance in Engliſh miles, multiply the degrees by 69½, and the product will by the number of miles required. Or, take the diſtance betwixt any two places with a pair of compaſſes, and apply that extent to the equator; the number of degrees, intercepted between the points of the compaſſes, is the diſtance in degrees of a great circle *; which may be reduced either to geographical miles, or to Engliſh miles, as above.

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* Any circle that divides the globe into two equal parts, is called a *great circle*, as the equator or meridian. Any circle that divides the globe into two unequal parts (which every parallel of latitude does) is called a *leſſer circle*. Now, as every circle, whether great or ſmall, contains 360 degrees, and a degree upon the equator or meridian contains 60 geographical miles, it is evident, that a degree of longitude upon the equator, is longer than a degree of longitude upon any parallel of latitude, and muſt therefore contain a greater number of miles. So that, although all the degrees of latitude are equally long upon an artificial globe (though not preciſely ſo upon the earth itſelf) yet the degrees of longitude decreaſe in length, as the latitude increaſes, but not in the ſame proportion. The following table ſhews the length of a degree of longitude, in geographical miles, and hundredth parts of a mile, for every degree of latitude, from the equator to the poles: a degree on the equator being 60 geographical miles.

Great circle.

Leſſer circle.

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PROBLEM VII.

A place on the globe being given, and its diſtance from any other place, to find all the other places upon the globe which are at the ſame diſtance from the given place.

Bring the given place to the braſen meridian, and ſcrew the quadrant of altitude to the meridian, directly over that place; then keeping the globe in that poſition, turn the quadrant quite round upon it, and the degree of the quadrant that touches the ſecond place, will paſs over all the other places which are equally diſtant with it from the given place.

This is the ſame as if one foot of a pair of compaſſes was ſet in the given place, and the other foot extended to the ſecond place, whoſe diſtance is known; for if the compaſſes be then turned round the firſt place as a center, the moving foot will go over all thoſe places which are at the ſame diſtance with the ſecond from it.

A TABLE

¶ TABLE *ſhewing the number of miles in a degree of longitude, in any given degree of latitude.*

Deg.	Miles. Parts.	Deg.	Miles. Parts.	Deg.	Miles. Parts.
1	59.99	31	51.43	61	29.09
2	59.96	32	50.88	62	28.17
3	59.92	33	50.32	63	27.24
4	59.85	34	49.74	64	26.30
5	59.77	35	49.15	65	25.36
6	59.67	36	48.54	66	24.41
7	59.56	37	47.92	67	23.44
8	59.42	38	47.28	68	22.48
9	59.26	39	46.63	69	21.50
10	59.09	40	45.97	70	20.52
11	58.89	41	45.28	71	19.53
12	58.69	42	44.59	72	18.54
13	58.46	43	43.88	73	17.54
14	58.22	44	43.16	74	16.53
15	57.95	45	42.43	75	15.52
16	57.67	46	41.68	76	14.51
17	57.38	47	40.92	77	13.50
18	57.06	48	40.15	78	12.48
19	56.73	49	39.36	79	11.45
20	56.38	50	38.57	80	10.42
21	56.02	51	37.76	81	9.38
22	55.63	52	36.94	82	8.35
23	55.23	53	36.11	83	7.32
24	54.81	54	35.27	84	6.28
25	54.38	55	34.41	85	5.24
26	53.93	56	33.55	86	4.20
27	53.46	57	32.68	87	3.15
28	52.96	58	31.79	88	2.10
29	52.47	59	30.90	89	1.05
30	51.99	60	30.00	90	0.00

PROBLEM VIII.

The hour of the day at any place being given, to find all thoſe places where it is noon at that time.

Bring the given place to the braſen meridian, and ſet the index to the given hour; this done, turn the globe until the index points to the upper XII, and then, all the places that lie under the braſen meridian have noon at that time.

N. B. The upper XII always ſtands for noon; and when the bringing of any place to the braſen meridian is mentioned, the ſide of that meridian on which the degrees are reckoned from the equator is meant, unleſs the contrary ſide be mentioned.

PROBLEM IX.

The hour of the day at any place being given to find what o'clock it then is at any other place.

Bring the given place to the braſen meridian, and ſet the index to the given hour; then turn the globe, until the place where the hour is required comes to the braſen meridian, and the index will point out the hour at that place.

PROBLEM X.

To find the ſun's place in the ecliptic, and his * *declination, for any given day of the year.*

Look on the horizon for the given day, and right againſt it you have the degree of the ſign in which the ſun is (or his place) on that day at noon. Find the ſame degree of that ſign in the ecliptic line upon the globe, and having brought it to the braſen meridian, obſerve what degree of the meridian ſtands over it; for that is the ſun's declination, reckoned from the equator.

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* The ſun's declination is his diſtance from the equinoctial in degrees, and is north or ſouth, as the ſun is between the equinoctial and the north or ſouth pole.

PROBLEM XI.

The day of the month being given, to find all thoſe places of the earth over which the ſun will paſs vertically on that day.

Find the ſun's place in the ecliptic for the given day, and having brought it to the braſen meridian, obſerve what point of the meridian is over it; then turning the globe round its axis; all thoſe places which paſs under that point of the meridian are the places required; for as their latitude is equal, in degrees and parts of a degree, to the ſun's declination, the ſun muſt be directly over head to each of them at its reſpective noon.

PROBLEM XII.

A place being given in the torrid zone, to find thoſe two days of the year, on which the ſun ſhall be vertical to that place.*

Bring the given place to the braſen meridian, and mark the degree of latitude that is exactly over it on the meridian; then turn the globe round its axis, and obſerve the two degrees of the ecliptic which paſs exactly under that degree of latitude: Laſtly, find on the wooden horizon the two days of the year on which the ſun is in thoſe degrees of the ecliptic, and they are the days required: for on them, and none elſe, the ſun's declination is equal to the latitude of the given place; and conſequently, he will then be vertical to it at noon.

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* The globe is divided into five zones; one torrid, two temperate, and two frigid. The *torrid zone* lies between the two tropics, and is 47 degrees in breadth, or 23½ on each ſide of the equator: the *temperate zones* lie between the tropics and polar circles, or from 23½ degrees of latitude, to 66½, on each

PROBLEM XIII.

To find all thoſe places of the north frigid zone, where the ſun begins to ſhine conſtantly without ſetting, on any given day, from the 20th of March, to the 23d of September.

On theſe two days, the ſun is in the equinoctial, and enlightens the globe exactly from pole to pole: therefore, as the earth turns round its axis, which terminates in the poles, every place upon it will go equally through the light and the dark, and ſo make the day and night equal to all places of the earth. But as the ſun declines from the equator, towards either pole, he will ſhine juſt as many degrees round that pole, as are equal to his declination from the equator; ſo that no place within that diſtance of the pole will then go through any part of the dark, and conſequently the ſun will not ſet to it. Now, as

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each ſide of the equator; and are each 43 degrees in breadth: the *frigid zones* are the ſpaces included within the polar circles, which being each $23\frac{1}{2}$ degrees from their reſpective poles, the breadth of each of theſe zones is 47 degrees. As the ſun never goes without the tropics, he muſt every moment be vertical to ſome place or other in the torrid zone.

the ſun's declination is northward, from the 21ſt of March to the 23d of September, he muſt conſtantly ſhine round the north pole all that time; and on the day that he is in the northern tropic, he ſhines upon the whole north frigid zone; ſo that no place within the north polar circle goes through any part of the dark on that day. Therefore,

Having brought the ſun's place for the given day to the braſen meridian, and found his declination (by Prob. IX.) count as many degrees on the meridian, from the north pole, as are equal to the ſun's declination from the equator, and mark that degree from the pole where the reckoning ends: then, turning the globe round its axis, obſerve what places in the north frigid zone paſs directly under that mark; for they are the places required.

The like may be done for the ſouth frigid zone, from the 23d of September to the 21ſt of March, during which time the ſun ſhines conſtantly on the ſouth pole.

PROBLEM XIV.

To find the place over which the ſun is vertical, at any hour of a given day.

Having found the ſun's declination for the given day (by Prob. IX.) mark it with a chalk on the braſen meridian: then bring the place where you are (ſuppoſe London) to the braſen meridian, and ſet the index to the given hour; which done, turn the globe on its axis, until the index points to XII at noon; and the place on the globe, which is then directly under the point *of the ſun's declination* marked upon the meridian, *has the ſun* that moment in the zenith, or *directly* overhead.

PROBLEM XV.

The day and hour at any place being given, to find all thoſe places where the ſun is then riſing, or ſetting, or on the meridian: conſequently, all thoſe places which are enlightened at that time, and thoſe which are in the dark.

This problem cannot be ſolved by any globe fitted up in the common way, with the hour circle fixed upon the braſs meridian; unleſs the ſun be on or near ſome of the tropics on the given day. But by a globe fitted up according to Mr. *Joſeph Harris*'s invention (already mentioned) where the hour-circle lies on the ſurface of the globe, below the meridian, it may be ſolved for, any day in the year, according to his method; which is as follows.

Having found the place to which the ſun is vertical at the given hour, if the place be in the northern hemiſphere, elevate the north pole as many degrees above the horizon, as are equal to the latitude of that place; if the place be in the ſouthern hemiſphere, elevate the ſouth pole accordingly; and bring the ſaid place to the braſen meridian. Then, all thoſe places which are in the weſtern ſemicircle of the horizon, have the ſun riſing to them at that time; and thoſe in the eaſtern ſemicircle have it ſetting: to thoſe under the upper ſemicircle of the braſs meridian, it is noon; and to thoſe under the lower ſemicircle, it is midnight. All thoſe places which are above the horizon, are enlightened by the ſun, and have the ſun juſt as many degrees high to them, as they themſelves are above the horizon: and this height may be known, by fixing the quadrant of altitude on the braſen meridian over the place to which the ſun is vertical; and then, laying it over any other place, obſerve what number of degrees on the quadrant are intercepted between the ſaid place and the horizon. In all thoſe places that are 18 degrees below the weſtern ſemicircle of the horizon, the morning twilight is juſt beginning; in all thoſe places that are 18 degrees below the eaſtern ſemicircle of the horizon, the evening twilight is ending; and all thoſe places that are lower than 18 degrees, have dark night.

If any place be brought to the upper ſemicircle of the braſen meridian, and the hour index be ſet to the upper XII or noon, and then the globe be turned round eaſtward on its axis; when the place comes to the weſtern ſemicircle of the horizon, the index will ſhew the time of ſun-riſing at that place; and when the ſame place comes to the eaſtern ſemicircle of the horizon, the index will ſhew the time of ſun-ſet.

To thoſe places which do not go under the horizon, the ſun ſets not on that day: and to thoſe which do not come above it, the ſun does not riſe.

PROBLEM XVI.

The day and hour of a lunar eclipſe being given; to find all thoſe places of the earth to which it will be viſible.

The moon is never eclipſed but when ſhe is full, and ſo directly oppoſite to the ſun, that the earth's ſhadow falls upon her. Therefore, whatever place of the earth the ſun is vertical to at that time, the moon muſt be vertical to the antipodes of that place: ſo that the ſun will be then viſible to one half of the earth, and the moon to the other.

Find the place to which the ſun is vertical at the given hour (by Prob. XIV.) elevate the pole to the latitude of that place, and bring the place to the upper part of the braſen meridian, as in the former problem: then, as the ſun will be viſible to all thoſe parts of the globe which are above the horizon, the moon will be viſible to all thoſe parts of the globe which are below it, at the time of her greateſt obſcuration.

But with regard to an eclipſe of the ſun, there is no ſuch thing as ſhewing to what places it will be viſible, with any degree of certainty, by a common globe; becauſe the moon's ſhadow covers but a ſmall portion of the earth's ſurface; and her latitude, or declination from the ecliptic, throws her ſhadow ſo variouſly upon the earth, that to determine the places on which it falls, recourſe muſt be had to long calculations.

PROBLEM XVII.

To rectify the globe for the latitude, the * *zenith, and the ſun's place.*

Find the latitude of the place (by Prob. I.) and if the place be in the northern hemiſphere, raiſe the north pole above the north point of the horizon, as many degrees (counted from the pole upon the braſen meridian) as are equal to the latitude of the place. If the place be in the ſouthern hemiſphere, raiſe the ſouth pole above the ſouth point of the horizon, as many degrees as are equal to the latitude. Then, turn the globe till the place comes under its latitude on the braſen meridian, and faſten the quadrant of altitude ſo, that the chamfered edge of its nut (which is even with the graduated edge) may be joined to the zenith, or point of latitude. This done, bring the ſun's place in the ecliptic for the given day, (found by Prob. X.) to the graduated ſide of the braſen meridian, and ſet the hour-index to XII. at noon, which is the uppermoſt XII on the hour-circle; and the globe will be rectified.

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* The *zenith*, in this ſenſe, is the higheſt point of the braſen meridian above the horizon; but in the proper ſenſe it is that point of the heaven which is directly vertical to any given place, at any given inſtant of time.

Remark. The latitude of any place is equal to the elevation of the neareſt pole of the heaven above the horizon of that place; and the poles of the heaven are directly over the poles of the earth, each 90 degrees from the equinoctial line. Let us be upon what place of the earth we will, if the limits of our view be not intercepted by hills, we ſhall ſee one half of the heaven, or 90 degrees every way round, from that point which is over our heads. Therefore, if we were upon the equator, the poles of the heaven would lie in our horizon, or limit of our view: if we go from the equator, towards either pole of the earth, we ſhall ſee the correſponding pole of the heaven riſing gradually above our horizon, juſt as many degrees as we have gone from the equator: and if we were at either of the earth's poles, the correſponding pole of the heaven would be directly over our head. Conſequently, the elevation or height of the pole in degrees above the horizon, is equal to the number of degrees that the place is from the equator.

PROBLEM XVIII.

*The latitude of any place, not exceeding *66½ degrees, and the day of the month, being given; to find the time of ſun-riſing and ſetting, and conſequently the length of the day and night.*

Having rectified the globe for the latitude, and for the ſun's place on the given day (as directed in the preceding problem) bring the ſun's place in the ecliptic to the eaſtern ſide of the horizon, and the hour-index will ſhew the time of ſun-riſing; then turn the globe on its axis, until the ſun's place comes to the weſtern ſide of the horizon, and the index will ſhew the time of ſun-ſetting.

The hour of ſun-ſetting doubled, gives the length of the day; and the hour of ſun-riſing doubled gives the length of the night.

PROBLEM XIX.

The latitude of any place, and the day of the month, being given; to find when the morning twilight begins, and the evening twilight ends, at that place.

This ploblem is often limited; for, when the ſun does not go 18 degrees below the horizon, the twilight continues the whole night; and for ſeveral nights together in ſummer, between 49 and 66½ degrees of latitude: and the nearer to 66½, the greater is the number of theſe nights. But when it does begin and end, the following method will ſhew the time for any given day.

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* All places whoſe latitude is more than 66½ degrees, are in the frigid zones: and to thoſe places the ſun does not ſet in ſummer, for a certain number of diurnal revolutions, which occaſions this limitation of latitude.

Rectify the globe, and bring the ſun's place in the ecliptic to the eaſtern ſide of the horizon; then mark that point of the ecliptic with a chalk which is in the weſtern ſide of the horizon, it being the point oppoſite to the ſun's place: this done, lay the quadrant of altitude over the ſaid point, and turn the globe eaſtward, keeping the quadrant at the chalk-mark, until it is juſt 18 degrees high on the quadrant; and the index will point out the time when the morning twilight begins: for the ſun's place will then be 18 degrees below the eaſtern ſide of the horizon. To find the time when the evening twilight ends, bring the ſun's place to the weſtern ſide of the horizon, and the point oppoſite to it, which was marked with the chalk, will be riſing in the eaſt: then, bring the quadrant over that point, and keeping it thereon, turn the globe weſtward, until the ſaid point be 18 degrees above the horizon on the quadrant, and the index will ſhew the time when the evening twilight ends; the ſun's place being then 18 degrees below the weſtern ſide of the horizon.

PROBLEM XX.

To find on what day of the year the ſun begins to ſhine conſtantly without ſetting, on any given place in the north frigid zone; and how long he continues to do ſo.

Rectify the globe to the latitude of the place, and turn it about until ſome point of the ecliptic, between *Aries* and *Cancer*, coincides with the north point of the horizon where the braſen meridian cuts it: then find, on the wooden horizon, what day of the year the ſun is in that point of the ecliptic; for that is the day on which the ſun begins to ſhine conſtantly on the given place, without ſetting. This done, turn the globe until ſome point of the ecliptic, between *Cancer* and *Libra*, coincides with the north point of the horizon, where the braſen meridian cuts it; and find, on the wooden horizon, on what day the ſun is in that point of the ecliptic; which is the day that the ſun leaves off conſtantly ſhining on the ſaid place, and riſes and ſets to it as to other places on the globe. The number of natural days, or complete revolutions of the ſun about the earth, between the two days above found, is the time that the ſun keeps conſtantly above the horizon without ſetting: for all the portion of the ecliptic, that lies between the two points which interſect the horizon in the very north, never ſets below it: and there is juſt as much of the oppoſite part of the ecliptic that never riſes; therefore, the ſun will keep as long conſtantly below the horizon in winter, as above it in ſummer.

Whoever conſiders the globe, will find, that all places of the earth do equally enjoy the benefit of the ſun, in reſpect of time, and are equally deprived of it. For, the days and nights are always equally long at the equator: and in all places that have latitude, the days at one time of the year are exactly equal to the nights at the oppoſite ſeaſon.

PROBLEM XXI.

To find in what latitude the ſun ſhines conſtantly without ſetting, for any length of time leſs than [*] 182½ *of our days and nights.*

Find a point in the ecliptic half as many degrees from the beginning of *Cancer* (either towards *Aries* or *Libra*) as there are [†] natural days in the time given; and bring that point to the north ſide of the braſen meridian, on which the degrees are numbered from the pole towards the equator: then, keep the globe from turning on its axis, and ſlide the meridian up or down, until the foreſaid point of the ecliptic comes to the north point of the horizon, and then, the elevation of the pole will be equal to the latitude required.

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[*] The reaſon of this limitation is, that 182½ of our days and nights make half a year, which is the longeſt time that the ſun ſhines without ſetting, even at the poles of the earth.

[†] A natural day contains the whole 24 hours: an artificial day, the time that the ſun is above the horizon.

PROBLEM XXII.

The latitude of a place, not exceeding 66½ degrees, and the day, of the month being given; to find the ſun's amplitude, or point of the compaſs on which he riſes or ſets on that day.

Rectify the globe, and bring the ſun's place to the eaſtern ſide of the horizon; then obſerve what point of the compaſs on the horizon ſtands right againſt the ſun's place, for that is his amplitude at riſing. This done, turn the globe weſtward, until the ſun's place comes to the weſtern ſide of the horizon, and it will cut the point of his amplitude at ſetting. Or, you may count the riſing amplitude in degrees, from the eaſt point of the horizon, to that point where the ſun's place cuts it; and the ſetting amplitude, from the weſt point of the horizon, to the ſun's place at ſetting.

PROBLEM XXIII.

The latitude, the ſun's place, and his[*] altitude, being given; to find the hour of the day, and the ſun's azimuth, or number of degrees that he is diſtant from the meridian.

Rectify the globe, and bring the ſun's place to the given height upon the quadrant of altitude; on the eaſtern ſide of the horizon, if the time be in the forenoon; or the weſtern ſide, if it be in the afternoon: then, the index will ſhew the hour; and the number of degrees in the horizon intercepted between the quadrant of altitude and the ſouth point, will be the ſun's true azimuth at that time.

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[*] The ſun's altitude, at any time, is his height in degrees above the horizon at that time.

N. B. Always when the quadrant of altitude is mentioned in working any problem, the graduated edge of it is meant.

If this be done at ſea, and compared with the ſun's azimuth, as ſhewn by the compaſs, if they agree, the compaſs has no variation in that place: but if they differ, the compaſs does vary; and the variation is equal to this difference.

PROBLEM XXIV.

The latitude, hour of the day, and the ſun's place, being given; to find the ſun's altitude and azimuth.

Rectify the globe, and turn it until the index points to the given hour; then lay the quadrant of altitude over the ſun's place in the ecliptic, and the degree of the quadrant cut by the ſun's place is his altitude at that time above the horizon; and the degree of the horizon cut by the quadrant is the ſun's azimuth, reckoned from the ſouth.

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PROBLEM XXV.

The latitude, the ſun's altitude, and his azimuth being given; to find his place in the ecliptic, the day of the month, and hour of the day, though they had all been loſt.

Rectify the globe for the latitude and * zenith, and ſet the quadrant of altitude to the given azimuth in the horizon; keeping it there, turn the globe on its axis until the ecliptic cuts the quadrant in the given altitude: that point of the ecliptic which cuts the quadrant there, will be the ſun's place; and the day of the month anſwering thereto, will be found over the like place of the ſun on the wooden horizon. Keep the quadrant of altitude in that poſition, and having brought the ſun's place to the braſen meridian, and the hour index to XII at noon, turn back the globe, until the ſun's place cuts the quadrant of altitude again, and the index will ſhew the hour.

Any two points of the ecliptic which are equidiſtant from the beginning of *Cancer* or of *Capricorn*, will have the ſame altitude and azimuth at the ſame hour, though the months be different; and therefore it requires ſome care in this problem, not to miſtake both the month, and the day of the month; to avoid which obſerve, that from the 20th of March to the 21ſt of June, that part of the ecliptic which is between the beginning of *Aries* and beginning of *Cancer* is to be uſed: from the 21ſt of June to the 23d of September, between the beginning of *Cancer* and beginning of *Libra*: from the 23d of September to the 21ſt of December, between the beginning of *Libra* and the beginning of *Capricorn*; and from the 21ſt of December to the 20th of March, between the beginning of *Capricorn* and beginning of *Aries*. And as one can never be at a loſs to know in what quarter of the year he takes the ſun's altitude and azimuth, the above caution with regard to the quarters of the ecliptic, will keep him right as to the month and day thereof.

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* By rectifying the globe for the zenith, is meant ſcrewing the quadrant of altitude to the given latitude on the braſs meridian.

PROBLEM XXVI.

To find the length of the longeſt day at any given place.

If the place be on the north ſide of the equator, find its latitude (by Prob. I.) and elevate the north pole to that latitude; then, bring the beginning of *Cancer* ♋ to the braſen meridian, and ſet the hour-index to XII at noon. But if the given place be on the ſouth ſide of the equator, elevate the ſouth pole to its latitude, and bring the beginning of *Capricorn* ♑ to the braſs meridian, and the hour-index to XII. This done, turn the globe weſtward, until the beginning of *Cancer* or *Capricorn* (as the latitude is north or ſouth) comes to the horizon; and the index will then point out the time of ſun-ſetting, for it will have gone over all the afternoon hours, between mid-day and ſun-ſet;

which length of time being doubled, will give the whole length of the day, from ſun-riſing to ſun-ſetting. For, in all latitudes, the ſun riſes as long before mid-day, as he ſets after it.

PROBLEM XXVII.

To find in what latitude the longeſt day is of any given length leſs than 24 *hours.*

If the latitude be north, bring the beginning of *Cancer* to the braſen meridian, and elevate the north pole to about 66½ degrees; but if the latitude be ſouth, bring the beginning of *Capricorn* to the meridian, and elevate the ſouth pole to about 66½ degrees; becauſe the longeſt day in north latitude, is when the ſun is in the firſt point of *Cancer*; and in ſouth latitude, when he is in the firſt point of *Capricorn*. Then ſet the hour-index to XII at noon, and turn the globe weſtward, until the index points at half the number of hours given: which done, keep the globe from turning on its axis, and ſlide the meridian down in the notches, until the aforeſaid point of the ecliptic (viz. *Cancer* or *Capricorn)* comes to the horizon; then, the elevation of the pole will be equal to the latitude required.

PROBLEM XXVIII.

The latitude of any place, not exceeding $66\frac{1}{2}$ degrees being given; to ſind in what [*] *climate the place is.*

Find the length of the longeſt day at the given place by Prob. XXVI. and whatever be the number of hours whereby it exceedeth twelve, double that number, and the ſum will anſwer to the climate in which the place is.

PROBLEM XXIX.

The latitude, and the day of the month, being given; to ſind the hour of the day when the ſun ſhines.

Set the wooden horizon truly level, and the braſen meridian due north and ſouth by a mariner's compaſs: then, having rectified the globe, ſtick a ſmall ſewing-needle into the ſun's place in the ecliptic, perpendicular to that part of the ſurface of the globe: this done, turn the globe on its axis, until the needle comes to the braſen meridian, and ſet the hour-index to XII at noon; then, turn the globe on its axis, until the needle points exactly towards the ſun (which it will do when it caſts no ſhadow on the globe) and the index will ſhew the hour of the day.

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[*] A *climate*, from the equator to either of the polar circles, is a tract of the earth's ſurface, included between two ſuch parallels of latitude, that the length of the longeſt day in the one exceeds that in the other by half an hour; but from the polar circles to the poles, where the ſun keeps long above the horizon without ſetting, each climate differs a whole month from the one next to it. There are twenty-four climates between the equator and each of the polar circles; and ſix from each polar circle to its reſpective pole.

PROBLEM XXX.

A pleaſant way of ſhewing all thoſe places of the earth which are enlightened by the ſun, and alſo the time of the day when the ſun ſhines.

Take the terreſtrial ball out of the wooden horizon, and alſo out of the braſen meridian; then ſet it upon a pedeſtal in ſun-ſhine, in ſuch a manner, that its north pole may point directly towards the north pole of the heaven, and the meridian of the place where you are be directly towards the ſouth. Then, the ſun will ſhine upon all the like places of the globe, that he does on the real earth, riſing to ſome when he is ſetting to others; as you may perceive by that part where the enlightened half of the globe is divided from the half in the ſhade, by the boundary of the light and darkneſs: all thoſe places, on which the ſun ſhines, at any time, having day; and all thoſe, on which he does not ſhine, having night.

If a narrow ſlip of paper be put round the equator, and divided into 24 equal parts, beginning at the meridian of your place, and the hours be ſet to thoſe diviſions in ſuch a manner, that one of the VI's may be upon your meridian; the ſun being upon that meridian at noon, will then ſhine exactly to the two XII's; and at one o'clock to the two I's, &c. So that the place, where the enlightened half of the globe is parted from the ſhaded half, in this circle of hours, will ſhew the time of the day.

The principles of dialing ſhall be explained farther on, by the terreſtrial globe. At preſent we ſhall only add the following obſervations upon it; and then proceed to the uſe of the celeſtial globe.

1. *The latitude of any place is equal to the elevation of the pole above the horizon of that place, and the elevation of the equator is equal to the complement of the latitude, that is, to what the latitude wants of* 90 *degrees.*

2. *Thoſe places which lie on the equator, have no latitude, it being there that the latitude begins; and thoſe places which lie on the firſt meridian have no longitude, it being there that the longitude begins. Conſequently,* that *particular place of the earth where the firſt meridian interſects the equator, has neither longitude nor latitude.*

3. *At all places of the earth, except the poles, all the points of the compaſs may be diſtinguiſhed in the horizon: but from the north pole, every place is ſouth; and from the ſouth pole, every place is north. Therefore, as the ſun is conſtantly above the horizon of each pole for half a year in its turn, he cannot be ſaid to depart from the meridian of either pole for half a year together. Conſequently, at the north pole it may be ſaid to be noon every moment for half a year; and let the winds blow from what part they will, they muſt always blow from the ſouth; and at the ſouth pole, from the north.*

4. *Becauſe one half of the ecliptic is above the horizon of the pole, and the ſun, moon, and planets move in (or nearly in) the ecliptic; they will all*

riſe and ſet to the poles. But, becauſe the ſtars never change their declinations from the equator (at leaſt not ſenſibly in one age) thoſe which are once above the horizon of either pole, never ſet below it; and thoſe which are once below it, never riſe.

5. *All places of the earth do equally enjoy the benefit of the ſun, in reſpect of time, and are equally deprived of it.*

6. *All places upon the equator have their days and nights equally long, that is,* 12 *hours each, at all times of the year. For although the ſun declines alternately, from the equator towards the north and towards the ſouth, yet, as the horizon of the equator cuts all the parallels of latitude and declination in halves, the ſun muſt always continue above the horizon for one half a diurnal revolution about the earth, and for the other half below it.*

7. *When the ſun's declination is greater than the latitude of any place, upon either ſide of the equator, the ſun will come twice to the ſame azimuth or point of the compaſs in the forenoon, at that place, and twice to a like azimuth in the afternoon; that is, he will go twice back every day, whilſt his declination continues to be greater than the latitude. Thus, ſuppoſe the globe rectified to the latitude of* Barbadoes, *which is* 13 *degrees north; and the ſun to be any where in the ecliptic, between the middle of Taurus and middle of Leo; if the quadrant of altitude be ſet to about* * 18 *degrees north of the eaſt in the horizon, the ſun's place be marked with a chalk upon the ecliptic, and the globe be then turned weſtward on its axis, the ſaid mark will riſe in the horizon a little to the north of the quadrant, and thence aſcending, it will croſs the quadrant towards*

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* From the middle of Gemini to the middle of Cancer, the quadrant may be ſet 20 degrees.

the ſouth; but before it arrives at the meridian, it will croſs the quadrant again, and paſs over the meridian northward of Barbadoes. *And if the quadrant be ſet about* 18 *degrees north of the weſt, the ſun's place will croſs it twice, as it deſcends from the meridian towards the horizon, in the afternoon.*

8. *In all places of the earth between the equator and poles, the days and nights are equally long, viz.* 12 *hours each, when the ſun is in the equinoctial: for, in all elevations of the pole, ſhort of* 90 *degrees (which is the greateſt) one half of the equator or equinoctial will be above the horizon, and the other half below it.*

9. *The days and nights are never of an equal length at any place between the equator and polar circles, but when the ſun enters the ſigns* ♈ *Aries and* ♎ *Libra. For in every other part of the ecliptic, the circle of the ſun's daily motion is divided into two unequal parts by the horizon.*

10. *The nearer that any place is to the equator, the leſs is the difference between the length of the days and nights in that place; and the more remote, the contrary. The circles which the ſun deſcribes in the heaven every* 24 *hours, being cut more nearly equal in the former caſe, and more unequally in the latter.*

11. *In all places lying upon any given parallel of latitude, however long or ſhort the day or night be at any one of theſe places, at any time of the year, it is then of the ſame length at all the reſt; for in turning the globe round its axis (when rectified according to the ſun's declination) all theſe places will keep equally long above or below the horizon.*

12. *The ſun is vertical twice a year to every place between the tropics; to thoſe under the tropics,*

once a year, but never any where elſe. For, there can be no place between the tropics, but that there will be two points in the ecliptic, whoſe declination from the equator is equal to the latitude of that place; and but one point of the ecliptic which has a declination equal to the latitude of places on the tropic which that point of the ecliptic touches; and as the ſun never goes without the tropics, he can never be vertical to any place that lies without them.

13. *To all places in the* * *torrid zone, the duration of the twilight is leaſt, becauſe the ſun's daily motion is the moſt perpendicular to the horizon. In the frigid* † *zones, greateſt; becauſe the ſun's daily motion is nearly parallel to the horizon; and therefore he is the longer of getting* 18 *degrees below it (till which time the twilight always continues.) And in the* ‡ *temperate zones it is at a medium between the two, becauſe the obliquity of the ſun's daily motion is ſo.*

14. *In all places lying exactly under the polar circles, the ſun, when he is in the neareſt tropic, continues* 24 *hours above the horizon without ſetting; becauſe no part of that tropic is below their horizon. And when the ſun is in the fartheſt tropic, he is for the ſame length of time without riſing; becauſe no part of that tropic is above their horizon. But, at all other times of the year, he riſes and ſets there, as in other places; becauſe all the circles that can be drawn parallel to the equator, between the tropics, are more or leſs cut by the horizon, as they are farther from, or nearer to, that tropic which is all above the horizon: and*

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* Between the tropics.
† Between the polar circles and poles.
‡ Between the tropics and polar circles.

when the ſun is not in either of the tropics, his diurnal courſe muſt be in one or other of theſe circles.

15. *To all places in the northern hemiſphere, from the equator to the polar circle, the longeſt day and ſhorteſt night is when the ſun is in the northern tropic; and the ſhorteſt day and longeſt night is when the ſun is in the ſouthern tropic; becauſe no circle of the ſun's daily motion is ſo much above the horizon, and ſo little below it, as the northern tropic; and none ſo little above it, and ſo much below it, as the ſouthern. In the ſouthern hemiſphere, the contrary.*

16. *In all places between the polar circles and poles, the ſun appears for ſome number of days (or rather diurnal revolutions) without ſetting; and at the oppoſite time of the year without riſing; becauſe ſome part of the ecliptic never ſets in the former caſe, and as much of the oppoſite part never riſes in the latter. And the nearer unto, or the more remote from the pole, theſe places are, the longer or ſhorter is the ſun's continuing preſence or abſence.*

17. *If a ſhip ſets out from any port, and ſails round the earth eaſtward to the ſame port again, let her take what time ſhe will to do it in, the people in that ſhip, in reckoning their time, will gain one complete day at their return, or count one day more than thoſe who reſide at the ſame port; becauſe, by going contrary to the ſun's diurnal motion, and being forwarder every evening than they were in the morning, their horizon will get ſo much the ſooner above the ſetting ſun, than if they had kept for a whole day at any particular place. And thus, by cutting off a part proportionable to their own motion, from the length of every day, they will gain a complete day of that ſort at their return; without gaining one moment of abſolute time more*

than is elapſed during their courſe, to the peopl at the port. If they ſail weſtward, they will reckon one day leſs than the people do who reſide a the ſaid port, becauſe by gradually following th apparent diurnal motion of the ſun, they will kee him each particular day ſo much longer above thei horizon, as anſwers to that day's courſe; and b that means, they cut off a whole day in reckonin at their return, without loſing one moment of abſolute time.

Hence, if two ſhips ſhould ſet out at the ſam time from any port, and ſail round the globe, on eaſtward and the other weſtward, ſo as to meet a the ſame port on any day whatever; they will diffe two days in reckoning their time, at their return. If they ſail twice round the earth, they will diffe four days; if thrice, then ſix, &c.

L E C T. IX.

The uſe of the celeſtial globe, and armillary ſphere.

The *celeſtial globe.*

HAVING done for the preſent with the terreſtrial globe, we ſhall proceed to the uſe of the celeſtial; firſt premiſing, that as the equator, ecliptic, tropics, polar circles, horizon, and braſen meridian, are exactly alike on both globes, all the former problems concerning the ſun are ſolved the ſame way by both globes. The method alſo of rectifying the celeſtial globe is the ſame as rectifying the terreſtrial, viz. Elevate the pole according to the latitude of your place, then ſcrew the quadrant of altitude to the zenith, on the braſs meridian; bring the ſun's place in the ecliptic to the graduated edge of the braſs meridian, on the ſide

To recti-*fy it.*

ſide which is above the ſouth point of the wooden horizon, and ſet the hour-index to the uppermoſt XII, which ſtands for noon.

N. B. The ſun's place for any day of the year ſtands directly over that day on the horizon of the celeſtial globe, as it does on that of the terreſtrial.

Latitude and longitude of the ſtars.

The *latitude* and *longitude* of the ſtars, and of all other celeſtial phenomena, are reckoned in a very different manner from the latitude and longitude of places on the earth: for all terreſtrial latitudes are reckoned from the equator; and longitudes from the meridian of ſome remarkable place, as of London by the Britiſh, and of Paris by the French; though moſt of the French maps begin their longitude at the meridian of the iſland *Ferro.*——But the aſtronomers of all nations agree in reckoning the *latitudes* of the moon, ſtars, planets, and comets, from the *ecliptic*; and their *longitudes* from the * *equinoctial colure*, in that ſemicircle of it which cuts the ecliptic at the beginning of *Aries* ♈; and thence eaſtward, quite round, to the ſame ſemicircle again. Conſequenlty thoſe ſtars which lie between the equinoctial and the northern half of the ecliptic, have north declination and ſouth latitude; thoſe which lie between the equinoctial and the ſouthern half of the ecliptic, have ſouth declination and north latitude; and

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Colures.

* *The great circle* that paſſes through the *equinoctial points* at the beginning of ♈ and ♎, and through the *poles of the world* (which are two oppoſite points, each 90 degrees from the equinoctial) is called the *equinoctial colure*: and the *great circle* that paſſes through the *beginning of* ♋ and ♑, and alſo through the *poles of the ecliptic*, and poles of the world, is called the *ſolſtitial colure.*

all thoſe which lie between the tropics and poles, have their declinations and latitudes of the ſame denomination.

There are ſix great circles on the celeſtial globe, which cut the ecliptic perpendicularly, and meet in two oppoſite points in the polar circles; which points are each ninety degrees from the ecliptic, and are called its poles. Theſe polar points divide thoſe circles into 12 ſemicircles; which cut the ecliptic at the beginnings of the 12 ſigns. They reſemble ſo many meridians on the terreſtrial globe; and as all places which lie under any particular meridian ſemicircle on that globe; have the ſame longitude, ſo all thoſe points of the heaven, through which any one of the above ſemicircles are drawn, have the ſame longitude.—And as the greateſt latitudes on the earth are at the north and ſouth poles of the earth, ſo the greateſt latitudes in the heaven, are at the north and ſouth poles of the ecliptic.

Conſtellations. In order to diſtinguiſh the ſtars, with regard to their ſituations and poſitions in the heaven, the ancients divided the whole viſible firmament of ſtars into particular ſyſtems, which they called *conſtellations*; and digeſted them into the forms of ſuch animals as are delineated upon the celeſtial globe. And thoſe ſtars which lie between the figures of thoſe imaginary animals, and could not be brought within the compaſs of any of them, were called *unformed ſtars.*

Becauſe the moon and all the planets were obſerved to move in circles or orbits which croſs the ecliptic (or line of the ſun's path) at ſmall angles, and to be on the north ſide of the ecliptic for one half of their courſe round the heaven of ſtars, and on the ſouth ſide of it for the other half, but never to go quite 8 degrees from it on either ſide, the ancients diſtinguiſhed that ſpace by two leſſer circles, parallel to the ecliptic (one on each ſide) at 8 degrees diſtance from it. And the ſpace included between the circles, they called the *zodiac*, becauſe moſt of the 12 conſtellations placed therein reſemble ſome living creature.—Theſe conſtellations are, 1. *Aries* ♈, the ram; 2. *Taurus* ♉, the bull; 3. *Gemini* ♊, the twins; 4. *Cancer* ♋, the crab; 5. *Leo* ♌, the lion; 6. *Virgo* ♍, the virgin: 7. *Libra* ♎, the balance; 8 *Scorpio* ♏, the ſcorpion; 9. *Sagittarius* ♐, the archer; 10. *Capricornus* ♑, the goat; 11. *Aquarius* ♒, the water bearer; and 12. *Piſces* ♓, the fiſhes.

It is to be obſerved, that in the infancy of aſtronomy, theſe twelve conſtellations ſtood at or near the places of the ecliptic, where the above characteriſtics are marked upon the globe: but now, each conſtellation has got a whole ſign forwarder, on account of the receſſion of the equinoctial points from their former places. So that the conſtellation of *Aries*, is now in the former place of *Taurus*; that of *Taurus*, in the former place of *Gemini*; and ſo on. **Remark.**

The ſtars appear of different magnitudes to the eye; probably becauſe they are at different diſtances from us. Thoſe which appear brighteſt and largeſt, are called *ſtars of the firſt magnitude*; the next to them in ſize and luſtre, are called *ſtars of the ſecond magnitude*; and ſo on to the *ſixth*, which are the ſmalleſt that can be diſcerned by the bare eye.

Some of the moſt remarkable ſtars have names given them, as *Caſtor* and *Pollux* in the heads of the *Twins*, *Sirius* in the mouth of the *Great Dog*, *Procyon* in the ſide of the *Little Dog*, *Rigel* in the left foot of *Orion*, *Arcturus* near the right thigh of *Bootes*, &c.

Theſe things being premiſed, which I think are all that the young *Tyro* need be acquainted with, before he begins to work any problem by this globe, we ſhall now proceed to the moſt uſeful of thoſe problems; omitting ſeveral which are of little or no conſequence.

PROBLEM I.

To find the[*] *right aſcenſion and*[†] *declination of the ſun, or any fixed ſtar.*

Bring the ſun's place in the ecliptic to the braſen meridian, then that degree in the equinoctial which is cut by the meridian, is the ſun's *right aſcenſion*; and that degree of the meridian which is over the ſun's place, is his *declination*. Bring any fixed ſtar to the meridian, and its *right aſcenſion* will be cut by the meridian in the equinoctial; and the degree of the meridian that ſtands over it, is its *declination*.

So that *right aſcenſion* and *declination*, on the celeſtial globe, are found in the ſame manner as *longitude* and *latitude* on the terreſtrial.

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[*] The degree of the equinoctial, reckoned from the beginning of *Aries*, that comes to the meridian with the ſun or ſtar, is its *right aſcenſion*.

[†] The diſtance of the ſun or ſtar in degrees from the equinoctial, towards either of the poles, north or ſouth, is its declination, which is north or ſouth accordingly.

PROBLEM II.

To find the latitude and longitude of any ſtar.

If the given ſtar be on the north ſide of the ecliptic, place the 90th degree of the quadrant of altitude on the north pole of the ecliptic, where the twelve ſemicircles meet; which divide the ecliptic into the 12 ſigns: but if the ſtar be on the ſouth ſide of the ecliptic, place the 90th degree of the quadrant on the ſouth pole of the ecliptic: keeping the 90th degree of the quadrant on the proper pole, turn the quadrant about, until its graduated edge cuts the ſtar: then, the number of degrees in the quadrant, between the ecliptic and the ſtar, is its latitude; and the degree of the ecliptic cut by the quadrant is the ſtar's longitude, reckoned according to the ſign in which the quadrant then is.

PROBLEM III.

To repreſent the face of the ſtarry firmament, as ſeen from any given place of the earth, at any hour of the night.

Rectify the celeſtial globe for the given latitude, the zenith, and ſun's place, in every reſpect, as taught by the 17th problem, for the terreſtrial; and turn it about, until the index points to the given hour: then, the upper hemiſphere of the globe will repreſent the viſible half of the heaven for that time: all the ſtars upon the globe being then in ſuch ſituations, as exactly correſpond to thoſe in the heaven. And if the globe be placed duly north and ſouth, by means of a ſmall ſea-compaſs, every ſtar on the globe will point toward the like ſtar in the heaven: by which means, the conſtellations and remarkable ſtars may be eaſily known. All thoſe ſtars which are in the eaſtern ſide of the horizon, are then riſing in the eaſtern ſide of the heaven; all in the weſtern, are ſetting in the weſtern ſide; and all thoſe under the upper part of the braſen meridian, between the ſouth point of the horizon and the north pole, are at their greateſt altitude, if the latitude of the place be north: but if the latitude be ſouth, thoſe ſtars which lie under the upper part of the meridian, between the north point of the horizon and the ſouth pole, are at their greateſt altitude.

PROBLEM IV.

The latitude of the place, and day of the month, being given; to find the time when any known ſtar will riſe, or be on the meridian, or ſet.

Having rectified the globe, turn it about until the given ſtar comes to the eaſtern ſide of the horizon, and the index will ſhew the time of the ſtar's riſing; then turn the globe weſtward, and when the ſtar comes to the braſen meridian, the index will ſhew the time of the ſtar's coming to the meridian of your place; laſtly, turn on, until the ſtar comes to the weſtern ſide of the horizon, and the index will ſhew the time of the ſtar's ſetting.

N. B. In northern latitudes, thoſe ſtars which are leſs diſtant from the north pole, than the quantity of its elevation above the north point of the horizon, never ſet; and thoſe which are leſs diſtant from the ſouth pole, than the number of degrees by which it is depreſſed below the horizon, never riſe: and *vice verſâ* in ſouthern latitudes.

PROBLEM V.

To find at what time of the year a given ſtar will be upon the meridian, at a given hour of the night.

Bring the given ſtar to the upper ſemicircle of the braſs meridian, and ſet the index to the given hour; then turn the globe, until the index points to XII at noon, and the upper ſemicircle of the meridian will then cut the ſun's place, anſwering to the day of the year ſought; which day may be eaſily found againſt the like place of the ſun among the ſigns on the wooden horizon.

PROBLEM VI.

The latitude, day of the month, and * *azimuth of any known ſtar being given; to find the hour of the night.*

Having rectified the globe for the latitude, zenith, and ſun's place; lay the quadrant of altitude to the given degree of azimuth in the horizon: then turn the globe on its axis, until the ſtar comes to the graduated edge of the quadrant; and when it does, the index will point out the hour of the night.

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* The number of degrees that the ſun, moon, or any ſtar, is from the meridian, either to the eaſt or weſt, is called its *azimuth.*

PROBLEM VII.

The latitude of the place, the day of the month, and altitude[*] *of any known ſtar, being given; to find the hour of the night.*

Rectify the globe as in the former problem, gueſs at the hour of the night, and turn the globe until the index points at the ſuppoſed hour; then lay the graduated edge of the quadrant of altitude over the known ſtar, and if the degree of the ſtar's height in the quadrant upon the globe, anſwers exactly to the degree of the ſtar's obſerved altitude in the heaven, you have gueſſed exactly: but if the ſtar on the globe is higher or lower than it was obſerved to be in the heaven, turn the globe backwards or forwards, keeping the edge of the quadrant upon the ſtar, until its center comes to the obſerved altitude in the quadrant; and then, the index will ſhew the true time of the night.

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[*] The number of degrees that the ſtar is above the horizon, as obſerved by means of a common quadrant, is called its *altitude*.

PROBLEM VIII.

An eaſy method for finding the hour of the night by any two known ſtars, without knowing either their altitude or azimuth; and then, of finding both their altitude and azimuth, and thereby the true meridian.

Tie one end of a thread to a common muſket bullet; and, having rectified the globe as above, hold the other end of the thread in your hand, and carry it ſlowly round betwixt your eye and the ſtarry heaven, until you find it cuts any two known ſtars at once. Then, gueſſing at the hour of the night, turn the globe until the index points to the time in the hour-circle; which done, lay the graduated edge of the quadrant over any one of theſe two ſtars on the globe, which the thread cut in the heaven. If the ſaid edge of the quadrant cuts the other ſtar alſo, you have gueſſed the time exactly; but if it does not, turn the globe ſlowly backwards or forwards, until the quadrant (kept upon either ſtar) cuts them both through their centers: and then, the index will point out the exact time of the night; the degree of the horizon, cut by the quadrant, will be the true azimuth of both theſe ſtars from the ſouth; and the ſtars themſelves will cut their true altitudes in the quadrant. At which moment, if a common azimuth compaſs be ſo ſet upon a floor or level pavement, that theſe ſtars in the heaven may have the ſame bearing upon it (allowing for the variation of the needle) as the quadrant of altitude has in the wooden horizon of the globe, a thread extended over the north and ſouth points of that compaſs will be directly in the plane of the meridian: and if a line be drawn upon the floor or pavement, along the courſe of the thread, and an upright wire be placed in the ſouthmoſt end of the line, the ſhadow of the wire will fall upon that line, when the ſun is on the meridian, and ſhines upon the pavement.

PROBLEM IX.

To find the place of the moon, or of any planet; and thereby to ſhew the time of its riſing, ſouthing, and ſetting.

Seek in *Parker*'s or *White*'s Ephemeris the [*] geocentric place of the moon or planet in the ecliptic, for the given day of the month, and, according to its longitude and latitude, as ſhewn by the Ephemeris, mark the ſame with a chalk upon the globe. Then, having rectified the globe, turn it round its axis weſtward; and as the ſaid mark comes to the eaſtern ſide of the horizon, to the braſen meridian, and to the weſtern ſide of the horizon, the index will ſhew at what time the planet riſes, comes to the meridian, and ſets, in the ſame manner as it would do for a fixed ſtar.

PROBLEM X.

To explain the phenomena of the harveſt moon.

In order to do this, we muſt premiſe the following things. 1. That as the ſun goes only once a year round the ecliptic, he can be but once a year in any particular point of it: and that his motion is almoſt a degree every 24 hours, at a mean rate. 2. That as the moon goes round the ecliptic once in 27 days and 8 hours, ſhe advances 13⅙ degrees in it, every day at a mean rate. 3. That as the ſun goes through part of the ecliptic in the time the moon goes round it, the moon cannot at any time be either in conjunction with the ſun, or oppoſite to him, in that part of the ecliptic where ſhe was ſo the laſt time before; but muſt travel as much forwarder, as the ſun has advanced in the ſaid time: which being 29½ days, makes almoſt a whole ſign. Therefore, 4. The moon can be but once a year oppoſite to the ſun, in any particular part of the ecliptic. 5. That the moon is never full but when ſhe is oppoſite to the ſun, becauſe at no other time can we ſee all that half of her, which the ſun enlightens. 6. That when any point of the ecliptic riſes, the oppoſite point ſets. Therefore, when the moon is oppoſite to the ſun, ſhe muſt riſe at * ſun ſet. 7. That the different ſigns of the ecliptic riſe at very different angles or degrees of obliquity with the horizon, eſpecially in conſiderable latitudes; and that the ſmaller this angle is, the greater is the portion of the ecliptic that riſes in any ſmall part of time; and *vice verſâ*. 8. That, in northern latitudes, no part of the ecliptic riſes at ſo ſmall an angle with the horizon, as *Piſces* and *Aries* do; therefore, a greater portion of the ecliptic riſes in one hour, about theſe ſigns, than about any of the reſt. 9. That the moon can never be full in *Piſces* and *Aries* but in our autumnal months, for at no other time of the year is the ſun in the oppoſite ſigns *Virgo* and *Libra.*

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[*] The place of the moon or planet, as ſeen from the earth, is called its geocentric place.

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* This is not always ſtrictly true, becauſe the moon does not keep in the ecliptic, but croſſes it twice every month. However, the difference need not be regarded in a general explanation of the cauſe of the harveſt moon.

Theſe things premiſed, take 13$\frac{1}{6}$ degrees of the ecliptic in your compaſſes, and beginning at *Piſces*, carry that extent all round the ecliptic, marking the places with a chalk, where the points of the compaſſes ſucceſſively fall. So you will have the moon's daily motion marked out for one complete revolution in the ecliptic (according to § 2 of the laſt paragraph.)

Rectify the globe for any conſiderable northern latitude, (as ſuppoſe that of London) and then, turning the globe round its axis, obſerve how much of the hour circle the index has gone over, at the riſing of each particular mark on the ecliptic; and you will find that ſeven of the marks (which take in as much of the ecliptic as the moon goes through in a week) will all riſe ſucceſſively about *Piſces* and *Aries* in the time that the index goes over two hours. Therefore, whilſt the moon is in *Piſces* and *Aries*, ſhe will not differ in general above two hours in her riſing for a whole week. But if you take notice of the marks on the oppoſite ſigns, *Virgo* and *Libra*, you will find that ſeven of them take nine hours to riſe; which ſhews, that when the moon is in theſe two ſigns, ſhe differs nine hours in her riſing within the compaſs of a week. And ſo much later as every mark is of riſing than the one that roſe next before it, ſo much later will the moon be of riſing on any day than ſhe was on the day before, in the correſponding part of the heaven. The marks about *Cancer* and *Capricorn* riſe at a mean difference of time between thoſe about *Aries* and *Libra*.

Now, although the moon is in *Piſces* and *Aries* every month, and therefore muſt riſe in thoſe ſigns within the ſpace of two hours later for a whole week, or only about 17 minutes later every day then ſhe did on the former; yet ſhe is never full in theſe ſigns, but in our autumnal months, *Auguſt* and *September*, when the ſun is in *Virgo* and *Libra*. Therefore, no full moon in the year will continue to riſe ſo near the time of ſun ſet for a week or ſo, as theſe two full moons do, which fall in the time of harveſt.

In the winter months, the moon is in *Piſces* and *Aries* about her firſt quarter; and as theſe ſigns riſe about noon in winter, the moon's riſing in them paſſes unobſerved. In the ſpring months, the moon changes in theſe ſigns, and conſequently riſes at the ſame time with the ſun; ſo that it is impoſſible to ſee her at that time. In the ſummer months ſhe is in theſe ſigns about her third quarter, and riſes not until mid-night, when her riſing is but very little taken notice of; eſpecially as ſhe is on the decreaſe. But in the harveſt months ſhe is at the full, when in theſe ſigns, and being oppoſite to the ſun, ſhe riſes when the ſun ſets (or ſoon after) and ſhines all the night.

In ſouthern latitudes, *Virgo* and *Libra* riſe at as ſmall angles with the horizon, as *Piſces* and *Aries* do in the northern; and as our ſpring is at the time of their harveſt, it is plain their harveſt full moons muſt be in *Virgo* and *Libra*; and will therefore riſe with as little difference of time, as ours do in *Piſces* and *Aries*.

For a fuller account of this matter, I muſt refer the reader to my Aſtronomy, in which it is deſcribed at large.

PROBLEM XI.

To explain the equation of time, or difference of time between well regulated clocks and true ſun-dials.

The earth's motion on its axis being perfectly equable, and thereby cauſing an apparent equable motion of the ſtarry heaven round the ſame axis, produced to the poles of the heaven; it is plain that equal portions of the celeſtial equator paſs over the meridian in equal parts of time, becauſe the axis of the world is perpendicular to the plane of the equator. And therefore, if the ſun kept his annual courſe in the celeſtial equator, he would always revolve from the meridian to the meridian again in 24 hours exactly, as ſhewn by a well-regulated clock.

But as the ſun moves in the ecliptic, which is oblique both to the plane of the equator and axis of the world, he cannot always revolve from the meridian to the meridian again in 24 equal hours; but ſometimes a little ſooner, and at other times a little latter, becauſe equal portions of the ecliptic paſs over the meridian in unequal parts of time on account of its obliquity. And this difference is the ſame in all latitudes.

To ſhew this by a globe, make chalk-marks all round the equator and ecliptic, at equal diſtances from one another (ſuppoſe 10 degrees) beginning at *Aries* or at *Libra*, where theſe two circles interſect each other. Then turn the globe

Fig. 2.

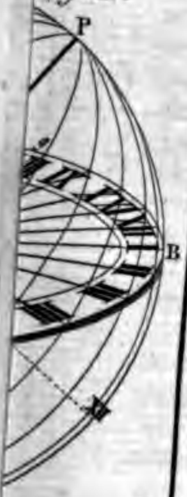
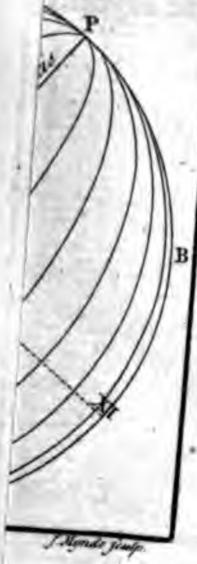


Fig. 3.



J. Mynde sculp.

globe round its axis, and you will ſee that all the marks in the firſt quadrant of the ecliptic, or from the beginning of *Aries* to the beginning of *Cancer*, come ſooner to the braſen meridian than their correſponding marks do on the equator: thoſe in the ſecond quadrant, or from the beginning of *Cancer* to the beginning of *Libra*, come later: thoſe in the third quadrant, from *Libra* to *Capricorn*, ſooner; and thoſe in the fourth, from *Capricorn* to *Aries*, later. But thoſe at the beginning of each quadrant come to the meridian at the ſame time with their correſponding marks on the equator.

Therefore, whilſt the ſun is in the firſt and third quadrants of the ecliptic, he comes ſooner to the meridian every day than he would do if he kept in the equator; and conſequently he is faſter than a well regulated clock, which always keeps equable or equatorial time: and whilſt he is in the ſecond and fourth quadrants, he comes latter to the meridian every day than he would do if he kept in the equator; and is therefore ſlower than the clock. But at the beginning of each quadrant, the ſun and clock are equal.

And thus, if the ſun moved equably in the ecliptic, he would be equal with the clock on four days of the year, which would have equal intervals of time between them. But as he moves faſter at ſome times than at others (being eight days longer in the northern half of the ecliptic than in the ſouthern) this will cauſe a ſecond inequality; which combined with the former, ariſing from the obliquity of the ecliptic to the equator, makes up that difference, which is ſhewn by the common equation tables to be between good clocks and true ſun-dials.

The deſcription and uſe of the armillary ſphere.

Plate XX. Fig. 1. Whoever has ſeen a common *armillary ſphere*, and underſtands how to uſe it, muſt be ſenſible that the machine here referred to, is of a very different, and much more advantageous conſtruction. And thoſe who have ſeen the curious glaſs ſphere invented by Dr. LONG, or the figure of it in his Aſtronomy, muſt know that the furniture of the terreſtrial globe in this machine, the form of the pedeſtal, and the manner of turning either the earthly globe or the circles which ſurround it, are all copied from the Doctor's glaſs ſphere; and that the only difference is, a parcel of rings inſtead of a glaſs celeſtial globe; and all the additions are a moon within the ſphere, and a ſemicircle upon the pedeſtal.

The armillary ſphere. The exterior parts of this machine are a compages of braſs rings, which repreſent the principal circles of the heaven, viz. 1. The equinoctial *A A*, which is divided into 360 degrees, (beginning at its interſection with the ecliptic in *Aries*) for ſhewing the ſun's right aſcenſion in degrees; and alſo into 24 hours, for ſhewing his right aſcenſion in time. 2. The ecliptic *B B*, which is divided into 12 ſigns, and each ſign into 30 degrees, and alſo into the months and days of the year; in ſuch a manner, that the degree or point of the ecliptic in which the ſun is, on any given day, ſtands over that day in the circle of months. 3. The tropic of *Cancer C C*, touching the ecliptic at the beginning of *Cancer* in *e*, and the tropic of *Capricorn D D*, touching the ecliptic at the beginning of *Capricorn* in *f*; each 23½ degrees from the equinoctial circle. 4. The arctic circle *E*, and the antarctic circle *F*, each 23½ degrees from its reſpective pole at *N* and *S*. 5. The equinoctial colure *G G*, paſſing through the north and ſouth poles of the heaven at *N* and *S*, and through the equinoctial points *Aries*, and *Libra* in the ecliptic. 6. The ſolſtitial colure *H H*, paſſing through the poles of the heaven, and through the ſolſtitial points *Cancer* and *Capricorn*, in the ecliptic. Each quarter of the former of theſe colures is divided into 90 degrees, from the equinoctial to the poles of the world, for ſhewing the declination of the ſun, moon, and ſtars; and each quarter of the latter, from the ecliptic at *e* and *f*, to its poles *b* and *d*, for ſhewing the latitude of the ſtars.

In the north pole of the ecliptic is a nut *b*, to which is fixed one end of a quadrantal wire, and to the other end a ſmall ſun *Y*, which is carried round the ecliptic *B B*, by turning the nut: and in the ſouth-pole of the ecliptic is a pin at *d*, on which is another quadrantal wire, with a ſmall moon *Z* upon it, which may be moved round by hand: but there is a particular contrivance for cauſing the moon to move in an orbit which croſſes the ecliptic at an angle of 5⅓ degrees, in two oppoſite points called the *moon's nodes*; and alſo for ſhifting theſe points backward in the ecliptic, as the *moon*'s *nodes* ſhift in the heaven.

Within theſe circular rings is a ſmall terreſtrial globe *I*, fixt on an axis *K K*, which extends from the north and ſouth poles of the globe at *n* and *s*, to thoſe of the celeſtial ſphere at *N* and *S*. On this axis is fixt the flat celeſtial meridian *L L*, which may be ſet directly over the meridian of any place on the globe, and then turned round with the globe, ſo as to keep over the ſame meridian upon it. This flat meridian is graduated the ſame way as the braſs meridian of a common globe, and its uſe is much the ſame. To this globe is fitted the moveable horizon *M M*, ſo as to turn upon two ſtrong wires proceeding from its eaſt and weſt points to the globe, and entering the globe at oppoſite points of its equator, which is a moveable braſs ring let into the globe in a groove all around its equator. The globe may be turned by hand within this ring, ſo as to place any given meridian upon it, directly under the celeſtial meridian *L L*. The horizon is divided into 360 degrees all around its outermoſt edge, within which are the points of the compaſs, for ſhewing the amplitude of the ſun and moon, both in degrees and points. The celeſtial meridian *L L* paſſes through two notches in the north and ſouth points of the horizon, as in a common globe: but here, if the globe be turned round, the horizon and meridian turn with it. At the ſouth pole of the ſphere is a circle of 24 hours, fixt to the rings, and on the axis is an index which goes round that circle, if the globe be turned round its axis.

The whole fabric is ſupported on a pedeſtal *N*, and may be elevated or depreſſed upon the joint *O*, to any number of degrees from 0 to 90, by means of the arc *P*, which is fixed in the ſtrong braſs arm *Q*, and ſlides in the upright piece *R*, in which is a ſcrew at *r*, to fix it at any proper elevation.

In the box *T* are two wheels (as in Dr. *Long*'s ſphere) and two pinions, whoſe axes come out at *V* and *U*; either of which may be turned by the ſmall winch *W*. When the winch is put upon the axis *V*, and turned backward, the terreſtrial globe, with its horizon and celeſtial meridian, keep at reſt; and the whole ſphere of circles turns round from eaſt, by ſouth, to weſt, carrying the ſun *Y*, and moon *Z*, round the ſame way, and cauſing them to riſe above and ſet below the horizon. But when the winch is put upon the axis *U*, and turned forward, the ſphere with the ſun and moon keep at reſt; and the earth, with its horizon and meridian, turn round from weſt, by ſouth, to eaſt; and bring the ſame points of the horizon to the ſun and moon, to which theſe bodies came when the earth kept at reſt, and they were carried round it; ſhewing that they riſe and ſet in the ſame points of the horizon, and at the ſame time in the hour-circle, whether the motion be in the earth or in the heaven. If the earthly globe be turned, the hour-index goes round its hour-circle; but if the ſphere be turned, the hour-circle goes round below the index.

And ſo, by this conſtruction, the machine is equally fitted to ſhew either the real motion of the earth, or the apparent motion of the heaven.

To rectify the ſphere for uſe, firſt ſlacken the ſcrew *r* in the upright ſtem *R*, and taking hold of the arm *Q*, move it up or down until the given degree of latitude for any place be at the ſide of the ſtem *R*; and then the axis of the ſphere will be properly elevated, ſo as to ſtand parallel to the axis of the world, if the machine be ſet north and ſouth by a ſmall compaſs: this done, count the latitude from the north-pole, upon the celeſtial meridian *L L*, down towards the north notch of the horizon, and ſet the horizon to that latitude; then, turn the nut *b* until the ſun *Y* comes to the given day of the year in the ecliptic, and the ſun will be at its proper place for that day: find the place of the moon's aſcending node, and alſo the place of the moon, by an Ephemeris, and ſet them right accordingly: laſtly, turn the winch *W*, until either the ſun comes to the meridian *L L*, or until the meridian comes to the ſun (according as you want the ſphere or earth to move) and ſet the hour-index to the XII, marked noon, and the whole machine will be rectified.——Then turn the winch, and obſerve when the ſun or moon riſe and ſet in the horizon, and the hour-index will ſhew the times thereof for the given day.

As thoſe who underſtand the uſe of the globes will be at no loſs to work many other problems by this ſphere, it is needleſs to enlarge any farther upon it.

LECT. X.

The principles and art of dialing.

Preliminaries.

A Dial is a plane, upon which lines are deſcribed in ſuch a manner, that the ſhadow of a wire, or of the upper edge of a plate ſtile, erected perpendicularly on the plane of the dial, may ſhew the true time of the day.

The edge of the plate by which the time of the day is found, is called the ſtile of the dial, which muſt be parallel to the earth's axis; and the line on which the ſaid plate is erected, is called the ſubſtile.

The angle included between the ſubſtile and ſtile, is called the elevation, or height of the ſtile.

Thoſe dials whoſe planes are parallel to the plane of the horizon, are called *horizontal dials*;

and thoſe dials whoſe planes are perpendicular to the plane of the horizon, are called vertical, or erect dials.

Thoſe erect dials, whoſe planes directly front the north or ſouth, are called direct north or ſouth dials; and all other erect dials are called decliners, becauſe their planes are turned away from the north or ſouth.

Thoſe dials whoſe planes are neither parallel nor perpendicular to the plane of the horizon, are called inclining, or reclining dials, according as their planes make acute or obtuſe angles with the horizon; and if their planes are alſo turned aſide from facing the ſouth or north, they are called declining-inclining, or declining-reclining dials.

The interſection of the plane of the dial, with that of the meridian, paſſing through the ſtile, is called the meridian of the dial, or the hour-line of XII.

Thoſe meridians, whoſe planes paſs through the ſtile, and make angles of 15, 30, 45, 60, 75, and 90 degrees with the meridian of the place (which marks the hour-line of XII) are called hour-circles; and their interſections with the plane of the dial, are called hour-lines.

In all declining dials, the ſubſtile makes an angle with the hour-line of XII; and this angle is called the diſtance of the ſubſtile from the meridian.

The declining plane's difference of longitude, is the angle formed at the interſection of the ſtile and plane of the dial, by two meridians; one of which paſſes through the hour-line of XII, and the other through the ſubſtile.

This much being premiſed concerning dials in general, we ſhall now proceed to explain the different methods of their conſtruction.

Plate XX. Fig. 2.

The univerſal principle on which dialing depends.

If the whole earth $a\ P\ c\ p$ were tranſparent, and hollow, like a ſphere of glaſs, and had its equator divided into 24 equal parts by ſo many meridian ſemicircles, a, b, c, d, e, f, g, &c. one of which is the geographical meridian of any given place, as London (which is ſuppoſed to be at the point a;) and if the hours of XII were marked at the equator, both upon that meridian and the oppoſite one, and all the reſt of the hours in order on the reſt of the meridians, thoſe meridians would be the hour-circles of London: then, if the ſphere had an opake axis, as $P\ E\ p$, terminating in the poles P and p, the ſhadow of the axis would fall upon every particular meridian and hour, when the ſun came to the plane of the oppoſite meridian, and would conſequently ſhew the time at London, and at all other places on the meridian of London.

Horizontal dial.

If this ſphere was cut through the middle by a ſolid plane $ABCD$, in the rational horizon of London, one half of the axis EP would be above the plane, and the other half below it; and if ſtraight lines were drawn from the center of the plane, to thoſe points where its circumference is cut by the hour-circles of the ſphere, thoſe lines would be the hour-lines of a horizontal dial for London: for the ſhadow of the axis would fall upon each particular hour-line of the dial, when it fell upon the like hour-circle of the ſphere.

Fig. 3.

If the plane which cuts the ſphere be upright, as $AFCG$, touching the given place (London) at F, and directly facing the meridian of London,

don, it will then become the plane of an erect direct ſouth dial: and if right lines be drawn from its center *E*, to thoſe points of its circumference where the hour-circles of the ſphere cut it, theſe will be the hour lines of a vertical or direct ſouth dial for London, to which the hours are to be ſet as in the figure (contrary to thoſe on a horizontal dial) and the lower half *E p* of the axis will caſt a ſhadow on the hour of the day in this dial, at the ſame time that it would fall upon the like hour-circle of the ſphere, if the dial-plane was not in the way. *Vertical dial.*

If the plane (ſtill facing the meridian) be made to incline, or recline, by any given number of degrees, the hour-circles of the ſphere will ſtill cut the edge of the plane in thoſe points to which the hour-lines muſt be drawn ſtraight from the center; and the axis of the ſphere will caſt a ſhadow on theſe lines at the reſpective hours. *Inclining and reclining dials.* The like will ſtill hold, if the plane be made to decline by any given number of degrees from the meridian, towards the eaſt or weſt: *Declining dials.* provided the declination be leſs than 90 degrees, or the reclination be leſs than the co-latitude of the place: and the axis of the ſphere will be a gnomon, or ſtile, for the dial. But it cannot be a gnomon, when the declination is quite 90 degrees, nor when the reclination is equal to the co-latitude; becauſe in theſe two caſes, the axis has no elevation above the plane of the dial.

And thus it appears, that the plane of every dial repreſents the plane of ſome great circle upon the earth; and the gnomon the earth's axis, whether it be a ſmall wire, as in the above figures, or the edge of a thin plate, as in the common horizontal dials.

The whole earth, as to its bulk, is but a point, if compared to its diſtance from the ſun: and therefore, if a ſmall ſphere of glaſs be placed upon any part of the earth's ſurface, ſo that its axis be parallel to the axis of the earth, and the ſphere have ſuch lines upon it, and ſuch planes within it, as above deſcribed; it will ſhew the hours of the day as truly as if it were placed at the earth's center, and the ſhell of the earth were as tranſparent as glaſs.

Fig. 2, 3. But becauſe it is impoſſible to have a hollow ſphere of glaſs perfectly true, blown round a ſolid plane; or if it was, we could not get at the plane within the glaſs to ſet it in any given poſition; we make uſe of a wire-ſphere to explain the principles of dialing, by joining 24 ſemicircles together at the poles, and putting a thin flat plate of braſs within it.

Dialing by the common *terreſtial globe.*

A common globe, of 12 inches diameter, has generally 24 meridian ſemicircles drawn upon it. If ſuch a globe be elevated to the latitude of any given place, and turned about until any one of theſe meridians cuts the horizon in the north point, where the hour of XII is ſuppoſed to be marked, the reſt of the meridians will cut the horizon at the reſpective diſtances of all the other hours from XII. Then, if theſe points of diſtance be marked on the horizon, and the globe be taken out of the horizon, and a flat board or plate be put into its place, even with the ſurface of the horizon; and if ſtraight lines be drawn from the center of the board, to thoſe points of diſtance on the horizon which were cut by the 24 meridian ſemicircles, theſe lines will be the hour-lines of a horizontal dial for that latitude, the edge of whoſe gnomon muſt be in the very ſame ſituation that the axis of the globe was, before it was taken out of the horizon: that is, the gnomon muſt make an angle with the plane of the dial, equal to the latitude of the place for which the dial is made.

If the pole of the globe be elevated to the [*] co-latitude of the given place, and any meridian be brought to the north point of the horizon, the reſt of the meridians will cut the horizon in the reſpective diſtances of all the hours from XII, for a direct ſouth dial, whoſe gnomon muſt make an angle with the plane of the dial, equal to the co-latitude of the place; and the hours muſt be ſet the contrary way on this dial, to what they are on the horizontal.

But if your globe have more than 24 meridian ſemicircles upon it, you muſt take the following method for making *horizontal* and *ſouth dials by it.*

To conſtruct a *horizontal dial.*

Elevate the pole to the latitude of your place, and turn the globe until any particular meridian (ſuppoſe the firſt) comes to the north point of the horizon and the oppoſite meridian will cut the horizon in the ſouth. Then, ſet the hour-index to the uppermoſt XII on its circle; which done, turn the globe weſtward until 15 degrees of the equator paſs under the braſen meridian, and then the hour-index will be at I, (for the ſun moves 15 degrees every hour) and the firſt meridian will cut the horizon in the number of degrees from the north point, that I is diſtant from XII. Turn on, until other 15 degrees of the equator paſs under the braſen meridian, and the hour-index will then be at II, and the firſt meridian will cut the horizon in the number of degrees that II is diſtant from XII: and ſo, by making 15 degrees of the equator paſs under the braſen meridian for every hour, the firſt meridian of the globe will cut the horizon in the diſtances of all the hours from XII to VI, which is juſt 90 degrees; and then you need go no farther, for the diſtances of XI, X, IX, VIII, VII, and VI, in the forenoon, are the ſame from XII, as the diſtances of I, II, III, IV, V, and VI, in the afternoon: and theſe hour-lines continued through the center, will give the oppoſite hour-lines on the other half of the dial: but no more of theſe lines need be drawn, than what anſwer to the ſun's continuance above the horizon of your place on the longeſt day, which may be eaſily found by the 26th problem of the foregoing lecture.

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[*] If the latitude be ſubtracted from 90 degrees, the remainder is called the co-latitude, or complement of the latitude.

Thus, to make a horizontal dial for the latitude of London, which is $51\frac{1}{2}$ degrees north, elevate the north pole of the globe $51\frac{1}{2}$ degrees above the north point of the horizon, and then turn the globe, until the firſt meridian (which is that of London on the Engliſh terreſtrial globe) cuts the north point of the horizon, and ſet the hour-index to XII at noon.

Then, turning the globe weſtward until the index points ſucceſſively to I, II, III, IV, V, and VI, in the afternoon; or until 15, 30, 45, 60, 75, and 90 degrees of the equator paſs under the braſen meridian, you will find that the firſt meridian of the globe cuts the horizon in the following numbers of degrees from the north towards the eaſt, viz. $11\frac{2}{3}$ $24\frac{1}{3}$, $38\frac{1}{12}$ $53\frac{1}{2}$, $71\frac{1}{15}$, and 90; which are the reſpective diſtances of the above hours from XII upon the plane of the horizon.

PLATE XXI.

Fig. 1.

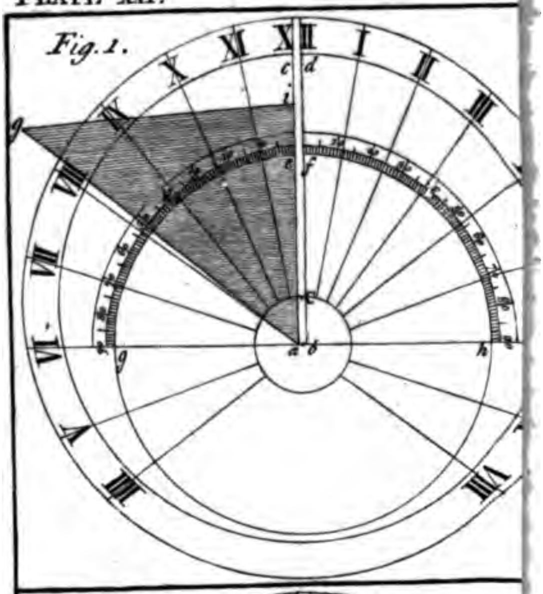

Fig. 3.

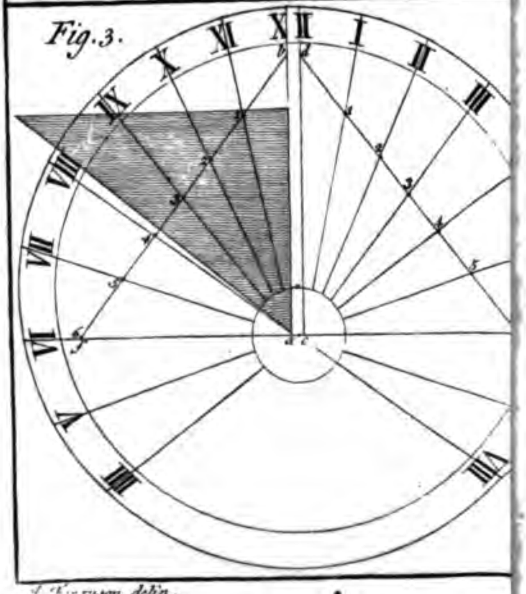

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Plate XXI Fig. 1.

To transfer theſe, and the reſt of the hours, to a horizontal plane, draw the parallel right lines *a c* and *b d* upon that plane, as far from each other as is equal to the intended thickneſs of the gnomon or ſtile of the dial, and the ſpace included between them will be the meridian or twelve o'clock line on the dial. Croſs this meridian at right angles with the ſix o'clock line *g h*, and ſetting one foot of your compaſſes in the interſection *a*, as a center, deſcribe the quadrant *g e* with any convenient radius or opening of the compaſſes: then, ſetting one foot in the interſection *b*, as a center, with the ſame radius deſcribe the quadrant *f h*, and divide each quadrant into 90 equal parts or degrees, as in the figure.

Becauſe the hour-lines are leſs diſtant from each other about noon, than in any other part of the dial, it is beſt to have the centers of theſe quadrants at a little diſtance from the center of the dial-plane, on the ſide oppoſite to XII, in order to enlarge the hour-diſtances thereabout under the ſame angles on the plane. Thus, the center of the plane is at *C*, but the centers of the quadrants at *a* and *b*.

Lay a ruler over the point *b* (and keeping it there for the center of all the afternoon hours in the quadrant *f h*) draw the hour-line of I, through 11⅔ degrees in the quadrant; the hour-line of II, through 24¼ degrees; of III, through 38 1/12 degrees; IIII, through 53½, and V through 71 1/15: and becauſe the ſun riſes about four in the morning, on the longeſt days at London, continue the hour-lines of IIII and V, in the afternoon, through the center *b* to the oppoſite ſide of the dial.—This done, lay the ruler to the center *a*, of the quadrant *e g*, and through the like diviſions or degrees of that quadrant, viz. $11\frac{2}{3}$, $24\frac{1}{4}$, $38\frac{1}{20}$, $53\frac{1}{2}$, and $71\frac{1}{10}$, draw the forenoon hour-lines of XI, X, IX, VIII, and VII; and becauſe the ſun ſets not before eight in the evening on the longeſt days, continue the hour-lines of VII and VIII in the forenoon, through the center *a*, to VII and VIII in the afternoon; and all the hour-lines will be finiſhed on this dial; to which the hours may be ſet, as in the figure.

Laſtly, through $51\frac{1}{2}$ degrees of either quadrant, and from its center, draw the right line *ag* for the hypothenuſe or axis of the gnomon *agi*; and from *g*, let fall the perpendicular *g i*, upon the meridian line *a i*, and there will be a triangle made, whoſe ſides are *a g*, *g i*, and *i a*. If a plate ſimilar to this triangle be made as thick as the diſtance between the lines *a c* and *b d*, and ſet upright between them, touching at *a* and *b*, its hypothenuſe *a g* will be parallel to the axis of the world, when the dial is truly ſet; and will caſt a ſhadow on the hour of the day.

N. B. The trouble of dividing the two quadrants may be ſaved, if you have a ſcale with a line of chords upon it, ſuch as that on the right hand of the plate: for if you extend the compaſſes from 0 to 60 degrees of the line of chords, and with that extent, as a radius, deſcribe the two quadrants upon their reſpective centers, the above diſtances may be taken with the compaſſes upon the line, and ſet off upon the quadrants.

Fig. 2. To conſtruct an *erect ſouth dial.*

To make an erect direct ſouth dial. Elevate the pole to the co-latitude of your place, and proceed in all reſpects as above taught for the horizontal dial, from VI in the morning to VI in the afternoon, only the hours muſt be reverſed, as in the figure; and the hypothenuſe *ag*, of the gnomon *agf*, muſt make an angle with the dial-plane equal to the co-latitude of the place. As the ſun can ſhine no longer on this dial, than from ſix in the morning until ſix in the evening, there is no occaſion for having any more than twelve hours upon it.

To conſtruct an *erect declining dial.*

To make an erect dial, declining from the ſouth towards the eaſt or weſt. Elevate the pole to the latitude of your place, and ſcrew the quadrant of altitude to the zenith. Then, if your dial declines towards the eaſt (which we ſhall ſuppoſe it to do at preſent) count in the horizon the degrees of declination, from the eaſt point towards the north, and bring the lower end of the quadrant to that degree of declination at which the reckoning ends. This done, bring any particular meridian of your globe (as ſuppoſe the firſt meridian) directly under the graduated edge of the upper part of the braſen meridian, and ſet the hour-index to XII at noon. Then, keeping the quadrant of altitude at the degree of declination in the horizon, turn the globe eaſtward on its axis, and obſerve the degrees cut by the firſt meridian in the quadrant of altitude (counted from the zenith) as the hour-index comes to XI, X, IX, &c. in the forenoon, or as 15, 30, 45, &c, degrees of the equator paſs under the braſen meridian at theſe hours reſpectively; and the degrees then cut in the quadrant by the firſt meridian, are the reſpective diſtances of the forenoon hours from XII on the plane of the dial.—Then, for the afternoon hours, turn the quadrant of altitude round the zenith until it comes to the degree in the horizon oppoſite to that where it was placed before; namely, as far from the weſt point of the horizon towards the ſouth, as it was ſet at firſt from the eaſt point towards the north; and turn the globe weſtward on its axis, until the firſt meridian comes to the braſen meridian again, and the hour-index to XII: then, continue to turn the globe weſtward, and as the index points to the afternoon hours I, II, III, &c. or as 15, 30, 45, &c. degrees of the equator paſs under the braſen meridian, the firſt meridian will cut the quadrant of altitude in the reſpective number of degrees from the zenith, that each of theſe hours is from XII on the dial.—And note, that when the firſt meridian goes off the quadrant at the horizon, in the forenoon, the hour-index ſhews the time when the ſun will come upon this dial: and when it goes off the quadrant in the afternoon, the index will point to the time when the ſun goes off the dial.

Having thus found all the hour-diſtances from XII, lay them down upon your dial-plate, either by dividing a ſemicircle into two quadrants of 90 degrees each (beginning at the hour-line of XII) or by the line of chords, as above directed.

In all declining dials, the line on which the ſtile or gnomon ſtands (commonly called the *ſubſtile-line)* makes an angle with the twelve o'clock line, and falls among the forenoon hour-lines, if the dial declines towards the eaſt; and among the afternoon hour-lines, when the dial declines towards the weſt; that is, to the left hand from the twelve o'clock line in the former caſe, and to the right hand from it in the latter.

To find the diſtance of the ſubſtile from the twelve o'clock line; if your dial declines from the ſouth towards the eaſt, count the degrees of that declination in the horizon from the eaſt point toward the north, and bring the lower end of the quadrant of altitude to that degree of declination where the reckoning ends: then, turn the globe until the firſt meridian cuts the horizon in the like number of degrees, counted from the ſouth point toward the eaſt; and the quadrant and firſt meridian will then croſs one another at right angles, and the number of degrees of the quadrant, which are intercepted between the firſt meridian and the zenith, is equal to the diſtance of the ſubſtile line from the twelve o'clock line; and the number of degrees of the firſt meridian, which are intercepted between the quadrant and the north pole, is equal to the elevation of the ſtile above the plane of the dial.

If the dial declines weſtward from the ſouth, count that declination from the eaſt point of the horizon towards the ſouth, and bring the quadrant of altitude to the degree in the horizon at which the reckoning ends; both for finding the forenoon hours, and the diſtance of the ſubſtile from the meridian: and for the afternoon hours, bring the quadrant to the oppoſite degree in the horizon, namely, as far from the weſt towards the north, and then proceed in all reſpects as above.

Thus, we have finiſhed our declining dial; and in ſo doing, we made four dials, viz.

1. A north dial, declining eaſtward by the ſame number of degrees. 2. A north dial, declining the ſame number weſt. 3. A ſouth dial, declining eaſt. And, 4. a ſouth dial declining weſt. Only, placing the proper number of hours, and the ſtile or gnomon reſpectively, upon each plane. For (as above-mentioned) in the ſouth weſt plane, the ſubſtilar-line falls among the afternoon hours; and in the ſouth-eaſt, of the ſame declination among the forenoon

hours, at equal diſtances from XII. And ſo, all the morning hours on the weſt decliner will be like the afternoon hours on the eaſt decliner: the ſouth-eaſt decliner will produce the north-weſt decliner; and the ſouth-weſt decliner, the north-eaſt decliner, by only extending the hour-lines, ſtile and ſubſtile, quite through the center: the axis of the ſtile (or edge that caſts the ſhadow on the hour of the day) being in all dials whatever parallel to the axis of the world, and conſequently pointing towards the north pole of the heaven in north latitudes, and towards the ſouth pole, in ſouth latitudes. *See more of this in the following lecture.*

An eaſy method for conſtructing of *dials.*

But becauſe every one who would like to make a dial, may perhaps not be provided with a globe to aſſiſt him, and may probably not underſtand the method of doing it by logarithmic calculation; we ſhall ſhew how to perform it by the plain dialing lines, or ſcale of latitudes and hours; ſuch as thoſe on the right hand of Fig. 4. in Plate XXI, or at the top of Plate XXII, and which may be had on ſcales commonly ſold by the mathematical inſtrument makers.

This is the eaſieſt of all mechanical methods, and by much the beſt, when the lines are truly divided: not only the half hours and quarters may be laid down by all of them, but every fifth minute by moſt, and every ſingle minute by thoſe where the line of hours is a foot in length.

Fig. 3.

Having drawn your double meridian line *a b*, *c d*, on the plane intended for a horizontal dial, and croſſed it at right angles by the ſix o'clock line *f e* (as in Fig. 1.) take the latitude of your place with the compaſſes, in the ſcale of latitudes, and ſet that extent from *c* to *e*, and from *a* to *f*, on the ſix o'clock line: then, taking the whole ſix hours between the points of the compaſſes in the ſcale of hours, with that extent ſet one foot in the point *e*, and let the other foot fall where it will upon the meridian line *cd*, as at *d*. Do the ſame from *f* to *b*, and draw the right lines *e d* and *fb*, each of which will be equal in length to the whole ſcale of hours. This done, ſetting one foot of the compaſſes in the beginning of the ſcale at XII, and extending the other to each hour on the ſcale, lay off theſe extents from *d* to *e* for the afternoon hours, and from *b* to *f* for thoſe of the forenoon: this will divide the lines *d e* and *b f* in the ſame manner as the hour-ſcale is divided, at 1, 2, 3, 4, 5 and 6; on which the quarters may alſo be laid down, if required. Then, laying a ruler on the point *c*, draw the firſt five hours in the afternoon, from that point, through the dots at the numeral figures 1, 2, 3, 4, 5, on the line *de*; and continue the lines of IIII and V through the center *c* to the other ſide of the dial, for the like hours of the morning; which done, lay the ruler on the point *a*, and draw the laſt five hours in the forenoon through the dots 5, 4, 3, 2, 1, on the line *f b*; continuing the hour-lines of VII and VIII through the center *a* to the other ſide uf the dial, for the like hours of the evening; and ſet the hours to their reſpective lines as in the figure. Laſtly, make the gnomon the ſame way as taught above for the horizontal dial, and the whole will be finiſhed.

To make an erect ſouth dial, take the co-latitude of your place from the ſcale of latitudes, and then proceed in all reſpects for the hour-lines, as in the horizontal dial; only reverſing the hours, as in Fig. 2; and making the angle of the ſtile's height equal to the co-latitude.

I have drawn out a ſet of dialing lines upon the top of the 22d Plate, large enough for making a dial of nine inches diameter, or more inches if required; and have drawn them tolerably exact for common practice, to every quarter of an hour. This ſcale may be cut off from the plate, and paſted on wood, or upon the inſide of one of the boards of this book; and then it will be ſomewhat more exact than it is on the plate, for being rightly divided upon the copper-plate, and printed off on wet paper, it ſhrinks as the paper dries: but when it is wetted again, it ſtretches to the ſame ſize as when newly printed; and if paſted on while wet, it will remain of that ſize afterwards.

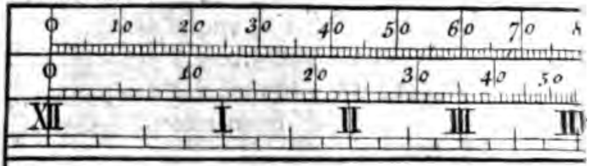
But leſt the young dialiſt ſhould have neither globe nor wooden ſcale, and ſhould tear or otherwiſe ſpoil the paper one in paſting, we ſhall now ſhew him how he may make a dial without any of theſe helps. Only, if he has not a line of chords, he muſt divide a quadrant into 90 equal parts or degrees for taking the proper angle of the ſtile's elevation, which is eaſily done.

Fig. 4.

Horizontal dial.

With any opening of the compaſſes, as *Z L*, deſcribe the two ſemicircles *L F k* and *L Q k*, upon the centers *Z* and *z*, where the ſix o'clock line croſſes the double meridian line, and divide each ſemicircle into 12 equal parts, beginning at *L* (though, ſtrictly ſpeaking, only the quadrants from *L* to the ſix o'clock line need be divided): then connect the diviſions which are equidiſtant from *L*, by the parallel lines *KM*, *IN*, *HO*, *GP*, and *FQ*, Draw *V Z* for the hypothenuſe of the ſtile, making the angle *V Z E* equal to the latitude of your place; and continue the line *V Z* to *R*. Draw the line *R r* parallel to the ſix o'clock line, and ſet off the diſtance *a K* from *Z* to *Y*, the

PLATE XXII.



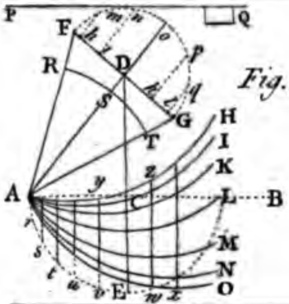


Fig. 3.

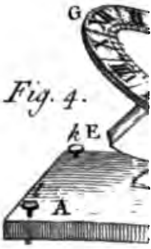


Fig. 4.

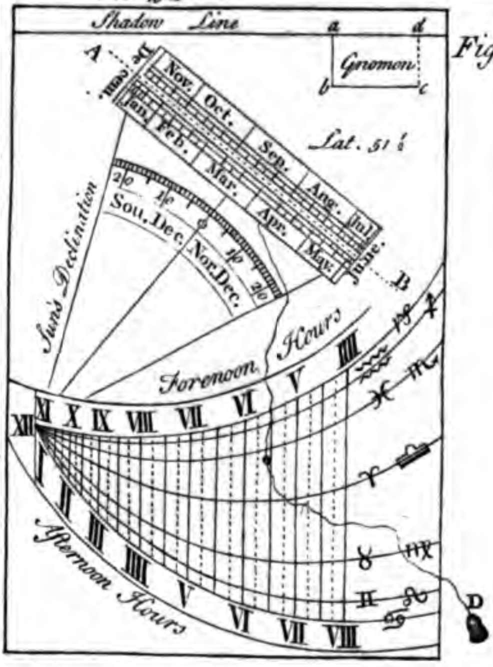
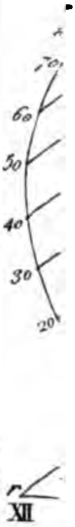


Fig. 2.



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diſtance *b I* from *Z* to *X*, *c H* from *Z* to *W*, from *Z* to *T*, and *e F* from *Z* to *S*. Then draw the lines *S s*, *T t*, *W w*, *X x*, and *Y y*, each parallel to *R r*. Set off the diſtance *y Y* from *a* to 11, and from *f* to 1; the diſtance *x X* from *b* to 10, and from *g* to 2; *w W* from *c* to 9, and from *h* to 3; *t T* from *d* to 8, and from *i* to 4; *s S* from *e* to 7, and from *n* to 5. Then laying a ruler to the center *Z*, draw the forenoon hour lines through the points 11, 10, 9, 8, 7; and laying it to the center *z*, draw the afternoon lines through the points 1, 2, 3, 4, 5; continuing the forenoon lines of VII and VIII through the center *Z*, to the oppoſite ſide of the dial, for the like afternoon hours; and the afternoon lines IIII and V through the center *z*, to the oppoſite ſide, for the like morning hours. Set the hours to theſe lines as in the figure, and then erect the ſtile or gnomon, and the horizontal dial will be finiſhed.

To conſtruct a ſouth dial, draw the line *V Z*, making an angle with the meridian *Z L* equal to the co-latitude of your place; and proceed in all reſpects as in the above horizontal dial for the ſame latitude, reverſing the hours as in Fig. 2. and making the elevation of the gnomon equal to the co-latitude. *South dial.*

Perhaps it may not be unacceptable to explain the method of conſtructing the dialing lines, and ſome others; which is as follows.

With any opening of the compaſſes, as *E A*, according to the intended length of the ſcale, deſcribe the circle *A D C B*, and croſs it at right angles by the diameters *C E A* and *D E B*. Divide the quadrant *A B* firſt into 9 equal parts, and then each part into 10; ſo ſhall the quadrant be divided into 90 equal parts or degrees. Draw the

Plate XXII. Fig. 1. *Dialing lines,* how conſtructed.

the right line *AFB* for the chord of this qua-rant, and ſetting one foot of the compaſſes i the point *A*, extend the other to the ſeveral diviſions of the quadrant, and transfer theſe diviſions to the line *AFB* by the arcs 10 10, 20 20, &c. and this will be a line of chords, divided into 90 unequal parts; which, if transferred from the line back again to the quadrant, will divide it equally. It is plain by the figure, that the diſtance from *A* to 60 in the line of chords, is juſt equal to *AE*, the radius of the circle from which that line is made; for if the arc 60 60 be continued, of which *A* is the center, it goes exactly through the center *E* of the arc *AB*.

And therefore, in laying down any number of degrees on a circle, by the line of chords, you muſt firſt open the compaſſes ſo, as to take in juſt 60 degrees upon that line, as from *A* to 60: and then, with that extent, as a radius, deſcribe a circle which will be exactly of the ſame ſize with that from which the line was divided: which done, ſet one foot of the compaſſes in the beginning of the chord line, as at *A*, and extend the other to the number of degrees you want upon the line, which extent, applied to the circle, will include the like number of degrees upon it.

Divide the quadrant *CD* into 90 equal parts, and from each point of diviſion draw right lines as *i*, *k*, *l*, &c. to the line *CE*; all perpendicular to that line, and parallel to *DE*, which will divide *EC* into a line of ſines; and although theſe are ſeldom put among the dialing lines on a ſcale, yet they aſſiſt in drawing the line of latitudes. For, if a ruler be laid upon the point *D*, and over each diviſion in the line of ſines, it will divide the quadrant *CB* into 90 unequal parts, as *B a*, *a b*, &c. ſhewn by the right lines 10 *a*, 20 *b*, 30 *c*, &c. drawn along the edge of the ruler. If the right line *B C* be drawn, ſubtending this quadrant, and the neareſt diſtances *B a*, *B b*, *B c*, &c. be taken in the compaſſes from *B*, and ſet upon this line in the ſame manner as directed for the line of chords, it will make a line of latitudes *B C*, equal in length to the line of chords *A B*, and of an equal number of diviſions, but very unequal as to their lengths.

Draw the right line *D G A*, ſubtending the quadrant *D A*; and parallel to it, draw the right line *r s*, touching the quadrant *D A* at the numeral figure 3. Divide this quadrant into ſix equal parts, as 1, 2, 3, &c. and through theſe points of diviſion draw right lines from the center *E* to the line *r s*, which will divide it at the points where the ſix hours are to be placed, as in the figure. If every ſixth part of the quadrant be ſubdivided into four equal parts, right lines drawn from the center through theſe points of diviſion, and continued to the line *r s*, will divide each hour upon it into quarters.

A dial on a card.

In Fig. 2. we have the repreſentation of a portable dial, which may be eaſily drawn on a card, and carried in a pocket-book. The lines *a d*, *a b* and *b c* of the gnomon muſt be cut quite through the card; and as the end *a b* of the gnomon is raiſed occaſionally above the plane of the dial, it turns upon the uncut line *c d* as on a hinge. The line dotted *A B* muſt be ſlit quite through the card, and the thread muſt be put through the ſlit, and have a knot tied behind, to keep it from being eaſily drawn out. On the other end of this thread is a ſmall plummet *D*, and on the middle of it a ſmall bead for ſhewing the time of the day.

To rectify this dial, ſet the thread in the ſc right againſt the day of the month, and ſtretch the thread from the day of the month over the angular point where the curve lines meet at XII; then ſhift the bead to that point on the thread, and the dial will be rectified.

To find the hour of the day, raiſe the gnomon (no matter how much or how little) and hold the edge of the dial next the gnomon towards the ſun, ſo as the uppermoſt edge of the ſhadow of the gnomon may juſt cover the *ſhadow-line*; and the bead then playing freely on the *face* of the dial, by the weight of the plummet, will ſhew the time of the day among the hour-lines, as it is forenoon or afternoon.

To find the time of ſun-riſing and ſetting, move the thread among the hour-lines, until it either covers ſome one of them, or lies parallel betwixt any two; and then it will cut the time of ſun-riſing among the forenoon hours, and of ſun-ſetting among the afternoon hours, on that day of the year for which the thread is ſet in the ſcale of months.

To find the ſun's declination, ſtretch the thread from the day of the month over the angular point at XII, and it will cut the ſun's declination, as it is north or ſouth, for that day, in the arched ſcale of north and ſouth declination.

To find on what days the ſun enters the ſigns: when the bead, as above rectified, moves along any of the curve lines which have the ſigns of the zodiac marked upon them, the ſun enters thoſe ſigns on the days pointed out by the thread in the ſcale of months.

The conſtruction of this dial is very eaſy, eſpecially if the reader compares it all along with Fig. 3. as he reads the following explanation of that figure.

Fig. 3. Draw the occult line *A B* parallel to the top of the card, and croſs it at right angles with the ſix o'clock line *E C D*; then upon *C*, as a center, with the radius *CA*, deſcribe the ſemicircle *AEL*, and divide it into 12 equal parts (beginning at *A*) as *A r*, *A s*, &c. and from theſe points of diviſion, draw the hour-lines *r*, *s*, *t*, *u*, *v*, *E*, *w*, and *x*, all parallel to the ſix o'clock line *E C*. If each part of the ſemicircle be ſubdivided into four equal parts, they will give the half-hour lines and quarters, as in Fig. 2. Draw the right line *A S D o*, making the angle *SAB* equal to the latitude of your place. Upon the center *A* deſcribe the arch *RST*, and ſet off upon it the arcs *S R* and *S T*, each equal to 23½ degrees, for the ſun's greateſt declination; and divide them into 23½ equal parts, as in Fig. 2. Through the interſection *D* of the lines *E C D* and *A D o*, draw the right line *F D G* at right angles to *A D o*. Lay a ruler to the points *A* and *R*, and draw the line *A R F* through 23½ degrees of ſouth declination in the arc *S R*; and then laying the ruler to the points *A* and *T*, draw the line *A T G* through 23½ degrees of north declination in the arc *S T*: ſo ſhall the lines *A R F* and *A T G* cut the line *F D G* in the proper length for the ſcale of months. Upon the center *D*, with the radius *D F*, deſcribe the ſemicircle *F o G*; and divide it into ſix equal parts, *F m*, *m n*, *n o*, &c. and from theſe points of diviſion draw the right lines *m h*, *n i*, *p k*, and *q l*, each parallel to *o D*. Then ſetting one foot of the compaſſes in the point *F*, extend the other to *A*, and deſcribe the arc *A z H* for the tropic of ♑: with the ſame extent, ſetting one foot in *G*, deſcribe the arc *A E O* for the tropic of ♋. Next ſetting one foot in the point *h*, and extending the other to *A*, deſcribe the arc *A C I* for the beginnings of the ſigns ♒ and ♐; and with the ſame extent, ſetting one foot in the point *l*, deſcribe the arc *A N* for the beginnings of the ſigns ♊ and ♌. Set one foot in the point *i*, and having extended the other to *A*, deſcribe the arc *A K* for the beginnings of the ſigns ♓ and ♏; and with the ſame extent, ſet one foot in *k*, and deſcribe the arc *A M* for the beginnings of the ſigns ♉ and ♍. Then, ſetting one foot in the point *D*, and extending the other to *A*, deſcribe the curve *A L* for the beginnings of ♈ and ♎; and the ſigns will be finiſhed. This done, lay a ruler from the point *A* over the ſun's declination in the arch *R S T* (found by the following table) for every fifth day of the year; and where the ruler cuts the line *F D G*, make marks; and place the days of the months right againſt theſe marks, in the manner ſhewn by Fig. 2. Laſtly, draw the ſhadow line *P Q* parallel to the occult line *A B*; make the gnomon, and ſet the hours to their reſpective lines, as in Fig. 2. and the dial will be finiſhed.

Fig. 4. There are ſeveral kinds of dials, which are called *univerſal*, becauſe they ſerve for all latitudes. Of theſe, the beſt one that I know, is Mr. *Pardie*'s, which conſiſts of three principal parts; the firſt whereof is called the *horizontal plane* (*A*), becauſe in the practice it muſt be parallel to the horizon. In this plane is fixt an upright pin, which enters into the edge of the ſecond part *B D*, called the *meridional plane*; which is made of two pieces, the loweſt whereof (*B*) is called the *quadrant*, becauſe it contains a quarter of a circle, divided into 90 degrees; and it

An *univerſal dial.*

it is only into this part, near *B*, that the pin enters. The other piece is a *ſemicircle* (*D*) adjuſted to the quadrant, and turning in it by a groove, for raiſing or depreſſing the diameter (*EF*) of the ſemicircle, which diameter is called the *axis* of the inſtrument. The third piece is a *circle* (*G*), divided on both ſides into 24 equal parts, which are the hours. This circle is put upon the meridional plane ſo, that the axis (*EF*) may be perpendicular to the circle; and the point *C* be the common center of the circle, ſemicircle, and quadrant. The ſtraight edge of the ſemicircle is chamfered on both ſides to a ſharp edge, which paſſes through the center of the circle. On one ſide of the chamfered part, the firſt ſix months of the year are laid down, according to the ſun's declination for their reſpective days, and on the other ſide the laſt ſix months. And againſt the days on which the ſun enters the ſigns, there are ſtraight lines drawn upon the ſemicircle, with the characters of the ſigns marked upon them. There is a black line drawn along the middle of the upright edge of the quadrant, over which hangs a thread (*H*), with its plummet (*I*), for levelling the inſtrument. *N. B.* From the 23d of September to the 20th of March, the upper ſurface of the circle muſt touch both the center *C* of the ſemicircle, and the line of ♈ and ♎; and from the 20th of March to the 23d of September, the lower ſurface of the circle muſt touch that center and line.

To find the time of the day by this dial. Having ſet it on a level place in ſun-ſhine, and adjuſted it by the leveling ſcrews *k* and *l*, until the plumb line hangs over the black line upon the edge of the quadrant, and parallel to the ſaid edge; move the ſemicircle in the quadrant, until the line of ♈ and ♎ (where the circle touches) comes to the latitude of your place in the quadrant: then, turn the whole meridional plane *B D*, with its circle *G*, upon the horizontal plane *A*, until the edge of the ſhadow of the circle falls preciſely on the day of the month in the ſemicircle; and then, the meridional plane will be due north and ſouth, the axis *E F* will be parallel to the axis of the world, and will caſt a ſhadow upon the true time of the day, among the hours on the circle.

N. B. As, when the inſtrument is thus rectified, the quadrant and ſemicircle are in the plane of the meridian, ſo the circle is then in the plane of the equinoctial. Therefore, as the ſun is above the equinoctial in ſummer (in nothern latitudes) and below it in winter; the axis of the ſemicircle will caſt a ſhadow on the hour of the day, on the upper ſurface of the circle, from the 20th of March to the 23d of September: and from the 23d of September, to the 20th of March, the hour of the day will be determined by the ſhadow of the ſemicircle, upon the lower ſurface of the circle. In the former caſe, the ſhadow of the circle falls upon the day of the month, on the lower part of the diameter of the ſemicircle; and in the latter caſe, on the upper part.

The method of laying down the months and ſigns upon the ſemicircle, is as follows. Draw Fig. 5. the right line *A C B*, equal the diameter of the ſemicircle *A D B*, and croſs it in the middle at right angles with the line *E C D*, equal in length to *A D B*; then *E C* will be the radius of the circle *F C G*, which is the ſame as that of the ſemicircle. Upon *E*, as a center, deſcribe the circle *F C G*, on which ſet off the arcs *C b* and *C i*, each equal to 23½ degrees, and divide them accordingly into that number, for the ſun's declination. Then, laying the edge of a ruler over the center *E*, and alſo over the ſun's declination for every[*] fifth day of each month (as in the card dial) mark the points on the diameter *AB* of the ſemicircle from *a* to *g*, which are cut by the ruler; and *there* place the days of the months accordingly, anſwering the ſun's declination. This done, ſetting one foot of the compaſſes in *C*, and extending the other to *a* or *g*, deſcribe the ſemicircle *a b c d e f g*; which divide into ſix equal parts, and through the points of diviſion draw right lines, parallel to *C D*, for the beginning of the ſines (of which one half are on one ſide of the ſemicircle, and the other half on the other ſide) and ſet the characters of the ſigns to their proper lines, as in the figure.

The following table ſhews the ſun's place and declination, in degrees and minutes, at the noon of every day of the ſecond year after leap-year; which is a mean between thoſe of leap-year itſelf, and the firſt and third years after. It is uſeful for inſcribing the months and their days on ſun-dials; and alſo for finding the latitudes of places, according to the methods preſcribed after the table.

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[*] The intermediate days may be drawn in by hand, if the ſpaces be large enough to contain them.

A Table ſhewing the ſun's place and declination.

January.					February.			
Days.	Sun's Pl.		Sun's Dec.		Days.	Sun's Pl.		Sun's D
	D.	M.	D.	M.		D.	M.	D.
1	11 ♑	5	23 S	1	1	12 ♒	38	17 S
2	12	6	22	55	2	13	39	16
3	13	8	22	49	3	14	40	16
4	14	9	22	43	4	15	41	16
5	15	10	22	37	5	16	41	15
6	16	11	22	29	6	17	42	15
7	17	12	22	22	7	18	43	15
8	18	13	22	14	8	19	43	14
9	19	14	22	5	9	20	44	14
10	20	16	21	56	10	21	45	14
11	21	17	21	47	11	22	45	13
12	22	18	21	37	12	23	46	13
13	23	19	21	27	13	24	46	13
14	24	20	21	17	14	25	47	12
15	25	21	21	6	15	26	47	12
16	26	22	20	54	16	27	48	12
17	27	24	20	43	17	28	48	11
18	28	25	20	30	18	29	48	11
19	29	26	20	18	19	0 ♓	49	11
20	0 ♒	27	20	5	20	1	49	10
21	1	28	19	52	21	2	50	10
22	2	29	19	38	22	3	50	10
23	3	30	19	24	23	4	50	9
24	4	31	19	10	24	5	51	9
25	5	32	18	55	25	6	51	9
26	6	33	18	40	26	7	51	8
27	7	34	18	24	27	8	51	8
28	8	35	18	9	28	9	51	7
29	9	35	17	53				
30	10	36	17	36				
31	11	37	17	19				

In theſe Tables N ſ
niſies north decli
tion, and S ſouth

A T

A Table ſhewing the ſun's place and declination.

March.					April.				
Days.	Sun's Pl.		Sun's Dec.		Days.	Sun's Pl.		Sun's Dec.	
	D.	M.	D.	M.		D.	M.	D.	M.
1	10 ♓	52	7 S	30	1	11 ♈	38	4 N	36
2	11	52	7	7	2	12	37	4	59
3	12	52	6	44	3	13	36	5	22
4	13	52	6	21	4	14	35	5	45
5	14	52	5	58	5	15	34	6	8
6	15	52	5	35	6	16	33	6	31
7	16	51	5	12	7	17	31	6	53
8	17	51	4	48	8	18	30	7	16
9	18	51	4	25	9	19	29	7	38
10	19	51	4	2	10	20	28	8	0
11	20	51	3	38	11	21	27	8	22
12	21	50	3	14	12	22	25	8	44
13	22	50	2	51	13	23	24	9	6
14	23	50	2	27	14	24	23	9	28
15	24	49	2	4	15	25	21	9	49
16	25	49	1	40	16	26	20	10	11
17	26	48	1	16	17	27	18	10	32
18	27	48	0	53	18	28	17	10	53
19	28	48	0	29	19	29	15	11	14
20	29	47	0	5	20	0 ♉	14	11	34
21	0 ♈	47	0 N	19	21	1	12	11	55
22	1	46	0	42	22	2	11	12	15
23	2	45	1	6	23	3	9	12	35
24	3	45	1	29	24	4	7	12	55
25	4	44	1	53	25	5	6	13	14
26	5	43	2	17	26	6	4	13	34
27	6	42	2	40	27	7	2	13	53
28	7	42	3	3	28	8	0	14	12
29	8	41	3	27	29	8	59	14	30
30	9	40	3	50	30	9	57	14	49
31	10	39	4	13	.				

A Table ſhewing the ſun's place and declination.

May.					June.			
Days.	Sun's Pl.		Sun's Dec.		Days.	Sun's Pl.		Sun's
	D.	M.	D.	M.		D.	M.	D.
1	10 ♉	55	15 N	7	1	10 ♊	44	22 N
2	11	53	15	25	2	11	41	22
3	12	51	15	43	3	12	39	22
4	13	49	16	0	4	13	36	22
5	14	47	16	18	5	14	34	22
6	15	45	16	35	6	15	31	22
7	16	43	16	51	7	16	28	22
8	17	41	17	8	8	17	26	22
9	18	39	17	24	9	18	23	22
10	19	36	17	40	10	19	20	23
11	20	34	17	55	11	20	18	23
12	21	32	18	10	12	21	15	23
13	22	30	18	25	13	22	12	23
14	23	28	18	40	14	23	9	23
15	24	25	18	54	15	24	7	23
16	25	23	19	8	16	25	4	23
17	26	21	19	22	17	26	1	23
18	27	19	19	35	18	26	58	23
19	28	16	19	48	19	27	56	23
20	29	14	20	1	20	28	53	23
21	0 ♊	11	20	13	21	29	50	23
22	1	9	20	25	22	0 ♋	47	23
23	2	7	20	37	23	1	45	23
24	3	4	20	48	24	2	42	23
25	4	2	20	59	25	3	39	23
26	4	59	21	10	26	4	36	23
27	5	57	21	20	27	5	33	23
28	6	54	21	30	28	6	31	23
29	7	52	21	39	29	7	28	23
30	8	49	21	49	30	8	25	23
31	9	47	21	57				

A Table ſhewing the ſun's place and declination,

July.					August.				
Days.	Sun's Pl.		Sun's Dec.		Days	Sun's Pl		Sun's Dec.	
	D.	M.	D.	M.		D.	M.	D.	M.
1	9 ♋	22	23 N	8	1	8 ♌	58	18 N	2
2	10	19	23	4	2	9	55	17	47
3	11	16	23	0	3	10	53	17	32
4	12	14	22	55	4	11	50	17	16
5	13	11	22	49	5	12	48	17	0
6	14	8	22	43	6	13	45	16	43
7	15	5	22	37	7	14	43	16	26
8	16	0	22	30	8	15	41	16	9
9	17	2	22	23	9	16	38	15	52
10	17	57	22	16	10	17	36	15	25
11	18	54	22	8	11	18	33	15	17
12	19	51	22	0	12	19	31	14	59
13	20	49	21	52	13	20	29	14	41
14	21	46	21	43	14	21	26	14	23
15	22	43	21	33	15	22	24	14	4
16	23	40	21	22	16	23	22	13	45
17	24	38	21	14	17	24	20	13	26
18	25	35	21	3	18	25	17	13	7
19	26	32	20	52	19	26	15	12	47
20	27	29	20	41	20	27	13	12	27
21	28	27	20	30	21	28	11	12	7
22	29	24	20	18	22	29	9	11	47
23	0 ♌	21	20	6	23	0 ♍	7	11	27
24	1	19	19	54	24	1	5	11	6
25	2	16	19	41	25	2	3	10	46
26	3	13	19	28	26	3	1	10	25
27	4	11	19	14	27	3	59	10	4
28	5	8	19	1	28	4	57	9	43
29	6	6	18	46	29	5	55	9	21
30	7	3	18	32	30	6	53	9	0
31	8	0	18	17	31	7	51	8	38

A Table ſhewing the ſun's place and declination.

September.					October.				
Days.	Sun's Pl.		Sun's Dec		Days.	Sun's Pl.		Sun's Dec.	
	D.	M.	D.	M.		D.	M.	D.	M.
1	8 ♍	49	8 N	16	1	8 ♎	8	3 S	14
2	9	47	7	55	2	9	7	3	37
3	10	46	7	33	3	10	7	4	1
4	11	44	7	10	4	11	6	4	24
5	12	42	6	48	5	12	5	4	47
6	13	40	6	26	6	13	4	5	10
7	14	39	6	3	7	14	4	5	33
8	15	37	5	41	8	15	3	5	56
9	16	35	5	18	9	16	3	6	19
10	17	34	4	55	10	17	2	6	42
11	18	32	4	32	11	18	1	7	5
12	19	31	4	9	12	19	1	7	27
13	20	29	3	46	13	20	0	7	50
14	21	28	3	23	14	21	0	8	12
15	22	26	3	0	15	22	0	8	35
16	23	25	2	37	16	23	0	8	57
17	24	24	2	14	17	23	59	9	19
18	25	22	1	50	18	24	59	9	41
19	26	21	1	27	19	25	58	10	3
20	27	20	1	4	20	26	58	10	24
21	28	19	0	40	21	27	58	10	46
22	29	17	0	17	22	28	58	11	7
23	0 ♎	16	0 S	6	23	29	58	11	28
24	1	15	0	30	24	0 ♏	58	11	49
25	2	14	0	53	25	1	58	12	10
26	3	13	1	17	26	2	58	12	31
27	4	12	1	40	27	3	58	12	51
28	5	11	2	4	28	4	58	13	12
29	6	10	2	27	29	5	58	13	32
30	7	9	2	50	30	6	58	13	51
					31	7	58	14	11

A Table ſhewing the ſun's place and declination.

November.					December.				
Days.	Sun's Pl.		Sun's Dec.		Days.	Sun's Pl.		Sun's Dec.	
	D.	M.	D.	M.		D.	M.	D.	M.
1	8 ♏	58	14 S	30	1	9 ♐	16	21 S	52
2	9	58	14	50	2	10	17	22	1
3	10	59	15	8	3	11	18	22	10
4	11	59	15	27	4	12	19	22	18
5	12	59	15	45	5	13	20	22	26
6	13	59	16	3	6	14	21	22	33
7	15	0	16	21	7	15	22	22	40
8	16	0	16	39	8	16	23	22	46
9	17	0	16	56	9	17	24	22	52
10	18	1	17	13	10	18	25	22	58
11	19	1	17	30	11	19	26	23	3
12	20	2	17	46	12	20	27	23	8
13	21	2	18	2	13	21	28	23	12
14	22	3	18	18	14	22	29	23	15
15	23	4	18	34	15	23	30	23	18
16	24	4	18	49	16	24	31	23	21
17	25	5	19	4	17	25	33	23	24
18	26	5	19	18	18	26	34	23	26
19	27	6	19	32	19	27	35	23	27
20	28	7	19	46	20	28	36	23	28
21	29	7	19	59	21	29	37	23	28
22	0 ♐	8	20	12	22	0 ♑	38	23	28
23	1	9	20	25	23	1	40	23	28
24	2	10	20	37	24	2	41	23	27
25	3	11	20	49	25	3	42	23	25
26	4	11	21	1	26	4	43	23	23
27	5	12	21	12	27	5	44	23	21
28	6	13	21	23	28	6	46	23	18
29	7	14	21	33	29	7	47	23	15
30	8	15	21	43	30	8	48	23	11
					31	9	49	23	6

To find the latitude of any place by obſervation.

The latitude of any place is equal to the elevation of the pole above the horizon of that place. Therefore it is plain, that if a ſtar was fixt in the pole, there would be nothing required to find the latitude, but to take the altitude of that ſtar with a good inſtrument. But although there is no ſtar in the pole, yet the latitude may be found by taking the greateſt and leaſt altitude of any ſtar that never ſets: for if half the difference between theſe altitudes be added to the leaſt altitude, or ſubtracted from the greateſt, the ſum or remainder will be equal to the altitude of the pole at the place of obſervation.

But becauſe the length of the night muſt be more than 12 hours, in order to have two ſuch obſervations; the ſun's meridian altitude and declination are generally made uſe of for finding the latitude, by means of its complement, which is equal to the elevation of the equinoctial above the horizon; and if this complement be ſubtracted from 90 degrees, the remainder will be the latitude: concerning which, I think, the following rules take in all the various caſes.

1. If the ſun has north declination, and is on the meridian, and to the ſouth of your place, ſubtract the declination from the meridian altitude (taken by a good quadrant) and the remainder will be the height of the equinoctial or complement of the latitude north.

EXAMPLE.

Suppoſe	The ſun's meridian altitude	42°	20′	South
	And his declination, ſubd.	10	15	North
Rem. the complement of the lat.		32	5	
Which ſubtract from — —		90	0	
And the remainder is the latitude		57	55	North

2. If the ſun has ſouth declination, and is ſouthward of your place at noon, add the declination to the meridian altitude; the ſum, if leſs than 90 degrees, is the complement of the latitude north: but if the ſum exceeds 90 degrees, the latitude is ſouth; and if 90 be taken from that ſum, the remainder will be the latitude.

EXAMPLES.

The ſun's meridian altitude —	65°	10′	South
The ſun's declination, add —	15	30	South
Complement of the latitude	80	40	
Subtract from — —	90	0	
Remains the latitude —	9	20	North

The ſun's meridian altitude	80°	40′	South
The ſun's declination, add —	20	10	South
The ſum is - - - - - -	100	50	
From which ſubtract - - -	90	0	
Remains the latitude — —	10	50	South.

3. If the ſun has north declination, and is on the meridian north of your place, add the declination to the north meridian altitude; the ſum, if leſs than 90 degrees, is the complement of the latitude ſouth: but if the ſum is more than 90 degrees, ſubtract 90 from it, and the remainder is the latitude north.

EXAMPLES.

Sun's meridian altitude —	60°	30′	North
Sun's declination, add —	20	10	North
Complement of the latitude - -	80	40	
Subtract from - - - - - -	90	0	
Remains the latitude - - -	9	20	South

Sun's meridian altitude - -	70°	20′	North
Sun's declination, add - -	23	20	North
The ſum is - - - - - -	93	40	
From which ſubtract - - -	90	0	
Remains the latitude - - -	3	40	North.

4. If the ſun has ſouth declination, and is north of your place at noon, ſubtract the declination from the north meridian altitude, and the remainder is the complement of the latitude ſouth.

EXAMPLE.

Sun's meridian altitude - -	50°	30′	North
Sun's declination, ſubtract -	20	10	South
Complement of the latitude -	32	20	
Subtract this from - - -	90	0	
And the remainder is the latitude	57	40	South

5. If the ſun has no declination, and is ſouth of your place at noon, the meridian altitude is the complement of the latitude north: but if the ſun be then north of your place, his meridian altitude is the complement of the latitude ſouth.

EXAMPLES.

Sun's meridian altitude - -	38°	30′	South
Subtract from - - - - - -	90	0	
Remains the latitude - -	51	30	North

Sun's meridian altitude - -	38°	30′	North
Subtract from - - - - - -	90	0	
Remains the latitude - -	51	30	South.

6. If you obſerve the ſun beneath the pole, ſubtract his declination from 90 degrees, and add the remainder to his altitude; and the ſum is the latitude.

EXAMPLE.

Sun's declination - - - -	20°	30′	
Subtract from - - - - - -	90	0	
Remains - - - - - - -	69	30	add
Sun's altitude below the pole -	10	20	
The ſum is the latitude - -	79	50.	

Which is north or ſouth, according as the ſun's declination is north or ſouth: for when the ſun has ſouth declination, he is never ſeen below the north pole; nor is he ever ſeen below the ſouth pole, when his declination is north.

7. If the ſun be in the zenith at noon, and at the ſame time has no declination, you are then under the equinoctial, and ſo have no latitude.

8. If the ſun be in the zenith at noon, and has declination, the declination is equal to the latitude, north or ſouth. Theſe two caſes are ſo plain, that they require no examples.

LECT. XI.

Of Dialing.

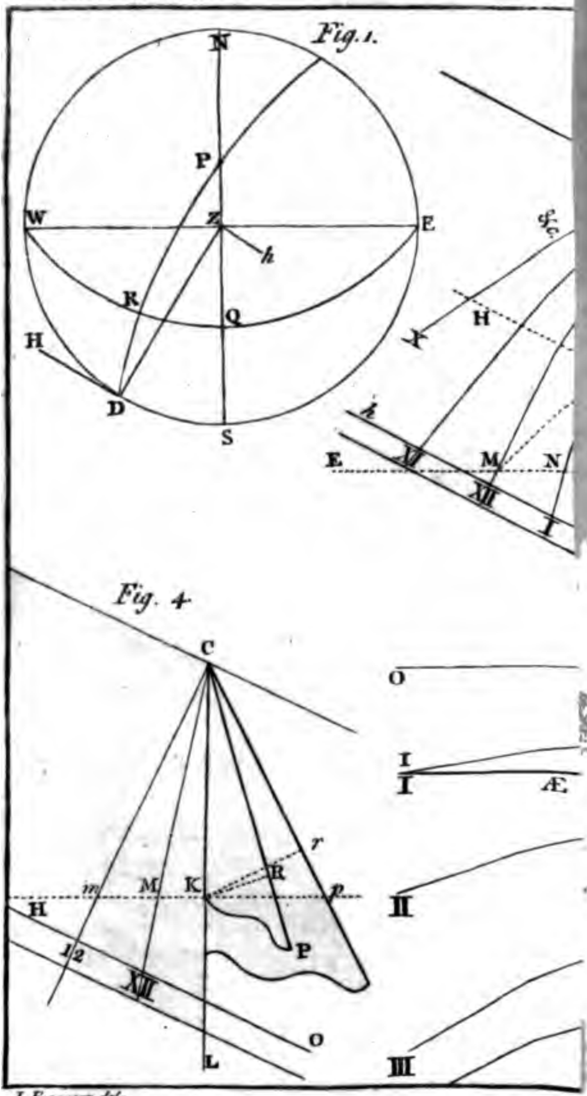
HAVING ſhewn in the preceding Lecture how to make ſun-dials by the aſſiſtance of a good globe, or of a dialing ſcale, we ſhall now proceed to the method of conſtructing dials arithmetically; which will be more agreeable to thoſe who have learnt the elements of trigonometry, becauſe globes and ſcales can never be ſo accurate as the logarithms, in finding the angular diſtances of the hours. Yet, as a globe may be found exact enough for ſome other requiſites in dialing, we ſhall take it in occaſionally.

The conſtruction of ſun-dials on all planes whatever, may be included in one general rule: intelligible, if that of a horizontal dial for any given latitude be well underſtood. For there is no plane, however obliquely ſituated with reſpect to any given place, but what is parallel to the horizon of ſome other place; and therefore, if we can find that other place by a problem on the terreſtrial globe, or by a trigonometrical calculation, and conſtruct a horizontal dial for it; that dial, applied to the plane where it is to ſerve, will be a true dial for that place.—Thus, an erect direct ſouth dial in 51½ degrees north latitude, would be a horizontal dial on the ſame meridian, 90 degrees ſouthward of 51½ degrees north latitude; which falls in with 38½ degrees of ſouth latitude. but if the upright plane declines from facing the ſouth at the given place, it would ſtill be a horizontal plane 90 degrees from that place; but for a different longitude: which would alter the reckoning of the hours accordingly.

CASE I.

1. Let us ſuppoſe that an upright plane at London declines 36 degrees weſtward from facing the ſouth; and that it is required to find a place on the globe, to whoſe horizon the ſaid plane is parallel; and alſo the difference of longitude between London and that place.

PLATE XXIII.



J. Ferguson del.

Which difference of longitude being converted into time, is 2 hours 51 minutes.

The vertical dial declining weſtward 36 degrees at London, is therefore to be drawn in all reſpects as a horizontal dial for ſouth latitude 10¼ degrees; ſave only, that the reckoning of the hours is to anticipate the reckoning on the horizontal dial, by 2 hours 51 minutes: for ſo much ſooner will the ſun come to the meridian of London, than to the meridian of any place whoſe longitude is 42¾ degrees weſt from London.

2. But to be more exact than the globe will ſhew us, we ſhall uſe a little trigonometry.

Plate XXIII. Fig. 1.

Let $NESW$ be the horizon of London, whoſe zenith is Z, and P the north pole of the ſphere; and let Zb be the poſition of a vertical plane at Z, declining weſtward from S (the ſouth) by an angle of 36 degrees; on which plane an erect dial for London at Z is to be deſcribed. Make the ſemidiameter ZD perpendicular to Zb, and it will cut the horizon in D, 36 degrees weſt of the ſouth S. Then, a plane in the tangent HD, touching the ſphere in D, will be parallel to the plane Zb; and the axis of the ſphere will be equally inclined to both theſe planes.

Let WQE be the equinoctial, whoſe elevation above the horizon of Z (London) is 38½ degrees; and PRD be the meridian of the place D, cutting the equinoctial in R. Then, it is evident, that the arc RD is the latitude of the place D (where the plane Zb would be horizontal) and the arc RQ is the difference of longitude of the planes Zb and DH.

In the ſpherical triangle WDR, the arc WD is given, for it is the complement of the plane's declination from S the ſouth; which complement is 54° (viz. 90°—36°:) the angle at R, in which the meridian of the place D cuts the equator, is a right angle; and the angle RWD meaſures the elevation of the equinoctial above the horizon of Z, namely, 38½ degrees. Say therefore, as radius is to the co-ſine of the plane's declination from the ſouth, ſo is the co-ſine of the latitude of Z to the ſine of RD the latitude of D: which is of a different denomination from the latitude of Z, becauſe Z and D are on different ſides of the equator.

As radius	- - - - - - -	10.00000
To co-ſine	36° 0′ $= RQ$	9.90796
So co-ſine	51° 30′ $= QZ$	9.79415

To ſine 30° 14′ $= DR$ (9.70211) = the latitude of D, whoſe horizon is parallel to the vertical plane Zb at Z.

N. B. When radius is made the firſt term, it may be omitted, and then, by ſubtracting it mentally from the ſum of the other two, the operation will be ſhortened. Thus, in the preſent caſe,

To the logarithmic ſine of $WR =$ *	54° 0′	9.90796
Add the logarithmic ſine of $RD =$ †	38° 30′	9.79415

Their ſum—radius - - - - 9.70211 gives the ſame ſolution as above. And we ſhall keep to this method in the following part of the work.

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* The co-ſine of 36° 0′, or of RQ.

† The co-ſine of 51° 30′, or of QZ.

To find the difference of longitude of the places *D* and *Z*, ſay, as radius is to the co-ſine of $38\frac{1}{2}$ degrees, the height of the equinoctial at *Z*, ſo is the co-tangent of 36 degrees, the plane's declination, to the co-tangent of the difference of longitudes. Thus,

To the logarithmic ſine of * 51° 30′ 9.89354
Add the logarithmic tang. of † 54° 0′ 10.13874

Their ſum—radius - - - - 10.03228
is the neareſt tangent of 47° 8′ = *W R*; which is the co-tangent of 42° 52′ = *R Q*, the difference of longitude ſought. Which difference, being reduced to time, is 2 hours $51\frac{1}{2}$ minutes.

3. And thus having found the exact latitude and longitude of the place *D*, to whoſe horizon the vertical plane at *Z* is parallel, we ſhall proceed to the conſtruction of a horizontal dial for the place *D*, whoſe latitude is 30° 14′ ſouth; but anticipating the time at *D* by 2 hours 51 minutes (neglecting the $\frac{1}{2}$ minute in practice) becauſe *D* is ſo far weſtward in longitude from the meridian of London; and this will be a true vertical dial at London, declining weſtward 36 degrees.

Aſſume any right line *C S L* for the ſubſtile of the dial, and make the angle *K C P* equal to the latitude of the place (viz. 30° 14′) to whoſe horizon the plane of the dial is parallel; then *C R P* will be the axis of the ſtile, or edge that caſts the ſhadow on the hours of the day, in the dial. This done, draw the contingent line *E Q*, cutting the ſubſtilar line at right angles in *K*; Fig. 2.

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* The co-ſine of 38° 30′, or of *W D R*.

† The co-tangent of 36°, or of *D W*.

and from *K* make *K R* perpendicular to the axis *C R P*. Then *K G* (=*K R*) being made radius, that is, equal to the chord of 60′ or tangent of 45° on a good ſector, take 42° 52′ (the difference of longitude of the places *Z* and *D*) from the tangents, and having ſet it from *K* to *M*, draw *C M* for the hour-line of XII. Take *K N* equal to the tangent of an angle leſs by 15 degrees than *K M*; that is, the tangent 27° 52′; and through the point *N* draw *C N* for the hour-line of I. The tangent of 12° 52′ (which is 15° leſs than 27° 52′) ſet off the ſame way, will give a point between *K* and *N*, through which the hour-line of II is to be drawn. The tangent of 2° 8′ (the difference between 45° and 42° 52′) placed on the other ſide of *C L*, will determine the point through which the hour-line of III is to be drawn: to which 2° 8′, if the tangent of 15° be added, it will make 17° 8′; and this ſet off from *K* towards Q on the line *E Q*, will give the point for the hour-line of IV: and ſo of the reſt.—The forenoon hour-lines are drawn the ſame way, by the continual addition of the tangents 15°, 30°, 45°, &c. to 42° 52′ (=the tangent of *K M*) for the hours of XI, X, IX, &c. as far as neceſſary; that is, until there be five hours on each ſide of the ſubſtile. The ſixth hour, accounted from that hour or part of the hour on which the ſubſtile falls, will be always in a line perpendicular to the ſubſtile, and drawn through the center *C*.

4. In all erect dials, *C M*, the hour-line of XII, is perpendicular to the horizon of the place for which the dial is to ſerve: for that line is the interſection of a vertical plane with the plane of the meridian of the place, both which are perpendicular to the plane of the

horizon: and any line *H O*, or *h o*, perpendicular to *C M*, will be a horizontal line on the plane of the dial, along which line the hours may be numbered: and *C M* being ſet perpendicular to the horizon, the dial will have its true poſition.

5. If the plane of the dial had declined by an equal angle toward the eaſt, its deſcription would have differed only in this, that the hour-line of XII would have fallen on the other ſide of the ſubſtile *C L*, and the line *H O* would have a ſubcontrary poſition to what it has in this figure.

6. And theſe two dials, with the upper points of their ſtiles turned toward the north pole, will ſerve for the other two planes parallel to them; the one declining from the north toward the eaſt, and the other from the north toward the weſt, by the ſame quantity of angle. The like holds true of all dials in general, whatever be their declination and obliquity of their planes to the horizon.

CASE II.

7. If the plane of the dial not only *declines*, but alſo *reclines*, or *inclines*. Suppoſe its declination from fronting the ſouth *S* be equal to the arc *S D* on the horizon; and its reclination be equal to the arc *D d* of the vertical circle *D Z*: then it is plain, that if the quadrant of altitude *Z d D*, on the globe, cuts the point *D* in the horizon, and the reclination is counted upon the quadrant from *D* to *d*; the interſection of the hour-circle *P R d*, with the equinoctial *W* ♎ *E*, will determine *R d*, the latitude of the place *d*, Fig. 3.

whoſe

whoſe horizon is parallel to the given plane *Z b* at *Z*; and *R Q* will be the difference in longitude of the planes at *d* and *Z*.

Trigonometrically thus: let a great circle paſs through the three points *W*, *d*, *E*; and in the triangle *W D d*, right-angled at *D*, the ſides *W D* and *D d* are given; and thence the angle *D W d* is found, and ſo is the hypothenuſe *W d*. Again, the difference, or the ſum, of *D W d* and *D W R*, the elevation of the equinoctial above the horizon of *Z*, gives the angle *d W R*; and the hypothenuſe of the triangle *W R d* was juſt now found; whence the ſides *R d* and *W R* are found, the former being the latitude of the place *d*, and the latter the complement of *R Q*, the difference of longitude ſought.

Thus, if the latitude of the place *Z* be 52° 10′ north; the declination *S D* of the plane *Z b* (which would be horizontal at *d*) be 36°, and the reclination be 15°, or equal to the arc *D d*; the ſouth latitude of the place *d*, that is, the arc *R d*, will be 15° 9′; and *R Q*, the difference of the longitude, 36° 2′. From theſe data, therefore, let the dial (Fig. 4.) be deſcribed, as in the former example.

8. Only it is to be obſerved, that in the reclining or inclining dials, the horizontal line will not ſtand at right angles to the hour-line of XII, as in erect dials; but its poſition may be found as follows.

Fig. 4. To the common ſubſtilar line *C K L*, on which the dial for the place *d* was deſcribed, draw the dial *C r p m* 12 for the place *D*, whoſe declination is the ſame as that of *d* (viz. the arc *S D*; and *H O*, perpendicular to *C m*, the hour-line of XII on this dial, will be a horizontal line on the dial *C P R M* XII. For the declination

of both dials being the ſame, the horizontal line remains parallel to itſelf, while the erect poſition of one dial is reclined or inclined with reſpect to the poſition of the other.

Or, the poſition of the dial may be found by applying it to its plane, ſo as to mark the true hour of the day by the ſun, as ſhewn by another dial; or by a clock, regulated by a true meridian line and equation table.

9. There are ſeveral other things requiſite in the practice of dialing; the chief of which I ſhall give in the form of arithmetical rules, ſimple and eaſy to thoſe who have learnt the elements of trigonometry. For in practical arts of this kind, arithmetic ſhould be uſed as far as it can go; and ſcales never truſted to, except in the final conſtruction, where they are abſolutely neceſſary in laying down the calculated hour-diſtances on the plane of the dial. And although the inimitable artiſts of this metropolis have no occaſion for ſuch inſtructions, yet they may be of ſome uſe to ſtudents, and to private gentlemen who amuſe themſelves this way.

RULE I.

To find the angles which the hour-lines on any dial make with the ſubſtile.

To the logarithmic ſine of the given latitude, or of the ſtile's elevation above the plane of the dial, add the logarithmic tangent of the hour * diſtance from the meridian, or from the

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* That is, of 15, 30, 45, 60, 75°, for the hours of I, II, III, IV, V in the afternoon: and XI, X, IX, VIII, VII in the forenoon.

[†] ſubſtile; and the ſum *minus* radius will be the logarithmic tangent of the angle ſought.

For, in Fig. 2. *K C* is to *K M* in the ratio compounded of the ratio of *K C* to *K G* (=*K R*) and of *K G* to *K M*; which, making *C K* the radius, 10,000000, or 10,0000, or 10, or 1, are the ratio of 10,000000, or of 10,0000, or of 10, or of 1, to *K G* × *K M*.

Thus, in a horizontal dial, for latitude 51° 30′, to find the angular diſtance of XI in the forenoon, or I. in the afternoon, from XII.

To the logarithmic ſine of 51 30′° 9.89354[‡]
Add the logarithmic tang. of 15° 0′ 9.42805

The ſum—radius is - - - 9.32159= the logarithmic tangent of 11° 50′, or of the angle which the hour-line of XI or I makes with the hour of XII.

And by computing in this manner, with the ſine of the latitude, and the tangents of 30, 45, 60, and 75°, for the hours of II, III, IV, and V in the afternoon; or of X, IX, VIII, and VII in the forenoon; you will find their angular diſtances from XII to be 24° 18′, 38° 3′, 53° 35′, and 71° 6′; which are all that there is occaſion to compute for.——And theſe diſtances may be ſet off from XII by a line of chords; or rather, by taking 1000 from a ſcale of equal parts, and ſetting that extent as a radius from *C* to XII; and then, taking 209 of the ſame parts (which, in the tables, are the natural tangent of 11° 50′) and ſetting them from XII to XI and to I, on the line *b o*, which Fig. 2. is perpendicular to *C* XII: and ſo for the reſt of the hour-lines, which in the table of natural tangents, againſt the above diſtances, are 451, 782, 1355, and 2920, of ſuch equal parts from XII, as the radius *C* XII contains 1000. And laſtly, ſet off 1257 (the natural tangent of 51° 30′) for the angle of the ſtile's height, which is equal to the latitude of the place.

[†] In all horizontal dials, and erect north or ſouth dials, the ſubſtile and meridian are the ſame: but in all declining dials, the ſubſtile line makes an angle with the meridian.

[‡] In which caſe, the radius *C K* is ſuppoſed to be divided into 1000000 equal parts.

The reaſon why I prefer the uſe of the tabular numbers, and of a ſcale decimally divided, to that of the line of chords, is becauſe there is the leaſt chance of miſtake and error in this way; and likewiſe, becauſe in ſome caſes it gives us the advantage of a *nonius*' diviſion.

In the univerſal ring-dial, for inſtance, the diviſions on the axis are the tangents of the angles, of the ſun's declination placed on either ſide of the center. But inſtead of laying them down from a line of tangents, I would make a ſcale of equal parts, whereof 1000 ſhould anſwer exactly to the length of the ſemi-axis, from the center to the inſide of the equinoctial ring; and then lay down 434 of theſe parts toward each end from the center, which would limit all the diviſions on the axis, becauſe 434 are the natural tangent of 23° 29′. And thus by a *nonius* affixed to the ſliding piece, and taking the ſun's declination from an Ephemeris, and the tangent of that declination from the table of natural tangents, the ſlider might be always ſet true to within two minutes of a degree.

And this ſcale of 434 equal parts might be placed right againſt the 23½ degrees of the ſun's declination, on the axis, inſtead of the ſun's place, which is there of very little uſe. For then, the ſlider might be ſet in the uſual way, to the day of the month, for common uſe; but to the natural tangent of the declination, when great accuracy is required.

The like may be done wherever a ſcale of ſines or tangents is required on any inſtrument.

RULE II.

The latitude of the place, the ſun's declination, and his hour diſtance from the meridian, being given; to find (1.) *his altitude*; (2.) *his azimuth.*

Fig. 3. 1. Let d be the ſun's place, dR, his declination; and in the triangle PZd, Pd the ſum, or the difference, of dR, and the quadrant PR being given by the ſuppoſition, as alſo the complement of the latitude PZ, and the angle dPZ, which meaſures the horary diſtance of d from the meridian; we ſhall (by Caſe 4. of *Keill*'s Oblique ſpheric Trigonometry) find the baſe Zd, which is the ſun's diſtance from the zenith, or the complement of his altitude.

And (2.) As ſine Zd : ſine Pd :: ſine dPZ : dZP, or of its ſupplement DZS, the azimuthal diſtance from the ſouth.

Or, the practical rule may be as follows.

Write A for the ſine of the ſun's altitude, L and l for the ſine and co-ſine of the latitude, D and d for the ſine and co-ſine of the ſun's declination, and H for the ſine of the horary diſtance from VI.

Then the relation of H to A will have three varieties.

1. When the declination is toward the elevated pole, and the hour of the day is between XII and VI; it is $A = LD + Hld$, and $H = \frac{A - LD}{ld}$.

2. When the hour is after VI, it is $A = LD - Hld$, and $H = \frac{LD + A}{ld}$.

3. When the declination is toward the depreſſed pole, we have $A = Hld - LD$, and $H = \frac{A + LD}{ld}$.

Which theorems will be found uſeful, and expeditious enough for ſolving thoſe problems in geography and dialing, which depend on the relation of the ſun's altitude to the hour of the day.

EXAMPLE. I.

Suppoſe the latitude of the place to be $51\frac{1}{2}$ degrees north; the time five hours diſtant from XII, that is, an hour after VI in the morning, or before VI in the evening; and the ſun's declination 20° north. *Required the ſun's altitude?*

Then, to log. L = log. ſin. 51° 30′ $\bar{1}$.89354*
add log. D = log. ſin. 20° 0′ $\bar{1}$.53405

Their ſum - - - - - - - - - $\bar{1}$.42759
gives LD = logarithm of 0.267664, in the natural ſines.

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* Here we conſider the radius as unity, and not 10,00000, by which, inſtead of the index 9 we have—1, as above: which is of no farther uſe, than making the work a little eaſier.

And, to log. $H =$ log. ſin. † 15° 0′ 1.41300
add { log. $l =$ log. ſin. ‡ 38° 0′ 1.79414
add { log. $d =$ log. ſin. ‖ 70° 0′ 1.97300

Their ſum - - - - - - - - 1.18015
gives $H\,l\,d =$ logarithm of 0. 151408, in the natural ſines.

And theſe two numbers of (0.267664 and 0.151408) make 0.419072 $= A$; which, in the table, is the neareſt natural ſine of 24° 47′, the ſun's altitude ſought.

The ſame hour-diſtance being aſſumed on the other ſide of VI, then $L\,D - H\,l\,d$ is 0.116256, the ſine of 6° 40′½; which is the ſun's altitude at V in the morning, or VII in the evening, when his north declination is 20°.

But when the declination is 20° ſouth (or towards the depreſſed pole) the difference $H\,l\,d - L\,D$ becomes negative, and thereby ſhew that, an hour before VI in the morning, or paſt VI in the evening, the ſun's center is 6° 40 ½ below the horizon.

EXAMPLE. II.

In the ſame latitude and north declination, from the given altitude to find the hour.

Let the altitude be 48°; and becauſe, in this caſe, $H = \frac{A - L\,D}{l\,d}$, and A (the natural ſine of 48°) $= .743145$, and $L\,D = .267664$, $A - L\,D$

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† The diſtance of one hour from VI.
‡ The co-latitude of the place.
‖ The co-declination of the ſun.

will be 0.475481, whoſe logarithmic
ſine is - - - - - - - - 1.6771331
from which taking the logarithmic
ſine of $l + d =$ - - - - - 1.7671354

Remains - - - - - - - - 1.9099977
the logarithmic ſine of the hour-diſtance ſought, viz. of 54° 22′; which, reduced to time, is 3 hours 37½ min. that is, IX h. 37½ min. in the forenoon, or II h. 22½ min. in the afternoon.

Put the altitude = 18°, whoſe natural ſine is .3090170; and thence $A - LD$ will be $= .0491953$; which divided by $l + d$, gives .0717179, the ſine of 4° 6′½, in time 16½ minutes nearly, before VI in the morning, or after VI in the evening, when the ſun's altitude is 18°.

And, if the declination 20° had been towards the ſouth pole, the ſun would have been depreſſed 18° below the horizon at 16½ minutes after VI in the evening; at which time, the twilight would end; which happens about the 22d of November, and 19th of January, in the latitude of 51°½ north. The ſame way may the end of twilight, or beginning of dawn, be found for any time of the year.

NOTE 1. If in theorem 2 and 3 (page 363) A is put $= 0$, and the value of H is computed, we have the hour of ſun-riſing and ſetting for any latitude, and time of the year. And if we put $H = 0$, and compute A, we have the ſun's altitude or depreſſion at the hour of VI. And laſtly, if H, A, and D are given, the latitude may be found by the reſolution of a quadratic equation; for $l = \sqrt{1 - L^2}$.

NOTE 2. When A is equal o, H is equal $\frac{LD}{ld} = TL \times TD$, the tangent of the latitude multiplied by the tangent of the declination.

As, if it was required, *what is the greateſt length of day in latitude* 51° 30′ ?

To the log. tangent of 51° 30′ 0.0993948
Add the log. tangent of 23° 29′ 1.6379563

Their ſum - - - - - - - 1.7373511 is the log. ſine of the hour-diſtance 33° 7′; in time 2 h. 12½ m. The longeſt day therefore is 12 h. + 4 h. 25 m. = 16 h. 25 m. And the ſhorteſt day is 12 h. — 4 h. 25 m. = 7 h. 35 m.

And if the longeſt day is given, the latitude of the place is found; $\frac{H}{TD}$ being equal to TL. Thus, if the longeſt day is 13½ hours = 2 × 6 h. + 45 m. and 45 minutes in time being equal to 11¼ degrees.

From the log. ſine of 11° 15′ 1.2902357
Take the log. tang. of 23° 29′ 1.6379562

Remains - - - - - - - 1.6522795
= the logarithmic tangent of lat. 24° 11′.

And the ſame way, the latitudes, where the ſeveral geographical *climates* and parallels begin, may be found; and the latitudes of places, that are aſſigned in authors from the length of their days, may be examined and corrected.

NOTE 3. The ſame rule for finding the longeſt day in a given latitude, diſtinguiſhes the hour-lines that are neceſſary to be drawn on any dial from thoſe which would be ſuperfluous.

In lat. 52° 10′ the longeſt day is 16 h. 32 m. and the hour-lines are to be marked from 44 m.

after III in the morning, to 16 m. after VIII in the evening.

In the ſame latitude, let the dial of Art. 7. Fig. 4. be propoſed; and the elevation of its ſtile (or the latitude of the place *d*, whoſe horizon is parallel to the plane of the dial) being 15° 9′; the longeſt day at *d*, that is, the longeſt time that the ſun can illuminate the plane of the dial, will (by the rule $H = T\,L \times T\,D$) be twice 6 hours 27 minutes = 12 h. 54 m. The difference of longitude of the planes *d* and *Z* was found in the ſame example to be 36° 2′; in time, 2 hours 24 minutes; and the declination of the plane was from the ſouth towards the weſt. Adding therefore 2 h. 24 min. to 5 h. 33 m. the earlieſt ſun-riſing on a horizontal dial at *d*, the ſum 7 h. 57 m. ſhews that the morning hours, or the parallel dial at *Z*, ought to begin at 3 men. before VIII. And to the lateſt ſun-ſetting at *d*, which is 6 h. 27 m. adding the ſame two h. 24 m. the ſum 8 h. 51 m. exceeding 6 h. 16 m. the lateſt ſun-ſetting at *Z*, by 35 m. ſhews that none of the afternoon hour-lines are ſuperfluous. And the 4 h. 13 m. from III h. 44 m. the ſun-riſing at *Z* to VII h. 57 m. the ſun-riſing at *d*, belong to the other face of the dial; that is, to a dial declining 36° from north to eaſt, and inclining 15°.

EXAMPLE III.

From the ſame *data* to find the ſun's *azimuth*.

If *H*, *L*, and *D* are given, then (by Art, 2.ᵒ of Rule II.) from *H* having found the altitude and its complement *Z d*; and the arc *P D* (the diſtance from the pole) being given; ſay, As the co-ſine of the altitude is to the ſine of the diſtance from the pole, ſo is the ſine of the hour-diſtance from the meridian to the ſine of the azimuth diſtance from the meridian.

Let the latitude be 51° 30′ north, the declination 15° 9′ ſouth, and the time II h. 24 m. in the afternoon, when the ſun begins to illuminate a vertical wall, and it is required to find the poſition of the wall.

Then, by the foregoing theorems, the complement of the altitude will be 81° 32′½, and *P d* the diſtance from the pole being 109° 5′, and the horary diſtance from the meridian, or the angle *d P Z*, 36°.

To log. ſin. 74° 51′ - - 1.98464
Add log. ſin. 36° 0′ - - 1.76922

And from the ſum - - 1.75386
Take the log. ſin. 81° 32′½ 1.99525
Remains - - - - - - 1.75861 = log. ſin. 35°, the azimuth diſtance ſouth.

When the altitude is given, find from thence the hour, and proceed as above.

This praxis is of ſingular uſe on many occaſions; in finding the declination of vertical planes more exactly than in the common way, eſpecially if the tranſit of the ſun's center is obſerved by applying a ruler with ſights, either plain or teleſcopical, to the wall or plane, whoſe declination is required.—In drawing a meridian-line, and finding the magnetic variation.—In finding the bearings of places in terreſtrial ſurveys; the tranſits of the ſun over any place, or his horizontal diſtance from it being obſerved, together with the altitude and hour.—And thence determining ſmall differences of longitude.—In obſerving the variation at ſea, &c.

The learned Mr. *Andrew Reid* invented an inſtrument ſeveral years ago, for finding the latitude at ſea from two altitudes of the ſun, obſerved on the ſame day, and the interval of the obſervations, meaſured by a common watch. And this inſtrument, whoſe only fault was that of its being ſomewhat expenſive, was made by Mr. *Jackſon.* Tables have been lately computed for that purpoſe.

But we may often, from the foregoing rules, reſolve the ſame problem without much trouble; eſpecially if we ſuppoſe the maſter of the ſhip to know within 2 or 3 degrees what his latitude is. Thus.

Aſſume the two neareſt probable limits of the latitude, and by the theorem $H=\frac{A+LD}{ld}$, compute the hours of obſervation for both ſuppoſitions. If one interval of thoſe computed hours coincides with the interval obſerved, the queſtion is ſolved. If not, the two diſtances of the intervals computed, from the true interval, will give a proportional part to be added to, or ſubtracted from, one of the latitudes aſſumed. And if more exactneſs is required, the operation may be repeated with the latitude already found.

But whichever way the queſtion is ſolved, a proper allowance is to be made for the difference of latitude ariſing from the ſhip's courſe in the time between the two obſervations.

Of the double horizontal dial; and the Babylonian *and* Italian *dials.*

To the *gnomonic* projection, there is ſometimes added a *ſtereographic* projection of the hour-circles, and the parallels of the ſun's declination, on the ſame horizontal plane; the upright ſide of the gnomon being ſloped into an edge, ſtanding perpendicularly over the center of the projection: ſo that the dial, being in its due poſition, the ſhadow of *that* perpendicular edge is a vertical circle paſſing through the ſun, in the ſtereographic projection.

The months being duly marked on the dial, the ſun's declination, and the length of the day at any time, are had by inſpection (as alſo his altitude, by means of a ſcale of tangents). But its chief property is, that it may be placed true, whenever the ſun ſhines, without the help of any other inſtrument.

Fig. 3. Let *d* be the ſun's place in the ſtereographic projection, *x d y z* the parallel of the ſun's declination, *Z d* a vertical circle through the ſun's center, *P d* the hour-circle; and it is evident, that the diameter *N S* of this projection being placed duly north and ſouth, theſe three circles will paſs through the point *d*. And therefore, to give the dial its due poſition, we have only to turn its gnomon toward the ſun, on a horizontal plane, until the hour on the common gnomonic projection coincides with that marked by the hour-circle *P d*, which paſſes through the interſection of the ſhadow *Z d* with the circle of the ſun's preſent declination.

The *Babylonian* and *Italian* dials reckon the hours, not from the meridian, as with us, but from the ſun's riſing and ſetting. Thus, in *Italy*, an hour before ſun-ſet is reckoned the 23d hour; two hours before ſun-ſet, the 22d hour; and ſo of the reſt. And the ſhadow that marks them on the hour-lines, is *that* of the point of a ſtile. This occaſions a perpetual variation between their dials and clocks, which they muſt correct from time to time, before it ariſes to any ſenſible quantity, by ſetting their clocks ſo much faſter or ſlower. And in *Italy*, they begin their day, and regulate their clocks, not from ſun-ſet, but from about mid-twilight, when the *Ave Maria* is ſaid; which corrects the difference that would otherwiſe be between the clock and the dial.

Plate XXIII.

The improvements which have been made in all ſorts of inſtruments and machines for meaſuring time, have rendered ſuch dials of little account. Yet, as the theory of them is ingenious, and they are really, in ſome reſpects, the beſt contrived of any for vulgar uſe, a general idea of their deſcription may not be unacceptable.

Let Fig. 5. repreſent an erect direct ſouth wall, on which a *Babylonian dial* is to be drawn, ſhewing the hours from ſun-riſing; the latitude of the place, whoſe horizon is parallel to the wall, being equal to the angle KCR. Make, as for a common dial $KG = KR$ (which is perpendicular to CR) the radius of the equinoctial Æ Q, and draw RS perpendicular to CK for the ſtile of the dial; the ſhadow of whoſe point R is to mark the hours, when SR is ſet upright on the plane of the dial.

Then it is evident, that in the contingent line Æ Q, the ſpaces $K\,1$, $K\,2$, $K\,3$, &c. being taken equal to the tangents of the hour-diſtances from the meridian, to the radius KG, one, two, three, &c. hours after ſun riſing, on the equinoctial day; the ſhadow of the point R will be foůnd, at theſe times, reſpectively in the points 1, 2, 3, &c.

Draw, for the like hours after ſun-riſing, when the ſun is in the tropic of Capricorn ♑ *v*, the like common lines *C D*, *C E*, *C F*, &c. and at theſe hours the ſhadow of the point *R* will be found in thoſe lines reſpectively. Find the ſun's altitudes above the plane of the dial at these hours, and with their co-tangents *S d*, *S e*, *S f*, &c. to radius *S R*, deſcribe arcs interſecting the hour-lines in the points *d*, *e*, *f*, &c. ſo ſhall the right lines 1 *d*, 2 *e*, 3 *f*, &c. be the lines of I, II, III, &c. hours after ſun-riſing.

The conſtruction is the ſame in every other caſe, due regard being had to the difference of longitude of the place at which the dial would be horizontal, and the place for which it is to ſerve. And likewiſe, taking care to draw no lines but what are neceſſary; which may be done partly by the rules already given for determining the time that the ſun ſhines on any plane; and partly from this, that on the tropical days, the hyperbola deſcribed by the ſhadow of the point *R*, limits the extent of all the hour-lines.

The moſt uſeful however, as well as the ſimpleſt of ſuch dials, is that which is deſcribed on the two ſides of a meridian plane.

That the *Babylonian* and *Italic* hours are truly enough marked by right lines, is eaſily ſhewn. Mark the three points on a globe, where the horizon cuts the equinoctial, and the two tropics, toward the eaſt or weſt; and turn the globe on its axis 15°, or 1 hour; and it is plain, that the three points which were in a great circle (viz. the horizon) will be in a great circle ſtill; which will be projected geometrically into a ſtraight line. But theſe three points are univerſally the ſun's places, one hour after ſun-ſet (or one hour before ſun-riſe) on the equinoctial and ſolſtitial days. The like is true of all other circles of declination, beſides the tropics; and therefore, the hours on ſuch dials are truly marked by ſtraight lines limited by the projections of the tropics; and which are rightly drawn, as in the foregoing example.

Note 1. The ſame dials may be delineated without the hour-lines *C D*, *C E*, *C F*, &c. by ſetting off the ſun's azimuths on the plane of the dial, from the center *S*, on either ſide of the ſubſtile *C S K*, and the correſponding co-tangents of altitude from the ſame center *S*, for I, II, III, &c. hours before or after the ſun is in the horizon of the place for which the dial is to ſerve, on the equinoctial and ſolſtitial days.

2. One of theſe dials has its name from the hours being reckoned from ſun-riſing, the beginning of the *Babylonian* day. But we are not thence to imagine that the *equal* hours, which it ſhews, were thoſe in which the aſtronomers of that country marked their obſervations. Theſe, we know with certainty, were unequal, like the *Jewiſh*, as being twelfth parts of the natural day: and an hour of the night was, in like manner, a twelfth part of the night; longer or ſhorter, according to the ſeaſon of the year. So that an hour of the day, and an hour of the night, at the ſame place, would always make $\frac{1}{12}$ of 24, or 2 equinoctial hours. In *Paleſtine*, among the *Romans*, and in ſeveral other countries, 3 of theſe unequal nocturnal hours were a *vigilia* or *watch*. And the reduction of equal and unequal hours into one another, is extremely eaſy. If, for inſtance, it is found, by a foregoing rule, that in a certain latitude, at a given time of the year, the

length of a day is 14 equinoctial hours, the unequal hour is then $\frac{14}{12}$ or $\frac{7}{6}$ of an hour, that is, 70 minutes; and the nocturnal hour is 50 minutes. The firſt watch begins at VII (ſunſet); the ſecond at three times 50 minutes after, viz. IX h. 30 m. the third always at midnight; the morning watch at $\frac{1}{2}$ hour paſt II.

If it were required to draw a dial for ſhewing theſe unequal hours, or 12th parts of the day, we muſt take as many declinations of the ſun as are thought neceſſary, from the equator towards each tropic: and having computed the ſun's altitude and azimuth for $\frac{1}{12}$, $\frac{2}{12}$, $\frac{3}{12}$th parts, &c. of each of the diurnal arcs belonging to the declinations aſſumed: by theſe, the ſeveral points in the circles of declination, where the ſhadow of the ſtile's point falls, are determined: and curve lines drawn through the points of an homologous diviſion will be the hour-lines required.

Of the right placing of dials, and having a true meridian line for the regulating of clocks and watches.

The plane on which the dial is to reſt, being duly prepared, and every thing neceſſary for fixing it, you may find the hour tolerably exact by a large equinoctial ring-dial, and ſet your watch to it. And then the dial may be fixed by the watch at your leiſure.

If you would be more exact, take the ſun's altitude by a good quadrant, noting the preciſe time of obſervation by a clock or watch. Then, compute the time for the altitude obſerved, (by the rule, page 364) and ſet the watch to agree with that time, according to the ſun. A *Hadley*'s quadrant is very convenient for this purpoſe; for, by it you may take the angle between the ſun and his image, reflected from a baſon of water: the half of which angle, ſubtracting the refraction, is the altitude required. This is beſt done in ſummer, and the nearer the ſun is to the prime vertical (the eaſt or weſt azimuth) when the obſervation is made, ſo much the better.

Or, in ſummer, take two equal altitudes of the ſun in the ſame day; one any time between 7 and 10 o'clock in the morning, the other between 2 and 5 in the afternoon; noting the moments of theſe two obſervations by a clock or watch: and if the watch ſhews the obſervations to be at equal diſtances from noon, it agrees exactly with the ſun; if not, the watch muſt be corrected by half the difference of the forenoon and afternoon intervals; and then the dial may be ſet true by the watch.

Thus, for example, ſuppoſe you have taken the ſun's altitude when it was 20 minutes paſt VIII in the morning by the watch; and found, by obſerving in the afternoon, that the ſun had the ſame altitude 10 minutes before IIII; then it is plain, that the watch was 5 minutes too faſt for the ſun: for 5 minutes after XII is the middle time between VIII h. 20 m. in the morning, and III h. 50 m. in the afternoon; and therefore, to make the watch agree with the ſun, it muſt be ſet back five minutes.

A meridian line.

A good *meridian line*, for regulating clocks or watches, may be had by the following method.

Make a round hole, almoſt a quarter of an inch diameter, in a thin plate of metal; and fix the plate in the top of a ſouth window, in ſuch a manner, that it may recline from the zenith an angle equal to the co-latitude of your place, nearly as you can gueſs; for then, the plate wil face the ſun directly at noon on the equinoctial days. Let the ſun ſhine freely through the hole into the room; and hang a plumb-line to the ceiling of the room; at leaſt five or ſix feet fro the window, in ſuch a place as that the ſun's rays, tranſmitted through the hole, may fall upon the line when it is noon by the clock; and having marked the ſaid place on the ceiling, take away the line.

Having adjuſted a ſliding bar to a dove-tail groove, in a piece of wood about 18 inches long, and fixed a hook into the middle of the bar, nail the wood to the above-mentioned place on the cieling, parallel to the ſide of the room in which the window is: the groove and bar being towards the floor. Then, hang the plumb-line upon the hook in the bar, the weight or plummet reaching almoſt to the floor; and the whole will be prepared for farther and proper adjuſtment.

This done, find the true ſolar time by either of the two laſt methods, and thereby regulate your clock. Then, at the moment of next noon by the clock, when the ſun ſhines, move the ſliding bar in the groove until the ſhadow of the plumb-line biſects the image of the ſun (made by his rays tranſmitted through the hole) on the floor, wall, or on a white ſcreen placed on the north ſide of the line; the plummet or weight at the end of the line hanging freely in a pail of water placed below it on the floor.—But becauſe this may not be quite correct for the firſt time, on account that the plummet will not ſettle immediately, even in water; it may be farther corrected on the following days, by the above method, with the ſun and clock; and ſo brought to a very great exactneſs.

N.B. The rays tranſmitted through the hole, will caſt but a faint image of the ſun, even on a white ſcreen, unleſs the room be ſo darkened that no ſun-ſhine may be allowed to enter, but what comes through the ſmall hole in the plate. And always, for ſome time before the obſervation is made, the plummet ought to be immerſed in a jar of water, where it may hang freely; by which means the line will ſoon become ſteady, which otherwiſe would be apt to continue ſwinging.

As this meridian line will not only be ſufficient for regulating of clocks and watches to the true time by equation tables, but alſo for moſt aſtronomical purpoſes, I ſhall ſay nothing of the magnificent and expenſive meridian lines at *Bologna* and *Rome*, nor of the better methods by which aſtronomers obſerve preciſely the tranſits of the heavenly bodies on the meridian.

LECT. XII.

Shewing how to calculate the mean time of any New or Full Moon, or Eclipſe, from the creation of the world to the year of CHRIST 5800.

IN the following tables, the mean lunation is about a 20th part of a ſecond of time longer than its meaſure as now printed in the third edition of my Aſtronomy; which makes a difference of an hour and 30 minutes in 8000 years.—But this is not material, when only the mean times are required.

PRECEPTS.

To find the mean time of any New or Full Moon i any given year and month after the Chriſtia Æra.

1. If the given year be found in the third column of the *Table of the moon's mean motion from the ſun,* under the title *Years before and after* CHRIST; write out that year, with the mean motions belonging to it, and thereto join the given month with its mean motions. But, if the given year be not in the table, take out the next leſſer one to it that you find, in the ſame column; and thereto add as many ***compleat years,*** as will make up the given year: then, join the given month, and all the reſpective mean motions.

2. Collect theſe mean motions into one ſum of ſigns, degrees, minutes, and ſeconds; remembering, that 60 ſeconds (″) make a minute, 60 minutes (′) a degree, 30 degrees (°) a ſign, and 12 ſigns (ˢ) a circle. When the ſigns exceed 12, or 24, or 36 (which are whole circles) reject them, and ſet down only the remainder; which, together with the odd degrees, minutes, and ſeconds already ſet down, muſt be reckoned the whole ſum of the collection.

3. Subtract the reſult, or ſum of this collection, rom 12 ſigns; and write down the remainder. Then, look in the table, under *Days,* for the next leſs mean motions to this remainder,

and ſubtract them from it, writing down their remainder.

This done, look in the table under *hours* (marked H.) for the next leſs mean motions to this laſt remainder, and ſubtract them from it, writing down their remainder.

Then, look in the table under *minutes* (marked M.) for the next leſs mean motions to this remainder, and ſubtract them from it, writing down their remainder.

Laſtly, look in the table under *ſeconds* (marked S.) for the next leſs mean motions to this remainder, either greater or leſs; and againſt it you have the ſeconds anſwering thereto.

4. And theſe times collected, will give the mean time of the *required new moon*; which will be right in common years, and alſo in January and February in leap-years; but always one day too late in leap-years after February.

EXAMPLE I.

Required the time of new moon in September, 1764?
(a year not inſerted in the table)

		Moon from ſun.			
		s	°	′	″
To the year after *Chriſt*'s birth — —	1753	10	9	24	56
Add compleat years (ſum 1764)	11	0	10	14	20
And join September - -		2	22	21	8
The ſum of theſe mean motions is		1	12	0	24
which, being ſub. from a circle, or		12	0	0	0
Leaves remaining - -		10	17	59	36
Next leſs mean mot. for 26 days, ſub. - - - - -		10	16	57	34
And there remains - - -			1	2	2
Next leſs mean mot. for 2 hours, ſub. - - - -			1	0	57
And the remainder will be - -				1	5
Next leſs mean mot. for 2 min. ſub. - - - - -				1	1
Remains the mean mot. of 12 ſec.					4

Theſe times, being collected, would ſhew the mean time of the required new moon in September 1764, to be on the 26th day, at 2 hours 2 min. 12 ſec. paſt noon. But, as it is in a leap-year, and after February, the time is one day too late. So, the true mean time is September the 25th, at 2 m. 12 ſec. paſt II in the afternoon.

N. B. The tables always begin the day at noon, and reckon thenceforward, to the noon of the day following.

To find the mean time of full moon in any given year and month after the Chriſtian Æra.

Having collected the moon's mean motion from the ſun for the beginning of the given year and month, and ſubtracted their ſum from 12 ſigns (as in the former example) add 6 ſigns to the remainder, and then proceed in all reſpects as above.

EXAMPLE II.

Required the mean time of full moon in September 1764?

		Moon from ſun.			
		s	°	′	″
To the year after *Chriſt's* birth — —	1753	10	9	24	56
Add compleat years -	11	0	10	14	20
(ſum 1764)					
And join September - -		2	22	21	8
The ſum of theſe mean motions is - - - - -		1	12	0	24
Which, being ſubt. from a circle, or - - -		12	0	0	0
Leaves remaining - - -		10	17	59	36
To which remainder add -		6	0	0	0
And the ſum will be - -		4	17	59	36

Brought

	Moon from ſun			
	s	°	′	″
Brought over — —	4	17	59	36
Next leſs mean mot. for 11 days, ſubt. - -	4	14	5	54
And there remains - -		3	53	42
Next leſs mean mot. for 7 hours, ſubt. - -		3	33	20
And the remainder will be -			20	22
Next leſs mean mot. for 40 min. ſubt. - - -			20	19
Remains the mean mot. for 8 ſec. - - - -				3

So, the mean time, according to the tables, is the 11th of September, at 7 hours 40 minutes 8 ſeconds paſt noon. One day too late, being after February in a leap-year.

And thus may the mean time of any new or full moon be found, in any year after the Chriſtian Æra.

To find the mean time of new or full moon in any given year and month before the Chriſtian Æra.

If the given year before the year of CHRIST 1 be found in the third column of the table, under the title *Years before and after* CHRIST, write it out, together with the given month, and join the mean motions. But, if the given year be not in the table, take out the next greater one to it that you find; which being ſtill farther back than the given year, add as many compleat years to it as will bring the time forward to the given year; then join the month, and proceed in all reſpects as above.

EXAM-

EXAMPLE III.

Required the mean time of new moon in May, the year before Chriſt 585?

The next greater year in the table is 600; which being 15 years before the given year, add the mean motions for 15 years to thoſe of 600, together with thoſe for the beginning of May.

	Moon from ſun.			
	s	°	′	″
To the year before *Chriſt* 600 -	5	11	6	16
Add compleat years motion 15	6	0	55	24
And the mean motions for May	0	22	53	23
The whole ſum is - -	0	4	55	3
Which, being ſubt. from a circle, or - - -	12	0	0	0
Leaves remaining - -	11	25	4	57
Next leſs mean mot. for 29 days, ſubt. - -	11	23	31	54
And there remains - -		1	33	3
Next leſs mean mot. for 3 hours ſubt. - - -		1	31	26
And the remainder will be -			1	37
Next leſs mean mot. for 3 min. ſubt. - - -			1	31
Rem. the mean mot. of 14 ſeconds - - -				6

So, the mean time by the tables, was the 29th of May, at 3 hours 3 min. 14 ſec. paſt noon. A day later than the truth, on account of its being in a leap-year. For as the year of CHRIST 1 was the firſt after a leap-year, the year 585 before the year 1 was a leap-year of courſe.

If the given year be after the Chriſtian Æra, divide its date by 4, and if nothing remains, it is a leap-year in the old ſtile. But if the given year was before the Chriſtian Æra (or Year of CHRIST 1) ſubtract one from its date, and divide the remainder by 4; then, if nothing remains, it was a leap-year; otherwiſe not.

To find whether the ſun is eclipſed at the time of any given change, or the moon at any given full.

Of *eclipſes.* From the *Table of the ſun's mean motion* (or diſtance) *from the moon's aſcending node*, collect the mean motions anſwering to the given time; and if the reſult ſhews the ſun to be within 18 degrees of either of the nodes at the time of new moon, the ſun will be eclipſed at that time. Or, if the reſult ſhews the ſun to be within 12 degrees of either of the nodes at the time of full moon, the moon will be eclipſed at that time, in or near the contrary node; otherwiſe not.

EXAM-

EXAMPLE IV.

The moon changed on the 26th of September 1764, at 2 h. 2 m. (neglecting the ſeconds) after noon (See Example I.) *Qu. Whether the ſun was eclipſed at that time?*

			Sun from node.		
		s	°	′	″
To the year after *Chriſt's* birth — —	1753	1	28	0	19
Add compleat years -	11	7	2	3	56
	(ſum 1764)				
And September - - -		8	12	22	49
26 days - - - -			27	0	13
2 hours - - -				5	12
2 minutes - - -					6
Sun's diſtance from the aſcending node - - -		6	9	32	34

Now, as the deſcending node is juſt oppoſite to the aſcending, (viz. 6 ſigns diſtant from it) and the tables ſhew only how far the ſun has gone from the aſcending node, which, by this example, appears to be 6 ſigns 9 degrees 32 minutes 34 ſeconds, it is plain that he muſt have then been eclipſed; as he was then only 9° 32′ 34″ ſhort of the deſcending node.

EXAMPLE V.

The moon was full on the 11*th of September,* 1764, *at* 7 *h.* 40 *min. paſt noon.* (See Example II.) *Qu. Whether ſhe was eclipſed at that time?*

		Sun from node.			
		s	°	′	″
To the year after *Chriſt*'s birth — —	1753	1	28	0	19
Add compleat years -	11	7	2	3	56
	(ſum 1764)				
And September - -		8	12	22	49
And 11 days - -			11	25	29
And 7 hours - - -				18	11
And 40 minutes - -				1	44
Sun's diſtance from the aſcending node - - - - -		5	24	12	28

Which being ſubtracted from 6 ſigns, leaves only 5° 47′ 32″ remaining; and this being all the ſpace that the ſun was ſhort of the deſcending node, it is plain that the moon muſt then have been eclipſed, becauſe ſhe was juſt as near the contrary node.

EXAM-

EXAMPLE VI.

Q. *Whether the ſun was eclipſed in May, the year before* CHRIST 585? (See Example III.)

		Sun from node.		
	s	°	′	″
To the year before *Chriſt* 600 -	9	9	23	51
Add the mean motion of 15 complete years - -	9	19	27	49
And May - - -	4	4	37	57
29 days - -	1	0	7	10
3 hours - -			7	48
3 minutes (neglecting the ſeconds) - -				8
Sun's diſtance from the aſcending node - . -	0	3	44	43

Which being leſs than 18 degrees, ſhews that the ſun was eclipſed at that time.

This eclipſe was foretold by *Thales*, and is thought to be the eclipſe which put an end to the war between the Medes and Lydians.

Thales's eclipſe.

The times of the *ſun's conjunction with the nodes*, and conſequently the *eclipſe months* of any given year, are eaſily found by the *Table of the ſun's mean motion from the moon's aſcending node*; and much in the ſame way as the mean conjunctions of the ſun and moon are found by the table of the moon's mean motion from the ſun. For, collect the ſun's mean motion from the node (which is the ſame as his diſtance gone from it) for the beginning of any given *year*, and ſubtract it from 12 ſigns; then, from the remainder,

When *eclipſes muſt happen.*

remainder, ſubtract the next leſs mean motio belonging to whatever month you find them i the table; and from their remainder ſubtract th next leſs mean motion for *days*, and ſo on fo *hours* and *minutes*: the reſult of all which wi ſhew the time of the ſun's mean conjunctio with the *aſcending node* of the moon's orbit.

EXAMPLE VII.

Required the time of the ſun's conjunction with th aſcending node in the year 1764?

		Sun from node			
		s	°	′	″
To the year after *Chriſt's* birth - -	1753	1	28	0	19
Add compleat years -	11	7	2	3	56
Mean diſt at beg. of A.D. 1764		9	0	4	15
Subtract this diſtance from a circle, or - - -		12	0	0	0
And there remains - -		2	29	55	45
Next leſs mean motion for March, ſubtract - -		2	1	16	39
And the remainder will be - -		0	28	39	6
Next leſs mean motion for 27 days, ſubtract - -		0	28	2	32
And there remains - -				36	34
Next leſs mean motion for 14 hours, ſubtracted - -				36	21
Remains (nearly) the mean motion of 5 minutes - -					13

Hence it appears, that the ſun will paſs by the moon's *aſcending node* on the 27th of March, at 14 hours 5 minutes paſt noon; viz. on the 28th day, at 5 minutes paſt II in the morning, according to the tables: but this being in a leap-year, and after February, the time is one day too late. Conſequently, the true time is at 5 min. paſt II in the morning on the 27th day; at which time, the deſcending node will be directly oppoſite to the ſun.

If 6 ſigns be added to the remainder ariſing from the firſt ſubtraction, (viz. from 12 ſigns) and then the work carried on as in the laſt example, the reſult will give the mean time of the ſun's conjunction with the deſcending node. Thus, in

EXAMPLE VIII.

To find when the ſun will be in conjunction with the deſcending node *in the year* 1764?

		Sun from node. s	°	′	″
To the year after *Chriſt's* birth - -	1753	1	28	0	19
Add compleat years -	11	7	2	3	56
M. d. fr. aſc. n. at beg. of	1764	9	0	4	15
Subtract this diſtance from a circle, or - - -		12	0	0	0
And the remainder will be -		2	29	55	45
To which add half a circle, or -		6	0	0	0
And the ſum will be - -		8	29	55	45

	Sun fr. nod			
	s	°	'	"
Brought over -	8	29	55	4
Next leſs mean mot. for Sept. ſubt.	8	12	22	4
And there remains - -	0	17	32	5
Next leſs mean mot. for 16 days, ſubt. - - - - -	0	16	37	4
And the remainder will be -			55	5
Next leſs mean mot. for 21 hours, ſubtracted - - -			54	3
Rem. (nearly) the mean mot. of 31 min. - - - -			1	20

So that, according to the tables, the ſun will be in conjunction with the *deſcending node* on the 16th of September, at 21 hours 31 minutes paſt noon: one day later than the truth, on account of the leap-year.

The limits of *eclipſes.*

When the moon changes within 18 days before or after the ſun's conjunction with either of the nodes, the ſun will be eclipſed at that change: and when the moon is full within 12 days before or after the time of the ſun's conjunction with either of the nodes, ſhe will be eclipſed at that full: otherwiſe not.

Their period and reſtitution.

If to the mean time of any eclipſe, either of the ſun or moon, we add 557 Julian years 21 days 18 hours 11 minutes and 51 ſeconds (in which there are exactly 6890 mean lunations) we ſhall have the mean time of another eclipſe. For at the end of that time, the moon will be either new or full, according as we add it to the time of new or full moon; and the ſun will be only 45″ farther from the ſame node, at the end of the ſaid time, than he was at the beginning of it; as appears by the following example[*].

The period.		Moon fr. ſun.				Sun fr. node.			
		s	°	′	″	s	°	′	″
Compleat Years	500	3	5	32	47	10	14	45	8
	40	8	26	50	37	1	23	58	49
	17	3	2	21	39	10	28	40	55
days	21	8	16	0	21		21	48	38
hours	18		9	8	35			46	44
minutes	11			5	35				29
ſeconds	51				26				2
Mean motions		0	0	0	0	0	0	0	45

And this period is ſo very near, that in 6000 years it will vary no more from the truth, as to the reſtitution of eclipſes, than 8¼ minutes of a degree; which may be reckoned next to nothing. It is the ſhorteſt in which, after many trials, I can find ſo near a conjunction of the ſun, moon, and the ſame node.

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[*] Dr. HALLEY's period of eclipſes contains only 18 years 11 days 7 hours 43 minutes 20 ſeconds; in which time, according to his tables, there are juſt 223 mean lunations: but, as in that time, the ſun's mean motion from the node is no more than $11^{s}\,29^{\circ}\,31'\,49''$, which wants $28'\,11''$ of being as nearly in conjunction with the ſame node at the end of the period as it was at the beginning; this period cannot be of conſtant duration for finding eclipſes, becauſe it will in time fall quite without their limits. The following tables make this period 31 ſeconds ſhorter, as appears by the following calculation.

The period.		Moon fr. Sun.				Sun fr. node.			
		s	°	′	″	s	°	′	″
Compleat years	18	7	11	59	4	11	17	46	18
days	11	4	14	5	54		11	25	29
hours	7		3	33	20			18	11
min.	42			21	20			1	49
ſec.	44				22				2
Mean motions		0	0	0	0	11	29	31	49

A Table of Mean Lunations.

This table is made by the continual addition of a mean lunation, viz. $29^{d}\ 12^{h}\ 44^{m}\ 3^{s}\ 6^{th}\ 21^{iv}\ 14^{v}\ 24^{vi}\ 0^{vii}$

Lun.		Days.	H.	M.	S.	Th.
1	Contain	29	12	44	3	6
2		59	1	28	6	13
3		88	14	12	9	19
4		118	2	56	12	25
5		147	15	40	15	32
6		177	4	24	18	3
7		206	17	8	21	44
8		236	5	52	24	51
9		265	18	3	27	57
10		295	7	20	31	3
20		590	14	41	2	7
30		885	22	1	33	11
40		1181	5	22	4	14
50		1470	12	42	35	18
100		2953	1	25	10	35
200		5906	2	50	21	11
300		8859	4	15	31	46
400		11812	5	40	42	2
500		14756	7	5	52	57
1000		29530	14	11	45	54
2000		59061	4	23	31	48
3000		88591	18	35	17	42
4000		118122	8	47	3	36
5000		117652	22	58	49	30
10000		295305	21	57	39	0
20000		590611	9	55	18	0
30000		885917	17	52	57	0
40000		81223	15	50	36	0
0000		1476529	13	48	15	0
100000		2953059	3	36	39	0

In 100000 mean lunations, there are 8085 Julian years 12 days 21 hours 36 minutes 30 ſeconds = 2953059 days 3 hours 36 minutes 30 ſeconds.

Proof of the Table.

	In	Moon from ſun. s	°	′	″
Jol. years.	4000	1	14	22	12
	4000	1	14	22	12
	80	5	23	41	15
	5	10	0	18	28
Days	12	4	26	17	20
Hours	21		10	40	1
Min.	56			18	17
Sec.	20				15
M fr ſun.		0	0	0	0

Having by the former precepts computed the mean time of new moon in January, for any given year, it is eaſy, by this Table, to find the mean time of new moon in January for any number of years afterward: and by means of a ſmall table of lunations for 12 or 3 months, to make a general table for finding the mean time of new or full moon in any given year and month whatever.

	D.	H.	M.	S.	Th.
In 11 lunations there are	324	20	4	34	10.
In 12 lunations ——	354	8	48	37	16.
In 14 lunations ——	383	21	32	40	23.

But then it would be beſt to begin the year with March, to avoid the inconvenience of loſing a day by miſtake in leap-year.

Years of the Julian period.	Years of the World.	Years before and after Christ.	Moon from sun. s	°	'	″
706	0	4008	5	28	1	17
714	8	4000	5	9	23	24
1714	1008	3000	11	20	28	57
2714	2008	2000	6	1	34	30
3714	3008	1000	0	12	40	3
3814	3108	900	10	19	46	36
3914	3208	800	8	26	53	9
4014	3308	700	7	3	59	43
4114	3408	600	5	11	6	16
4214	3508	500	3	18	12	49
4314	3608	400	1	25	19	23
4414	3708	300	0	2	25	56
4514	3808	200	10	9	32	29
4614	3908	100	8	16	39	3
4714	4008	1	6	23	45	36
4814	4108	101	5	0	52	9
4914	4208	201	3	7	58	43
5014	4308	301	1	15	5	16
5114	4408	401	11	22	11	49
5214	4508	501	9	29	18	23
5714	5008	1001	1	4	51	9
6414	5708	1701	0	24	37	2
6466	5760	1753	10	9	24	56
6514	5808	1801	6	5	26	15

Before the year of Christ 1. | After Christ.

The 4008th year before the year of Christ 1, was the 4007th year before the year of his birth; and is supposed to have been the year of the creation.

Compleat years.	Moon from sun. s	°	'	″
1	4	9	37	24
2	8	19	14	48
3	0	28	52	13
4	5	20	41	4
5	10	0	18	28
6	2	9	55	52
7	6	19	33	17
8	11	11	22	7
9	3	20	59	32
10	8	0	36	55

Julian years, 3 of which have 365 days, and the 4th 366.

Compleat years.	Moon from sun. s	°	'	″
11	0	10	14	20
12	5	2	3	11
13	9	11	40	35
14	1	21	18	0
15	6	0	55	24
16	10	22	44	15
17	3	2	21	39
18	7	11	59	4
19	11	21	36	27
20	4	13	25	19
40	8	26	50	37
60	1	10	15	56
80	5	23	41	15
100	10	7	6	33
200	8	14	13	7
300	6	21	19	40
400	4	28	26	13
500	3	5	32	47
1000	6	11	5	33
2000	0	22	11	6
3000	7	3	16	39
4000	1	14	22	12

Months.	Moon from sun. s	°	'	″
Jan.	0	0	0	0
Feb.	0	17	54	48
Mar.	11	29	15	16
April	0	17	10	3
May	0	22	53	23
June	1	10	48	11
July	1	16	31	32
Aug.	2	4	26	20
Sept.	2	22	21	8
Oct.	2	28	4	29
Nov.	3	15	59	17
Dec.	3	21	42	7

This table agrees with the *old ſtile* until the year 1753; and after that, with the *new*.

A Table of the Moon's mean Motion from the Sun.

Days.	Moon from ſun.				Moon from ſun.				Moon from ſun.			
	s	°	'	"	H. M. S.	° ' "	' " ‴	" ‴ ⁗	M. S. Th.	' " ‴	" ‴ ⁗	‴ ⁗ v
1	0	12	11	27	1	0	30	29	31	15	44	47
2	0	24	22	53	2	1	0	57	32	16	15	16
3	1	6	34	20	3	1	31	26	33	16	45	44
4	1	18	45	47	4	2	1	54	34	17	16	13
5	2	0	57	13	5	2	32	23	35	17	46	42
6	2	13	8	40	6	3	2	52	36	18	17	10
7	2	25	20	7	7	3	33	20	37	18	47	39
8	3	7	31	34	8	4	3	49	38	19	18	7
9	3	19	43	0	9	4	34	18	39	19	48	36
10	4	1	54	27	10	5	4	46	40	20	19	5
11	4	14	5	54	11	5	35	15	41	20	49	33
12	4	26	17	20	12	6	5	43	42	21	20	2
13	5	8	28	47	13	6	36	12	43	21	50	31
14	5	20	40	14	14	7	6	41	44	22	20	59
15	6	2	51	40	15	7	37	9	45	22	51	28
16	6	15	3	7	16	8	7	38	46	23	21	56
17	6	27	14	34	17	8	38	6	47	23	52	25
18	7	9	26	0	18	9	8	35	48	24	22	54
19	7	21	37	27	19	9	39	4	49	24	53	22
20	8	3	48	54	20	10	9	32	50	25	23	51
21	8	16	0	21	21	10	40	1	51	25	54	19
22	8	28	11	47	22	11	10	30	52	26	24	48
23	9	10	23	14	23	11	40	58	53	26	55	17
24	9	22	34	41	24	12	11	27	54	27	25	45
25	10	4	46	7	25	12	41	55	55	27	56	14
26	10	16	57	34	26	13	12	24	56	28	26	43
27	10	29	9	1	27	13	42	53	57	28	57	11
28	11	11	20	27	28	14	13	21	58	29	27	40
29	11	23	31	54	29	14	43	50	59	29	58	8
30	0	5	43	21	30	15	14	18	60	30	28	37
31	0	17	54	47								
32	1	0	6	15								

1 Lunation = 29 12^{h} 44^{m} 3^{s} 6^{th} 21^{f} 14^{v} 24^{vi} 0^{vii}

In leap years, after February, a day and its motion muſt be added to the time for which the moon's mean diſtance from the ſun is given. But, when the mean time of any new or full moon is required in leap-year after February, a day muſt be ſubtracted from the mean time thereof, as found by the tables. In common years they give the day ight.

Years

A Table of the Sun's mean Motion from the Moon's Aſcending Node.

Years of the Julian period.	Years of the World.	Years before and after CHRIST.	Sun from node.			
			s	°	′	″
706	0	4008	7	6	17	9
714	8	4000	0	11	4	55
1714	1008	3000	9	10	35	11
2714	2008	2000	6	10	5	28
3714	3008	1000	3	9	35	44
3814	3108	900	7	24	32	46
3914	3208	800	0	9	29	48
4014	3308	700	4	24	26	49
4114	3408	600	9	9	23	51
4214	3508	500	1	24	20	53
4314	3608	400	6	9	17	54
4414	3708	300	10	24	14	56
4514	3808	200	3	9	11	58
4614	3908	100	7	24	8	59
4714	4008	1	0	9	6	1
4814	4108	101	4	24	3	3
4914	4208	201	9	9	0	4
5014	4308	301	1	23	57	6
5114	4408	401	6	8	54	8
5214	4508	501	10	23	51	9
5714	5008	1001	9	8	36	18
6414	5708	1701	4	23	15	30
6466	5760	1753	1	28	0	19
6514	5808	1801	8	25	44	44

Before the year of CHRIST 1.—After CHRIST.

The 4008th year before the year of CHRIST 1, was the 4007th year before the year of his birth; and is ſuppoſed to have been the year of the creation.

Com-pleat years.	Sun from node.			
	s	°	′	″
11	7	2	3	56
12	7	22	11	39
13	8	11	17	2
14	9	0	22	25
15	9	19	27	49
16	10	9	35	31
17	10	28	40	55
18	11	17	46	18
19	0	6	51	43
20	0	26	59	24
40	1	23	58	49
60	2	20	58	13
80	3	17	56	37
100	4	14	57	2
200	8	29	54	3
300	1	14	51	5
400	5	29	48	7
500	10	14	45	8
1000	8	29	30	17
2000	5	29	0	33
3000	2	28	30	50
4000	11	28	1	6

Compleat Years.	Sun from node.			
	s	°	′	″
1	0	19	5	23
2	1	8	10	47
3	1	27	16	10
4	2	17	23	53
5	3	6	29	16
6	2	25	34	40
7	4	14	40	3
8	5	4	47	46
9	5	23	53	9
10	6	12	58	33

Julian years, 3 of which have 365 days, and the 4th 366.

Months.	Sun from node.			
	s	°	′	″
Jan.	0	0	0	0
Feb.	1	2	11	48
Mar.	2	1	16	39
April	3	3	28	27
May	4	4	37	57
June	5	6	49	45
July	6	7	59	14
Aug.	7	9	11	1
Sept.	8	12	22	49
Oct.	9	13	32	18
Nov.	10	15	44	5
Dec.	11	16	53	34

This table agrees with the *old ſtile* until the year 1753; and after that, with the *new*.

A Table of the Sun's mean Motion from the Moon's Ascending Node.

Days.	Sun from node.				Sun from node.				Sun from node.			
	s	°	'	"	H. M. '	° ' "	' " ‴	" ‴ ⁗	M. ' Th	' " ‴	" ‴ ⁗	‴ ⁗ v
1	0	1	2	19								
2	0	2	4	38								
3	0	3	6	57	1	0	2	36	31	1	20	31
4	0	4	9	16	2	0	5	12	32	1	23	7
5	0	5	11	36	3	0	7	48	33	1	25	43
6	0	6	13	54	4	0	10	23	34	1	28	9
7	0	7	16	13	5	0	12	59	35	1	31	55
8	0	8	18	32	6	0	15	35	36	1	33	31
9	0	9	20	51	7	0	18	11	37	1	36	6
10	0	10	23	10	8	0	20	47	38	1	38	42
11	0	11	25	29	9	0	23	23	39	1	41	18
12	0	12	27	48	10	0	25	58	40	1	43	54
13	0	13	30	7	11	0	28	33	41	1	46	36
14	0	14	32	26	12	0	31	9	42	1	49	5
15	0	15	34	15	13	0	33	45	43	1	51	41
16	0	16	37	4	14	0	36	21	44	1	54	17
17	0	17	39	23	15	0	38	57	45	1	56	53
18	0	18	41	41	16	0	41	32	46	1	59	29
19	0	19	44	0	17	0	44	8	47	2	2	5
20	0	20	46	19	18	0	46	44	48	2	4	41
21	0	21	48	38	19	0	49	20	49	2	7	17
22	0	22	50	57	20	0	51	56	50	2	9	53
23	0	23	53	16	21	0	54	32	51	2	12	29
24	0	24	55	35	22	0	57	8	52	2	15	5
25	0	25	57	54	23	0	59	43	53	2	17	41
26	0	27	0	13	24	1	2	19	54	2	20	17
27	0	28	2	32	25	1	4	55	55	2	22	53
28	0	29	4	51	26	1	7	31	56	2	25	29
29	1	0	7	10	27	1	10	7	57	2	28	4
30	1	1	9	29	28	1	12	43	58	2	30	40
31	1	2	11	48	29	1	15	9	59	2	33	16
32	1	3	14	7	30	1	17	55	60	2	35	52

In leap years, after February, add one day and one day's motion to the time at which the ſun's mean diſtance from the aſcending node is required.

INDEX.

INDEX.

A

ADulteration of metals, to detect page 159

Air, its properties 168

Air-pump 179

——— experiments upon it 182—200

Alderſea (Mr.) his engine for raiſing water 125

Angle, of incidence 205

——— of reflection 240

——— of refraction 205

Antœci 267

Apparent motion of the heavens 250

Archimedes, his propoſition for finding the area of a circle, and the ſolidity of a cylinder raiſed upon that circle 139

—— for finding the deceit in king *Hiero*'s crown 153

Armillary ſphere 312

Atmoſphere, its whole weight upon the earth 170

Attraction, of coheſion 6

——— of gravitation 8

——— of magnetiſm 17

——— of electricity 18

Azimuth 303

B

Balance 50

Barometer 172

Bodies moving in orbits have a tendency to fly off from their orbits 30

Bodies move faſter in ſmall orbits than in large ones 31

——— their centrifugal forces ib.

Burning-glaſſes, the force of their heat 208

C

Camera obſcura 238

Carteſian vortexes, abſurd 35

Center of gravity 13

——— the curves deſcribed by bodies moving round it 25

Central forces 19—41

Circles of the ſphere 251

Climate 289

Coloured

Coloured bodies, which are tranſparent, become opake if put together 245
Colures 297
Combined forces, their effect 21
Common pump 117
Conſtellations 208
Cranes 84—89

D

Damps 175
Danger of people's riſing haſtily in a coach or boat when it is likely to be overſet 14
Days lengthened by the refraction of the ſun's rays 204
Declination of the ſun and ſtars 300
Deſcending velocity, gives a power of equal aſcent 12
Dialing 316—377
Double projectile force, a balance to a quadruple power of gravity 27

E

Earth, its motion demonſtrated 45
——proof of its being globular 247
Earthquakes 178
Eclipſes 384
Ecliptic 249
Electricity 18
Engine (any mechanical) how to compute its power 48
——for working pumps by water 125
——for raiſing water by the ſtrength of horſes 128
Equation of time 310
Equinoctial 249
Eye, deſcribed 212

F

Face of the heaven and earth, how repreſented in a machine 248
Fermentations 176
Fire damps 177
Fluids, their preſſure 109
Fire engine 143
Forces, central 19—41
——combined 21
Forcing pump 122
Foundation principle of all mechanics 47
Fountain at command 116
Frigid zones 274

Gold,

INDEX.

G

Gold, how much heavier than its bulk of water 153
Gold-beaters, to what a prodigious extent they can hammer out gold 4
Glaſs, the ſhapes into which it is generally ground for optical uſes 207
Globes, their uſe 247
Gravity 8
—decreaſes as the ſquare of the diſtance increaſes 10

H

Hand-mill 81
Harveſt moon 306
Heavens, their apparent motion 250
Horizon, ſenſible and rational 251
Horſe-mill 82
——pump 126
Hour-circles 253
Hydraulic engines 112—134
Hydroſtatics 101—110
Hydroſtatic balance 154
————paradox 104
————bellows 107
————tables 135—146

I

Inactivity of matter 2
Inclined plane 59
Infinite diviſibility of matter 5
Intermitting ſprings 117

K

Kepler's problem concerning the ſquares of the periods and cubes of the diſtances of the planets 34

L

Latitude, how found 346
Laws of the planets motions 23
Lead, how it may be made to ſwim in water 109
Lewenhoek, his account of the number and ſize of the ſmall animals in the milt of a cod-fiſh 4
Lenſes, their properties 207
Lever, its uſe 49
Light, the amazing ſmallneſs of its particles 201
—— reflected 202
—— refracted 203
Line of direction 13
Loadſtone, its properties 17

Long

Long (Rev. Dr.) his curious experiment with a concave mirrour 227
——— his glaſs ſphere 312
Looking-glaſs 240
——— need be only half the length and half the breadth of a man, to ſhew him his whole image 241

M

Magnetiſm 17
Man, how he may raiſe himſelf up by his breath 109
——— of a middle ſize, how much he is preſſed by the atmoſphere 171
Matter, its properties 1—19
Mechanical powers 49
——— all combined in one engine 70
Metals, expand by heat 15
——— their ſpecific gravities 158
Microſcope, ſingle 218
——— double ib.
———ſolar 219
Mills for grinding corn 71—80
Mirrours, how they reflect the light 221
Monſoons 174
Moon, the law of her motion 3, 24, 25
Motion (all) naturally rectilineal 20
Multiplying glaſs 237
Newſham's engine for extinguiſhing fire 124

O

Objects, how their images are formed by means of glaſſes 210
———why they appear erect, notwithſtanding their images are inverted in the eye 214
———why they appear coloured when ſeen through ſome teleſcopes 232
Opera-glaſs 240
Optic nerve, why that part of the image which falls upon it is loſt 215

P

Padmore (Mr.) his improvement of cranes 89
Periœci 268
Perſian wheel 152
Pile engine 98
Planetary motions (the laws thereof) 23

Poles of the earth and heavens 249—252
Polar circles 256
Poroſity of bodies 15
Precepts for calculating the mean times of new and full moons and eclipſes 377
Priſmatic colours 243
————make a white when blended together 245
Pulley 57
Pump, common 117
———forcing 122
———engine to work by water 125
——————by horſes 126
Pyrometer 16

Q

Quantity of matter in bodies, is in exact proportion to their weight 10
Quickſilver its weight 171

R

Rain-bow 246
Rays of light 201
Repulſion 7
Right aſcenſion 309
Running water, its weight 141

S

Sails of a wind-mill, their proper form and angle 82
——————— their incredible velocity 83, 84
Screw, its power 68
——— ſhewn by a machine ib.
Seaſons, how they may be ſhewn by a ſmall globe 257
Signs of the zodiac 259
Silver, how much heavier than its bulk of water 155
Slare (Dr.) his dangerous experiment 209
Solidity of matter 1
Specific gravities of bodies 153
Spectacles, why ſome eyes require them 217
Spirituous liquors, to know whether they are genuine or not 169
Spouting fluids 114
Steam (or fire) engine 143
Steelyard 51
Sun, appears above the horizon when he is really below it 294
Syphon 113

Tantalus's

INDEX.

T

Tantalus's cup 115
Table for mill-wrights 80
——of the quantity of water that may be raiſed to any given height by a common pump 122
—— of ſines for the elevation of water-pipes 132
—— of the quantity and weight of water in a pipe of a given length, and diameter of bore 135—146
——of the power of the ſteam-engine 151
——of ſpecific gravities 158, 59
——of troy weight reduced to avoirdupoiſe 165
——of avoirdupoiſe weight reduced to troy 166
——of the rarity and expanſion of air 170
——of the miles in a degree of longitude in all latitudes 271
——of the ſun's place and declination 340—345
Tables for calculating new and full moons and eclipſes 384
Teleſcopes, refracting and reflecting 227—232
Temperate zones 274
Thermometer 161
Thunder and lightning 176
Toricellian experiment 170
Torrid zone 274
Trade winds 173
Tropics 256

U V

Up and down, only relative terms 248
Velocity of ſpouting fluids 111
Viſion, how cauſed 211
Vivifying ſpirit in air 175

W

Water, how conveyed over hills and valleys 113
Water-mills 71
Wedge 62
Wheel and axle 55
Wheel-carriages 90
Whirling table 28
Winds, the cauſe of 173
Wind-mill 82
Wood, though light, may be made to lie at the bottom of water 110
World, has a tendency of itſelf to come to an end 42, 43

Z

Zodiac 299
Zones 274

A
SUPPLEMENT
TO THE PRECEDING
LECTURES.

A SUPPLEMENT TO THE PRECEDING LECTURES.

MECHANICS.

The Deſcription of a new and ſafe Crane, which has four different Powers, adapted to different Weights.

THE common crane conſiſts only of a large wheel and axle; and the rope, by which goods are drawn up from ſhips, or let down from the quay to them, winds or coils round by the axle, as the axle is turned by men walking in the wheel. But, as theſe engines have nothing to ſtop the weight from running down, if any of the men happen to trip or fall in the wheel, the weight deſcends, and turns the wheel rapidly backward, and toſſes the men violently about within it; which has produced melancholy inſtances, not only of limbs

to think of a method of remedying th
for that purpoſe, I contrived a cra
proper ſtop to prevent the danger,
different powers ſuited to different v
that there might be as little loſs of ti
ſible: and alſo, that when heavy go
down into ſhips, the deſcent may be
deliberate.

This crane has four different powe
believe, it might be built in a room e
width: the gib being on the outſ
room.

Three trundles, with different n
ſtaves, are applied to the cogs of a
wheel with an upright axle; and the
draws up the weight, coils round the
wheel has 96 cogs, the largeſt trundl
the next largeſt has 12, and the ſma
So that the largeſt trundle makes 4
for one revolution of the wheel; the
8, and the ſmalleſt makes 16.
occaſionally put upon the axis of eith
trundles, for turning it; the trundle
uſed that gives a power beſt ſuited to
and the handle of the winch deſcribe

length of the winch doubles the power gained by each trundle.

As the power gained by any machine, or engine whatever, is in direct proportion as the velocity of the power is to the velocity of the weight; the powers of this crane are eaſily eſtimated, and they are as follows.

If the winch be put upon the axle of the largeſt trundle, and turned four times round, the wheel and axle will be turned once round: and the circle deſcribed by the power that turns the winch, being, in each revolution, double the circumference of the axle, when the thickneſs of the rope is added thereto; the power goes through eight times as much ſpace as the weight riſes through: and therefore (making ſome allowance for friction) a man will raiſe eight times as much weight by the crane as he would by his natural ſtrength without it: the power, in this caſe, being as eight to one.

If the winch be put upon the axis of the next trundle, the power will be as ſixteen to one, becauſe it moves 16 times as faſt as the weight moves.

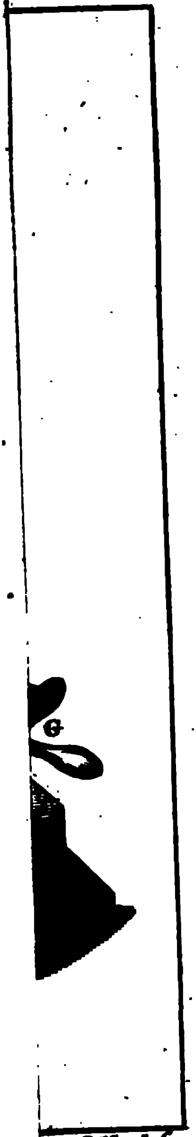
If the winch be put upon the axis of the ſmalleſt trundle, and turned round; the power will be as 32 to one.

But, if the weight ſhould be too great, even for this power to raiſe, the power may be doubled by drawing up the weight by one of the parts of a double rope, going under a pulley in the moveable block, which is hooked to the weight below the arm of the gib; and then the power will be as 64 to one. That is, a man could then raiſe 64 times as much weight by the crane as he could raiſe by his natural ſtrength without it; becauſe, for every inch that the weight riſes, the working power will move through 64 inches.

By hanging a block with two pullies to the arm of the gib, and having two pullies in the moveable block that riſes with the weight, the rope being doubled over and under theſe pullies, the power of the crane will be as 128 to one. And ſo, by increaſing the number of pullies, the power may be increaſed as much as you pleaſe: always remembering, that the larger the pullies are, the leſs is their friction.

Whilſt the weight is drawing up, the ratch-teeth of a wheel ſlip round below a catch or click that falls ſucceſſively into them, and ſo hinders the crane from turning backward, and detains the weight in any part of its aſcent, if the man who works at the winch ſhould accidentally happen to quit his hold, or chooſe to reſt himſelf before the weight be quite drawn up.

In order to let down the weight, a man pulls down one end of a lever of the ſecond kind, which lifts the catch of the ratchet-wheel, and gives the weight liberty to deſcend. But, if the deſcent be too quick, he pulls the lever a little farther down, ſo as to make it rub againſt the outer edge of a round wheel; by which means he lets down the weight as ſlowly as he pleaſes: and, by pulling a little harder, he may ſtop the weight, if needful, in any part of its deſcent.



J. Mynde sc.

If he accidentally quits hold of the lever, the catch immediately falls, and ſtops both the weight and the whole machine.

This crane is repreſented in PLATE I. where *A* is the great wheel, and *B* its axle on which the rope *C* winds. This rope goes over a pulley *D* in the end of the arm of the gib *E*, and draws up the weight *F*, as the winch *G* is turned round. *H* is the largeſt trundle, *I* the next, and *K* is the axis of the ſmalleſt trundle, which is ſuppoſed to be hid from view by the upright ſupporter *L*. A trundle *M* is turned by the great wheel, and on the axis of this trundle is fixed the ratchet-wheel *N*, into the teeth of which the catch *O* falls. *P* is the lever, from which goes a rope *Q Q*, over a pulley *R* to the catch; one end of the rope being fixed to the lever, and the other end to the catch. *S* is an elaſtic bar of wood, one end of which is ſcrewed to the floor: and, from the other end goes a rope (out of ſight in the figure) to the further end of the lever, beyond the pin or axis on which it turns in the upright ſupporter *T*. The uſe of this bar is to keep up the lever from rubbing againſt the edge of the wheel *U*, and to let the catch keep in the teeth of the ratchet-wheel: But a weight hung to the farther end of the lever would do full as well as the elaſtic bar and rope.

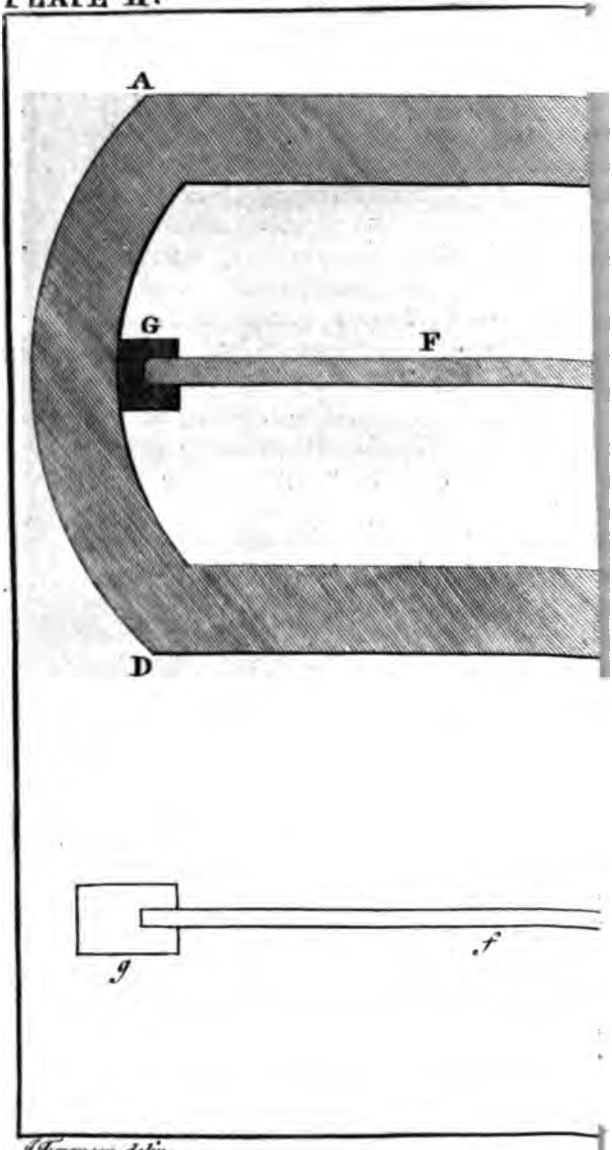
When the lever is pulled down, it lifts the catch out of the ratchet-wheel, by means of the rope *Q Q*, and gives the weight *F* liberty to deſcend: but if the lever *P* be pulled a little farther down than what is ſufficient to lift the catch *O* out of the ratchet-wheel *N*, it will rub againſt the edge of the wheel *U*, and thereby hinder the too quick deſcent of the weight; and will quite ſtop the weight if pulled hard. And if the man who pulls the lever, ſhould happen inadvertently to let it go; the elaſtic bar will ſuddenly pull it up, and the catch will fall down and ſtop the machine.

W W are two upright rollers above the axis or upper gudgeon of the gib *E*: their uſe is to let the rope *C* bend upon them, as the gib is turned to either ſide, in order to bring the weight over the place where it is intended to be let down.

N. B. The rollers ought to be ſo placed, that if the rope *C* be ſtretched cloſe by their utmoſt ſides, the half thickneſs of the rope may be perpendicularly over the center of the upper gudgeon of the gib. For then, and in no other poſition of the rollers, the length of the rope between the pulley in the gib and the axle of the great wheel will be always the ſame, in all poſitions of the gib: and the gib will remain in any poſition to which it is turned.

When either of the trundles is not turned by the winch in working the crane, it may be drawn off from the wheel, after the pin near the axis of the trundle is drawn out, and the thick piece of wood is raiſed a little behind the outward ſupporter of the axis of the trundle. But this is not material: for, as the trundle has no friction on its axis but what is occaſioned by its weight, it will be turned by the wheel without any ſenſible reſiſtance in working the crane.

PLATE II.



J. Ferguson delin.

A *Pyrometer, that makes the Expanſion of Metals by Heat viſible to the five and forty thouſandth Part of an Inch.*

The upper ſurface of this machine is repreſented by *Fig.* 1. of Plate II. Its frame *ABCD* is made of mahogany wood, on which is a circle divided into 360 equal parts; and within that circle is another, divided into 8 equal parts. If the ſhort bar *E* be puſhed one inch forward (or toward tne center of the circle) the index *e* will be turned 125 times round the circle of 360 parts or degrees. As 125 times 360 is 45,000, 'tis evident, that if the bar *E* be moved only the 45,000dth part of an inch, the index will move one degree of the circle. But as in my pyrometer, the circle is 9 inches in diameter, the motion of the index is viſible to half a degree, which anſwers to the ninety thouſandth part of an inch in the motion or puſhing of the ſhort bar *E*.

One end of a long bar of metal *F* is laid into a hollow place in a piece of iron *G*, which is fixed to the frame of the machine; and the other end of this bar is laid againſt the end of the ſhort bar *E*, over the ſupporting croſs bar *H I:* and, as the end *f* of the long bar is placed cloſe againſt the end of the ſhort bar, 'tis plain, that if *F* expands, it will puſh *E* forward, and turn the index *e*.

The machine ſtands on four ſhort pillars, high enough from a table, to let a ſpirit-lamp be put on the table under the bar *F*; and when that is done, the heat of the flame of the lamp expands the bar, and turns the index.

There are bars of different metals, as ſilver, braſs, and iron; all of the ſame length as the bar *F*, for trying experiments on the different expanſions of different metals, by equal degrees of heat applied to them for equal lengths of time; which may be meaſured by a pendulum, that ſwings ſeconds. Thus,

Put on the braſs bar *F*, and ſet the index to the 360th degree: then put the lighted lamp under the bar, and count the number of ſeconds in which the index goes round the plate, from 360 to 360 again; and then blow out the lamp, and take away the bar.

This done, put on an iron bar *F* where the braſs one was before, and then ſet the index to the 360th degree again. Light the lamp, and put it under the iron bar, and let it remain juſt as many ſeconds as it did under the braſs one; and then blow it out, and you will ſee how many degrees the index has moved in the circle: and by that means you will know in what proportion the expanſion of iron is to the expanſion of braſs; which I find to be as 210 is to 360, or as 7 is to 12.——By this method, the relative expanſions of different metals may be found.

The bars ought to be exactly of equal ſize; and to have them ſo, they ſhould be drawn, like wire, through a hole.

When the lamp is blown out, you will ſee the index turn backward; which ſhews that the metal contracts as it cools.

The inſide of this pyrometer is conſtructed as follows.

In *Fig.* 2. *A a* is the ſhort bar, which moves between rollers; and, on the ſide *a* it has 15 teeth in an inch, which take into the leaves of a pinion *B* (12 in number) on whoſe axis is the wheel *C* of 100 teeth, which take into the 10 leaves of the pinion *D*, on whoſe axis is the wheel *E* of 100 teeth, which take into the 10 leaves of the pinion *F*, on the top of whoſe axis is the index above mentioned.

Now, as the wheels *C* and *E* have 100 teeth each, and the pinions *D* and *F* have ten leaves each; 'tis plain, that if the wheel *C* turn once round, the pinion *F* and the index on its axis will turn 100 times round. But, as the firſt pinion *B* has only 12 leaves, and the bar *A a* that turns it has 15 teeth in an inch, which is 12 and a fourth part more; one inch motion of the bar will cauſe the laſt pinion *F* to turn an hundred times round, and a fourth part of an hundred over and above, which is 25. So that, if *A a* be puſhed one inch, *F* will be turned 125 times round.

A ſilk thread *b* is tied to the axis of the pinion *D*, and wound ſeveral times round it; and the other end of the thread is tied to a piece of ſlender watch-ſpring *G* which is fixed into the ſtud *H*. So that, as the bar *f* expands, and puſhes the bar *A a* forward, the thread winds round the axle, and draws out the ſpring; and as the bar contracts, the ſpring pulls back the thread, and turns the work the contrary way, which puſhes back the ſhort bar *A a* againſt the long bar *f*. This ſpring always keeps the teeth of the wheels in contact with the leaves of the pinions, and ſo prevents any ſhake in the teeth.

In *Fig.* 1. the eight diviſions of the inner circle are ſo many thouſandth parts of an inch in the expanſion or contraction of the bars; which is juſt one thouſandth part of an inch for each diviſion moved over by the index.

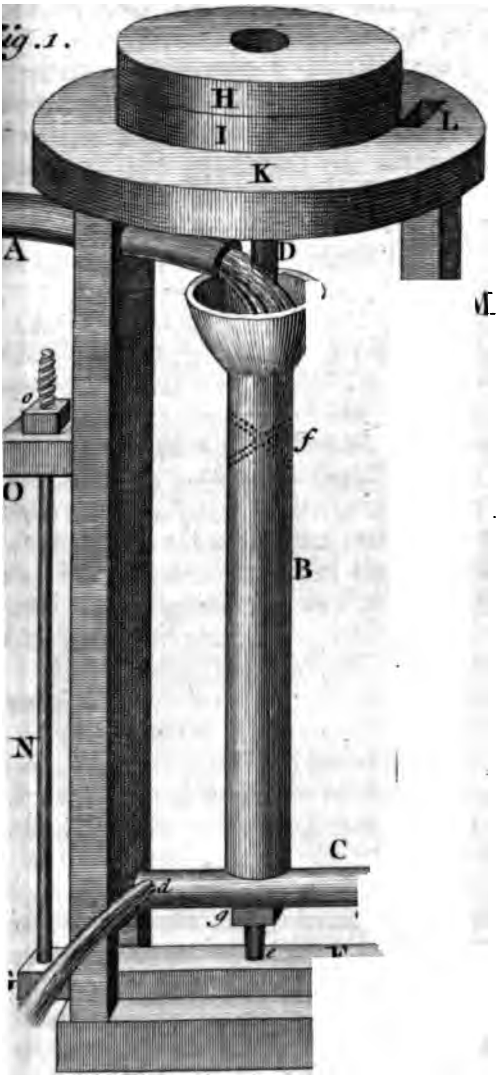
A Water-Mill, invented by Dr. Barker, *that has neither Wheel nor Trundle.*

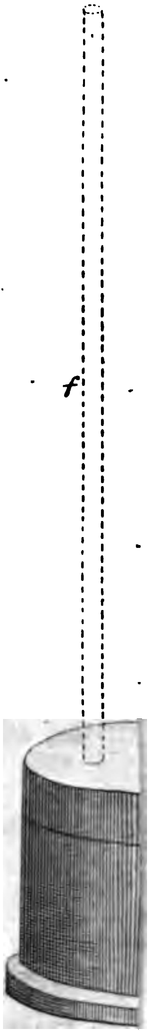
This machine is repreſented by *Fig.* 2. of Plate III. in which, *A* is a pipe or channel that brings water to the upright tube *B*. The water runs down the tube, and thence into the horizontal trunk *C*, and runs out through holes at *d* and *e* near the ends of the trunk on the contrary ſides thereof.

The upright ſpindle *D* is fixt in the bottom of the trunk, and ſcrewed to it below by the nut *g*; and is fixt into the trunk by two croſs bars at *f*: ſo that, if the tube *B* and trunk *C* be turned round, the ſpindle *D* will be turned alſo.

The top of the ſpindle goes ſquare into the rynd of the upper mill-ſtone *H*, as in common mills; and, as the trunk, tube, and ſpindle turn round, the mill-ſtone is turned round thereby. The lower, or quieſcent mill-ſtone is repreſented by *I*; and *K* is the floor on which it reſts, and wherein is the hole *L* for letting the meal

ATE III.





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HYDROSTATICS.

A Machine for demonſtrating that, on equal Bottoms, the Preſſure of Fluids is in Proportion to their perpendicular Heights, without any regard to their Quantities.

THIS is termed *The Hydroſtatical Paradox:* and the machine for ſhewing it is repreſented in *Fig.* 2. of Plate III. In which *A* is a box that holds about a pound of water, *a b c d e* a glaſs-tube fixt in the top of the box, having a ſmall wire within it; one end of the wire being hooked to the end *F* of the beam of a balance, and the other end of the wire fixt to a moveable bottom, on which the water lies, within the box; the bottom and wire being of equal weight with an empty ſcale (out of ſight in the figure) hanging at the other end of the balance. If this ſcale be pulled down, the bottom will be drawn up within the box, and that motion will cauſe the water to riſe in the glaſs-tube.

Put one pound weight into the ſcale, which will move the bottom a little, and cauſe the water to appear juſt in the lower end of the tube at *a*; which ſhews that the water preſſes with the force of one pound on the bottom: put another pound into the ſcale, and the water will riſe from *a* to *b* in the tube, juſt twice as high above the bottom as it was when at *a*; and then, as its preſſure on the bottom ſupports two pound weight in the ſcale, 'tis plain that the preſſure on the bottom is then equal to two pounds. Put a third pound weight in the ſcale, and the water will be raiſed from *b* to *c* in the tube, three times as high above the bottom as when it began to appear in the tube at *a*; which ſhews, that the ſame quantity of water that preſſed, but with the force of one pound on the bottom, when raiſed no higher than *a*, preſſes with the force of three pounds on the bottom when raiſed three times as high to *c* in the tube. Put a fourth pound weight into the ſcale, and it will cauſe the water to riſe in the tube from *c* to *d*, four times as high as it was when it was all contained in the box, which ſhews that its preſſure then upon the bottom is four times as great as when it lay all within the box. Put a fifth pound weight into the ſcale, and the water will riſe in the tube from *d* to *e*, five times as high as it was above the bottom before it roſe in the tube; which ſhews that its preſſure on the bottom is then equal to five pounds, ſeeing that it ſupports ſo much weight in the ſcale. And ſo on, if the tube was ſtill longer; for it would ſtill require an additional pound put into the ſcale, to raiſe the water in the tube to an additional height equal to the ſpace *d e*; even if the bore of the tube was ſo ſmall as only to let the wire move freely within it, and leave room for any water to get round the wire.

Hence we infer, that if a long narrow pipe or tube was fixed in the top of a caſk full of liquor, and if as much liquor was poured into the tube as would fill it, even though it were ſo ſmall as not to hold an ounce weight of liquor; the preſſure ariſing from the liquor in the tube would be as great upon the bottom, and be in as much danger of burſting it out, a if the caſk was continued up, in its full ſize, to the height of the tube, and filled with liquor.

In order to account for this ſurpriſing affair, we muſt conſider that fluids preſs equally in all manner of directions; and conſequently that they preſs juſt as ſtrongly upward as they do downward. For, if another tube, as *f*, be put into a hole made into the top of the box, and the box be filled with water; and then, if water be poured in at the top of the tube *a b c d e*, it will riſe in the tube *f* to the ſame height as it does in the other tube: and if you leave off pouring, when the water is at *c*, or any other place in the tube *a b c d e*, you will find it juſt as high in the tube *f*: and if you pour in water to fill the firſt tube, the ſecond will be filled alſo.

Now it is evident that the water riſes in the tube *f*, from the downward preſſure of the water in the tube *a b c d e*, on the ſurface of the water, contiguous to the inſide of the top of the box; and as it will ſtand at equal heights in both tubes, the upward preſſure in the tube *f* is equal to the downward preſſure in the other tube. But, if the tube *f* were put in any other part of the top of the box, the riſing of the water in it would ſtill be the ſame: or, if the top was full of holes, and a tube put into each of them, the water would riſe as high in each tube as it was poured into the tube *a b c d e*; and then the moveable bottom would have the weight of the water in all the tubes to bear, beſides the weight of all the water in the box.

And ſeeing that the water is preſſed upward into each tube, 'tis evident that, if they be all taken away, excepting the tube *a b c d e*, and the holes in which they ſtood be ſtopt up; each part, thus ſtopt, will be preſſed as much upward as was equal to the weight of water in each tube. So that, the upward preſſure againſt the inſide of the top of the box, on every part equal in breadth to the width of the tube *a b c d e*, will be preſſed upward with a force equal to the whole weight of water in the tube. And conſequently, the whole upward preſſure againſt the top of the box, ariſing from the weight or downward preſſure of the water in the tube, will be equal to the weight of a column of water of the ſame height with that in the tube, and of the ſame thickneſs as the width of the inſide of the box: and this upward preſſure againſt the top will re-act downward againſt the bottom, and be as great thereon, as would be equal to the weight of a column of water as thick as the moveable bottom is broad, and as high as the water ſtands in the tube. And thus, the paradox is ſolved.

The moveable bottom has no friction againſt the inſide of the box, nor can any water get between it and the box. The method of making it ſo, is as follows:

In *Fig.* 3. *A B C D* repreſents a ſection of the box, and *a b c d* is the lid or top thereof, which goes on tight, like the lid of a common paper ſnuff-box. *E* is the moveable bottom, with a groove around its edge, and it is put into a bladder *f g*, which is tied cloſe around it in the

groove

groove by a ſtrong waxed thread; the bladder coming up like a purſe within the box, and put over the top of it at *a* and *d* all round, and then the lid preſſed on. So that, if water be poured in through the hole *l l* of the lid, it will lie upon the bottom *E*, and be contained in the ſpace *f E g h* within the bladder; and the bottom may be raiſed by pulling the wire *i*, which is fixed to it at *E*: and by thus pulling the wire, the water will be lifted up in the tube *k*, and as the bottom does not touch againſt the inſide of the box, it moves without friction.

Now, ſuppoſe the diameter of this round bottom to be three inches (in which caſe, the area thereof will be 9 circular inches) and the diameter of the bore of the tube to be a quarter of an inch; the whole area of the bottom will be 144 times as great as the area of the top of a pin that would fill the tube like a cork.

And hence it is plain, that if the moveable bottom be raiſed only the 144th part of an inch, the water will thereby be raiſed a whole inch in the tube; and conſequently, that if the bottom be raiſed one inch, it would raiſe the water to the top of a tube 144 inches, or 12 feet, in height.

N. B. The box muſt be open below the moveable bottom, to let in the air. Otherwiſe, the preſſure of the atmoſphere would be ſo great upon the moveable bottom, if it be three inches in diameter, as to require 108 pounds in the ſcale, to balance that preſſure, before the bottom could begin to move.

A Machine,

Fig. 1.

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A Machine, to be ſubſtituted in Place of the common Hydroſtatical Bellows.

In *Fig.* 1. of PLATE IV. *ABCD* is an oblong ſquare box, in one end of which is a round groove, as at *a*, from top to bottom, for receiving the upright glaſs tube *I*, which is bent to a right angle at the lower end (as at *i* in *Fig.* 2.) and to that part is tied the neck of a large bladder *K*, (Fig. 2.) which lies in the bottom of the box. Over this bladder is laid the moveable board *L* (*Fig.* 1, and 3.) in which is fixt an upright wire *M*; and leaden weights, *N N*, to the amount of 16 pounds, with holes in their middle, which are put upon the wire, over the board, and preſs upon it with all their force.

The croſs bar *p* is then put on, to ſecure the tube from falling, and keep it in an upright poſition: And then the piece *E F G* is to be put on, the part *G* ſliding tight into the dove-tail'd groove *H*, to keep the weights *N N* horizontal, and the wire *M* upright; there being a round hole *e* in the part *E F* for receiving the wire.

There are four upright pins in the four corners of the box within, each almoſt an inch long, for the board *L* to reſt upon; to keep it from preſſing the ſides of the bladder below it cloſe together at firſt.

The whole machine being thus put together, pour water into the tube at top; and the water will run down the tube into the bladder below the board; and after the bladder has been filled up to the board, continue pouring water into the tube, and the upward preſſure which it will excite in the bladder, will raiſe the board with all the weight upon it, even though the bore of the tube ſhould be ſo ſmall, that [illegible] than an ounce of water would fill it.

This machine acts upon the ſame principle as the one laſt deſcribed, concerning the *Hydroſtatical paradox.* For, the upward preſſure againſt every part of the board (which the bladder touches) equal in area to the area of the bore of the tube, will be preſſed upward with a force equal to the weight of the water in the tube; and the ſum of all theſe preſſures, againſt ſo many areas of the board, will be ſufficient to raiſe it with all the weights upon it.

In my opinion, nothing can exceed this ſimple machine, in making the upward preſſure of fluids evident to ſight.

The Cauſe of reciprocating Springs, and of ebbing and flowing Wells, explained.

In *Fig.* 1. of PLATE V. Let *a b c d* be a hill; within which is a large cavern *A A* near the top, filled or fed by rains and melted ſnow on the top *a*, making their way through chinks and crannies into the ſaid cavern, from which proceeds a ſmall ſtream *C C* within the body of the hill, and iſſues out in a ſpring at *G* on the ſide of the hill, which will run conſtantly whilſt the cavern is fed with water.

From the ſame cavern *A A*, let there be a ſmall channel *D*, to carry water into the cavern

B,

PL

23

full to the top *e*, the water will run over the bend *e*, down the part *e F*, and go off at the mouth *G*; which will make a great ſtream at *G*: and that ſtream will continue, till the ſyphon has carried off all the water from the well *B*; the ſyphon carrying off the water faſter from *B* than the pipe *D* brings water to it: and then the ſwell at *G* will ceaſe, and only the water from the ſmall pipe *C C* will run off at *G*, till the pipe *D* fills the well *B* again; and then the ſyphon will run, and make a ſwell at *G* as before.

And thus, we have an artificial repreſentation of an ebbing and flowing well, and of a reciprocating ſpring, in a very natural and ſimple manner.

HYDRAULICS.

An Account of the Principles by which Mr. Blakey *propoſes to raiſe Water from Mines, or from Rivers, to ſupply Towns and Gentlemen's Seats, by his new invented Fire-Engine, for which he has received* His MAJESTY's *Patent.*

ALTHOUGH I am not at liberty to deſcribe the whole of this ſimple engine, yet I have the patentee's leave to deſcribe ſuch a one as will ſhew the principles by which it acts.

In *Fig.* 4. of PLATE IV. let *A* be a large, ſtrong, cloſe veſſel; immerſed in water up to the cock *b*, and having a hole in the bottom, with a valve *a* upon it, opening upward within the veſſel. A pipe *B C* riſes from the bottom of this veſſel, and has a cock *c* in it near the top, which is ſmall there, for playing a very high jet *d*. *E* is the little boiler (not ſo big as a common tea-kettle) which is connected with the veſſel *A* by the ſteam pipe *F*; and *G* is a funnel, through which a little water muſt be occaſionally poured into the boiler, to yield a proper quantity of ſteam. And a ſmall quantity of water will do for that purpoſe, becauſe ſteam poſſeſſeth upwards of 14,000 times as much ſpace or bulk as the water does from which it proceeds.

The veſſel *A* being immerſed in water up to the cock *b*, open that cock, and the water will ruſh in, through the bottom of the veſſel at *a*, and fill it as high up as the water ſtands on its outſide; and the water, coming into the veſſel, will drive the air out of it (as high as the water riſes within it) through the cock *b*. When the water has done ruſhing into the veſſel, ſhut the cock *b*, and the valve *a* will fall down, and hinder the water from being puſhed out that way, by any force that preſſeth on its ſurface. All the part of the veſſel above *b*, will be full of common air, when the water riſes to *b*.

Shut the cock *c*, and open the cocks *d* and *e*; then pour as much water into the boiler *E* (through the funnel *G*) as will about half fill the boiler; and then ſhut the cock *d*, and leave the cock *e* open.

This done, make a fire under the boiler *E*, and the heat thereof will raiſe a ſteam from the water in the boiler; and the ſteam will make its way thence, through the pipe *F*, into the veſſel *A*; and the ſteam will compreſs the air (above *b*) with a very great force upon the ſurface of the water in *A*.

When the top of the veſſel *A* feels very hot by the ſteam under it, open the cock *c* in the pipe *C*; and the air being ſtrongly compreſſed in *A*, between the ſteam and the water therein, will drive all the water out of the veſſel *A*, up the pipe *B C*, from which it will fly up in a jet to a very great height.——In my fountain, which is made in this manner after Mr. Blakey's, three tea-cup-fulls of water in the boiler will afford ſteam enough to play a jet 30 feet high.

When all the water is out of the veſſel *A*, and the compreſſed air begins to follow the jet, open the cocks *b* and *d* to let the ſteam out of the boiler *E* and veſſel *A*, and ſhut the cock *e* to prevent any more ſteam from getting into *A*; and the air will ruſh into the veſſel *A* through the cock *b*, and the water through the valve *a*; and ſo the veſſel will be filled up with water to the cock *b* as before. Then ſhut the cock *b* and the cocks *c* and *d*, and open the cock *e*; and then, the next ſteam that riſes in the boiler will make its way into the veſſel *A* again; and the operation will go on, as above.

When all the water in the boiler *E* is evaporated, and gone off into ſteam, pour a little more into the boiler, through the funnel *G*.

In order to make this engine raiſe water to any gentleman's houſe; if the houſe be on the bank of a river, the pipe *B C* may be continued up to the intended height, in the direction *H I*. Or, if the houſe be on the ſide or top of a hill, at a diſtance from the river, the pipe, through which the water is forced up, may be laid along on the hill, from the river or ſpring to the houſe.

The boiler may be fed by a ſmall pipe *K*, from the water that riſes in the main pipe *B C H I*; the pipe *K* being of a very ſmall bore, ſo as to fill the funnel *G* with water in the time that the boiler *E* will require a freſh ſupply. And then, by turning the cock *d*, the water will fall from the funnel into the boiler. The funnel ſhould hold as much water as will about half fill the boiler.

When either of theſe methods of raiſing water, perpendicularly or obliquely, is uſed, there will be no occaſion for having the cock *c* in the main pipe *B C H I*: for ſuch a cock is requiſite only, when the engine is uſed as a fountain.

A contrivance may be very eaſily made, from a lever to the cocks *b*, *d*, and *e*; ſo that, by pulling the lever, the cocks *b* and *d* may be opened when the cock *e* muſt be ſhut; and the cock *e* be opened when *b* and *d* muſt be ſhut.

The boiler *E* ſhould be incloſed in a brick wall, at a little diſtance from it, all around; to give liberty for the flames of the fire under the boiler to aſcend round about it. By which means, (the wall not covering the funnel *G*) the force of the ſteam will be prodigiouſly increaſed by the heat round the boiler; and the funnel and water in it will be heated from the boiler; ſo that, the boiler will not be chilled by letting cold water into it; and the riſing of the ſteam will be ſo much the quicker.

Mr. Blakey is the only perſon who ever thought of making uſe of air as an intermediate body between ſteam and water: by which means, the ſteam is always kept from touching the water, and conſequently from being condenſed by it. And, on this new principle, he has obtained a patent: ſo that no one (vary the engine how he will) can make uſe of air between ſteam and water, without infringing on the patent, and being ſubject to the penalties of the law.

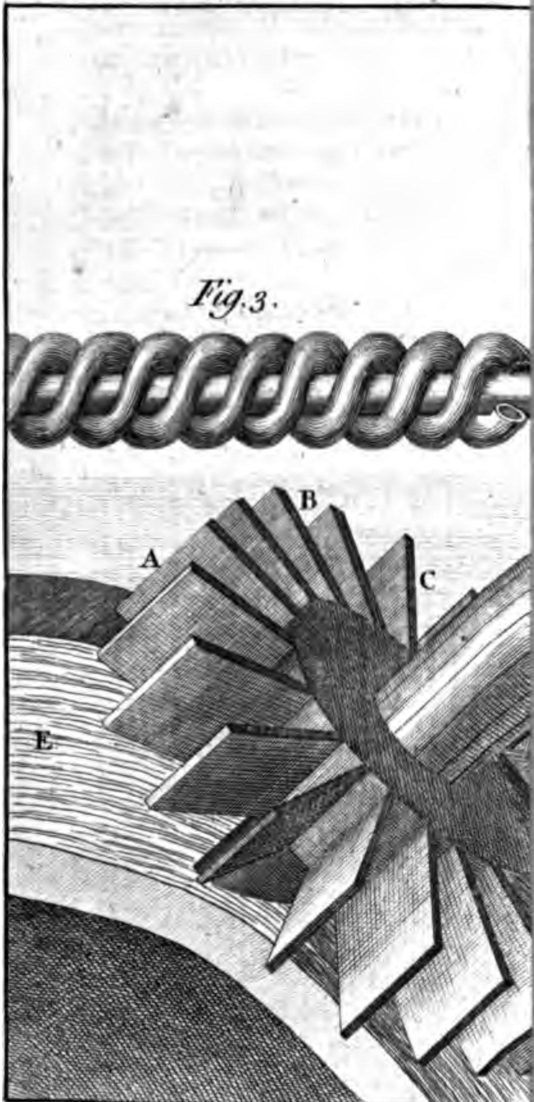
This engine may be built for a trifling expence, in compariſon of the common fire engine now in uſe: it will ſeldom need repairs, and will not conſume half ſo much fuel. And as it has no pumps with piſtons, it is clear of all their friction: and the effect is equal to the whole ſtrength or compreſſive force of the ſteam: which the effect of the common fire engine never is, on account of the great friction of the piſtons in their pumps.

ARCHIMEDES's *Screw Engine for raiſing Water.*

In *Fig.* 1. of PLATE VI. *ABCD* is a wheel, which is turned round, according to the order of the letters, by the fall of water *E F*, which need not be more than three feet. The axle *G* of the wheel is elevated ſo, as to make an angle of about 44 degrees with the horizon; and on the top of that axle is a wheel *H*, which turns ſuch another wheel *I* of the ſame number of

PLATE VI.

Fig. 3.



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of teeth: the axle *K* of this laſt wheel being parallel to the axle *G* of the two former wheels.

The axle *G* is cut into a double-threaded ſcrew (as in Fig. 2.) exactly reſembling the ſcrew on the axis of the fly of a common jack, which muſt be (what is called) a right-handed ſcrew, like the wood-ſcrews, if the firſt wheel turns in the direction *ABCD*; but muſt be a left-handed ſcrew, if the ſtream turns the wheel the contrary way. And, which-ever way the ſcrew on the axle *G* be cut, the ſcrew on the axle *K* muſt be cut the contrary way; becauſe theſe axles turn in contrary directions.

The ſcrews being thus cut, they muſt be covered cloſe over with boards, like thoſe of a cylindrical caſk; and then they will be ſpiral tubes. Or, they may be made of tubes of ſtiff leather, and wrapt round the axles in ſhallow grooves cut therein; as in *Fig.* 3.

The lower end of the axle *G* turns conſtantly in the ſtream that turns the wheel, and the lower ends of the ſpiral tubes are open into the water. So that, as the wheel and axle are turned round, the water riſes in the ſpiral tubes, and runs out at *L*, through the holes *M*, *N*, as they come about below the axle. Theſe holes (of which there may be any number, as four or ſix) are in a broad cloſe ring on the top of the axle, into which ring, the water is delivered from the upper open ends of the ſcrew-tubes, and falls into the open box *N*.

The lower end of the axle *K* turns on a gudgeon, in the water in *N*; and the ſpiral tubes in that axle take up the water from *N*, and deliver it into ſuch another box under the top of *K*; on which there may be ſuch another wheel as *I*, to turn a third axle by ſuch a wheel upon it.——And in this manner, water may be raiſed to any given height, when there is a ſtream ſufficient for that purpoſe to act on the broad float boards of the firſt wheel.

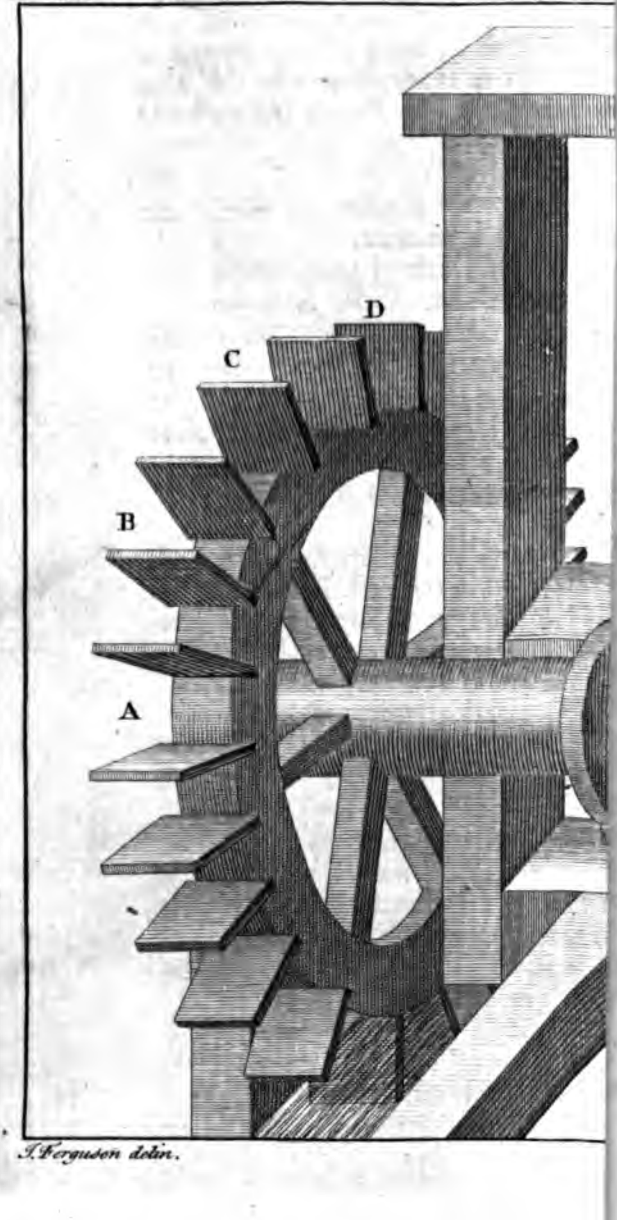
A quadruple Pump-Mill for raiſing Water.

This engine is repreſented in PLATE VII. In which *ABCD* is a wheel, turned by water according to the order of the letters. On the horizontal axis are four ſmall wheels, toothed almoſt half round: and the parts of their edges on which there are no teeth are cut down ſo, as to be even with the bottoms of the teeth where they ſtand.

The teeth of theſe four wheels take alternately into the teeth of four racks, which hang by two chains over the pullies *Q* and *L*; and to the lower ends of theſe racks there are four iron rods fixed, which go down into the four forcing pumps, *S*, *R*, *M* and *N*. And, as the wheels turn, the racks and pump-rods are alternately moved up and down.

Thus, ſuppoſe the wheel *G* has pulled down the rack *I*, and drawn up the rack *K* by the chain: as the laſt tooth of *G* juſt leaves the uppermoſt tooth of *I*, the firſt tooth of *H* is ready to take into the lowermoſt tooth of the rack *K* and pull it down as far as the teeth go; and

PLATE VII.



J. Ferguson delin.

and then the rack *I* is pulled upward through the whole ſpace of its teeth, and the wheel *G* is ready to take hold of it, and pull it down again, and ſo draw up the other.——In the ſame manner, the wheels *E* and *F* work the racks *O* and *P*.

Theſe four wheels are fixed on the axle of the great wheel in ſuch a manner, with reſpect to the poſitions of their teeth; that, whilſt they continue turning round, there is never one inſtant of time in which one or other of the pump-rods is not going down, and forcing the water. So that, in this engine, there is no occaſion for having a general air-veſſel to all the pumps, to procure a conſtant ſtream of water flowing from the upper end of the main pipe.

The piſtons of theſe pumps are ſolid plungers, the ſame as deſcribed in the fifth Lecture of my book, to which this is a Supplement. *See* PLATE XI, *Fig. 4. of that book, with the deſcription of the figure.*

From each of theſe pumps, near the loweſt end, in the water, there goes off a pipe, with a valve on its fartheſt end from the pump; and theſe ends of the pipes all enter one cloſe box, into which they deliver the water: and into this box, the lower end of the main conduct pipe is fixed. So that, as the water is forced or puſhed into this box, it is alſo puſhed up the main pipe to the height that it is intended to be raiſed.

There is an engine of this ſort, deſcribed in *Ramelli's* work: but I can truly ſay, that I never ſaw it till ſome time after I had made thi model.

The ſaid model is not above twice as big as the figure of it, here deſcribed. I turn it by a winch fixed on the gudgeon of the axle behind the water wheel; and, when it was newly made, and the piſtons and valves in good order, I put tin pipes 15 feet high upon it, when they were joined together, to ſee what it could do. And I found, that in turning it moderately by the winch, it would raiſe a hogſhead of water in an hour, to the height of 15 feet.

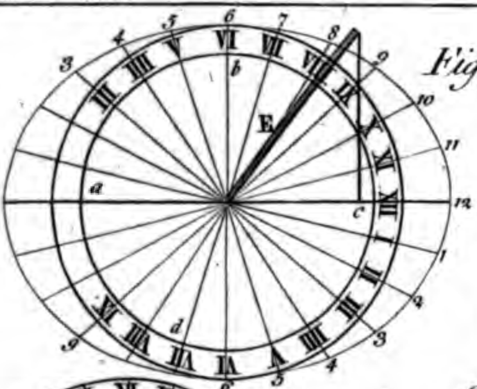
DIALLING.

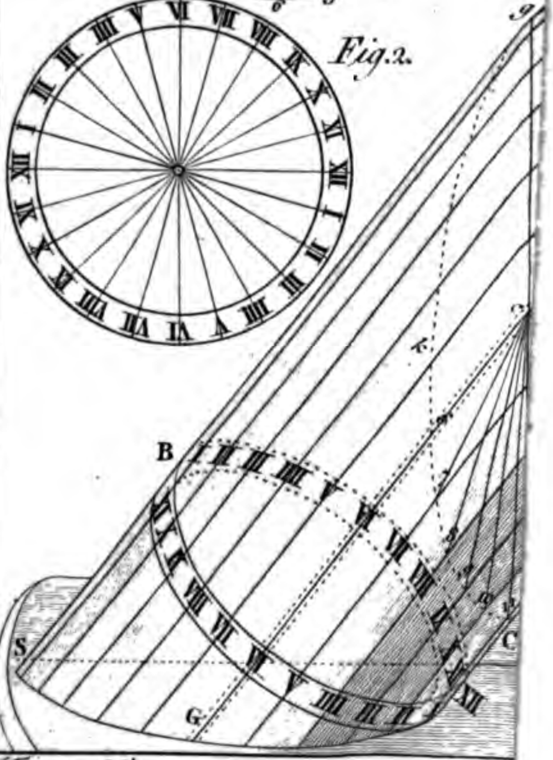
The univerſal Dialling Cylinder.

IN *Fig.* 1. of PLATE VIII. *ABCD* repreſents a cylindrical glaſs tube, cloſed at both ends with braſs plates, and having a wire or axis *EFG* fixt in the centers of the braſs plates at top and bottom. This tube is fixed to a horizontal board *H*, and its axis makes an angle with the board equal to the angle of the earth's axis with the horizon of any given place, for which the cylinder is to ſerve as a dial. And it muſt be ſet with its axis parallel to the axis of the world in that place; the end *E* pointing to the elevated pole. Or, it may be made to move upon a joint; and then it may be elevated for any particular latitude.

There are 24 ſtraight lines, drawn with a diamond, on the outſide of the glaſs, equidiſtant from each other, and all of them parallel to the axis. Theſe are the hour-lines; and the hours are

PLATE VIII.





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re ſet to them as in the figure: the XII next *B* ands for midnight, and the oppoſite XII, next he board *H*, ſtands for mid-day or noon.

The axis being elevated to the latitude of the place, and the foot-board ſet truly level, with the black line along its middle in the plane of the meridian, and the end *N* toward the north; the axis *E F G* will ſerve as a ſtile or gnomon, and caſt a ſhadow on the hour of the day, among the parallel hour lines when the ſun ſhines on the machine. For, as the ſun's apparent diurnal motion is equable in the heavens, the ſhadow of the axis will move equably in the tube; and will always fall upon *that* hour-line which is oppoſite to the ſun, at any given time.

The braſs plate *A D*, at the top, is parallel to the equator, and the axis *EFG* is perpendicular to it. If right lines be drawn from the center of this plate, to the upper ends of the equidiſtant parallel lines on the outſide of the tube; theſe right lines will be the hour-lines on the equinoctial dial *A D*, at 15 degrees diſtance from each other: and the hour-letters may be ſet to them as in the figure. Then, as the ſhadow of the axis within the tube comes on the hour-lines of the tube, it will cover the like hour-lines on the equinoctial plate *A D*.

If a thin horizontal plate *e f* be put within the tube, ſo as its edge may touch the tube all around; and right lines be drawn from the center of that plate to thoſe points of its edge which are cut by the parallel hour-lines on the tube; theſe right lines will be the hour-lines of a horizontal dial, for the latitude to which the tube is elevated. For, as the ſhadow of the axis comes ſucceſſively to the hour-lines of the tube, and covers them, it will then cover the like hour-lines on the horizontal plate *e f*, to which the hours may be ſet; as in the figure.

If a thin vertical plate *g C*, be put within the tube, ſo as to front the meridian or 12 o'clock line thereof, and the edge of this plate touch the tube all around; and then, if right lines be drawn from the center of the plate to thoſe points of its edge which are cut by the parallel hour-lines on the tube; theſe right lines will be the hour-lines of a vertical ſouth-dial: and the ſhadow of the axis will cover them at the ſame times when it covers thoſe of the tube.

If a thin plate be put within the tube ſo, as to decline, or incline, or recline, by any given number of degrees; and right lines be drawn from its center to the hour-lines of the tube; theſe right lines will be the hour-lines of a declining, inclining, or reclining dial, anſwering to the like number of degrees, for the latitude to which the tube is elevated.

And thus, by this ſimple machine, all the principles of dialling are made very plain, and evident to the ſight. And the axis of the tube (which is parallel to the axis of the world in every latitude to which it is elevated) is the ſtile or gnomon for all the different kinds of ſun-dials.

And laſtly, if the axis of the tube be drawn out, with the plates *A D*, *e f*, and *g C* upon it; and ſet it up in ſun-ſhine, in the ſame poſition as they were in the tube; you will have an equinoctial dial *A D*, a horizontal dial *e f*, and a vertical ſouth dial *g C*; on all which, the time of the day will be ſhewn by the ſhadow of the axis or gnomon *E F G*.

Let us now ſuppoſe that, inſtead of a glaſs tube, *A B C D* is a cylinder of wood; on which the 24 parallel hour-lines are drawn all around, at equal diſtances from each other; and that, from the points at top, where theſe lines end, right lines are drawn toward the center, on the flat ſurface *A D*: Theſe right lines will be the hour-lines on an equinoctial dial, for the latitude of the place to which the cylinder is elevated above the horizontal foot or pedeſtal *H*; and they are equidiſtant from each other, as in *Fig.* 2. which is a full view of the flat ſurface or top *A D* of the cylinder, ſeen obliquely in *Fig.* 1. And the axis of the cylinder (which is a ſtraight wire *E F G* all down its middle) is the ſtile or gnomon; which is perpendicular to the plane of the equinoctial dial, as the earth's axis is perpendicular to the plane of the equator.

To make a horizontal dial, by the cylinder, for any latitude to which its axis is elevated; draw out the axis and cut the cylinder quite through, as at *e b f g*, parallel to the horizontal board *H*, and take off the top part *e A D f e*; and the ſection *e b f g e* will be of an elliptical form, as in *Fig.* 3. Then, from the points of this ſection (on the remaining part *e B C f*) where the parallel lines on the outſide of the cylinder meet it, draw right lines to the center of the ſection; and they will be the true hour-lines for a horizontal dial, as *a b c d a* in *Fig.* 3. which may be included in a circle drawn on that ſection.

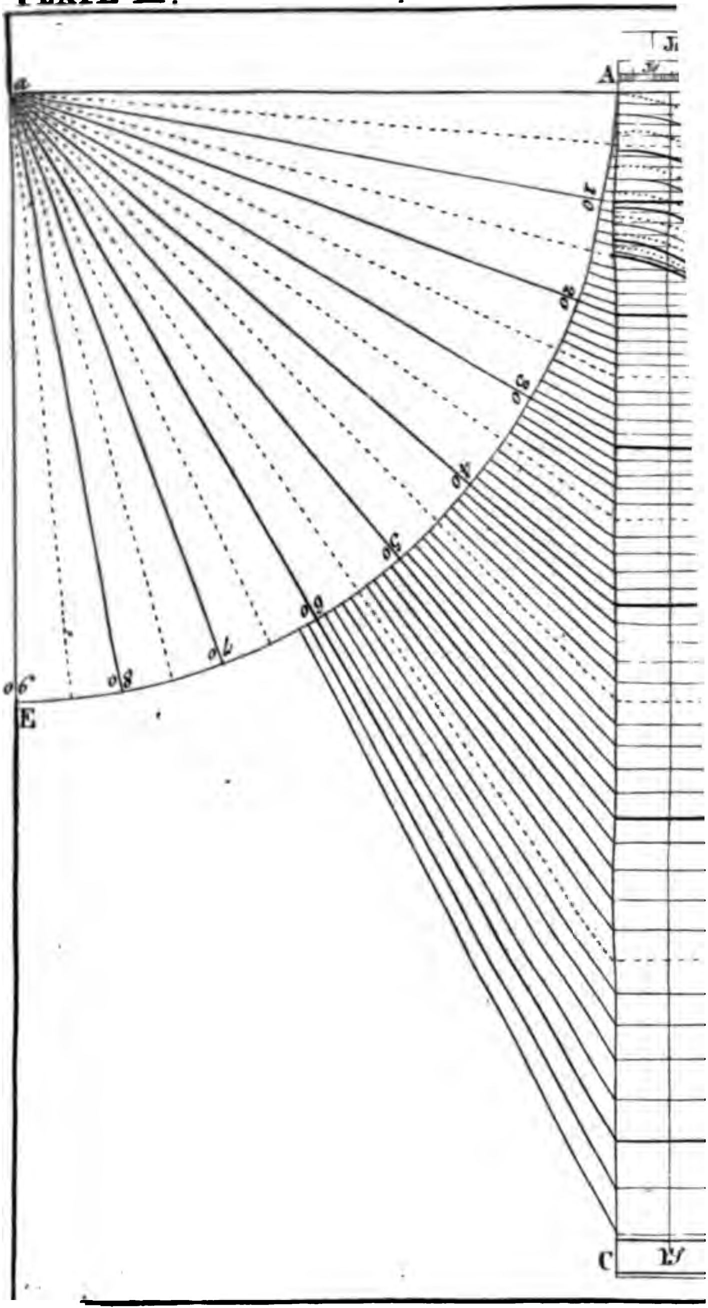
Then put the wire into its place again, and it will be a ſtile for caſting a ſhadow on the time of the day, on that dial. So, *E* (*Fig.* 3.) is the ſtile of the horizontal dial, parallel to the axis of the cylinder.

To make a vertical ſouth dial by the cylinder, draw out the axis, and cut the cylinder perpendicularly to the horizontal board *H*, as at *g i C k g*, beginning at the hour line (*B g e A*) of XII. and making the ſection at right angles to the line *S H N* on the horizontal board. Then, take off the upper part *g A D C*, and the face of the ſection thereon will be elliptical, as ſhewn in *Fig.* 4. From the points in the edge of this ſection, where the parallel hour-lines on the round ſurface of the cylinder meet it, draw right lines to the center of the ſection; and they will be the true hour-lines on a vertical direct ſouth dial, for the latitude to which the cylinder was elevated: and will appear as in *Fig.* 4. on which the vertical dial may be made of a circular ſhape, or of a ſquare ſhape as repreſented in the figure. And *F* will be its ſtile parallel to the axis of the cylinder.

And thus, by cutting the cylinder any way, ſo as its ſection may either incline, or decline, or recline, by any given number of degrees; and from thoſe points in the edge of the ſection where the outſide parallel hour-lines meet it, draw right lines to the center of the ſection; and they will be the true hour-lines, for the like declining, reclining, or inclining dial: And the axis of the cylinder will always be the gnomon or ſtile of the dial. For, which-ever way the plane of the dial lies, its ſtile (or the edge thereof

that

PLATE IX.



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ſts the ſhadow on the hours of the day)
be parallel to the earth's axis, and point
l the elevated pole of the heavens.

ineate a Sun-Dial on Paper; which, when 'ed round a Cylinder of Wood, ſhall ſhew the e of the Day, the Sun's Place in the Ecliptic, his Altitude, at any Time of Obſervation.

PLATE IX.

w the right line *a A B*, parallel to the top
paper; and, with any convenient opening
compaſſes, ſet one foot in the end of the
a, as a center, and with the other foot de-
the quadrantal arc *A E*, and divide it into
ual parts or degrees. Draw the right line
at right angles to *a A B*, and touching the
ant *A E* at the point *A*. Then, from the
a, draw right lines through as many de-
of the quadrant, as are equal to the ſun's
le at noon, on the longeſt day of the year,
place for which the dial is to ſerve;
altitude, at London, is 62 degrees: and
ue theſe right lines till they meet the tan-
ine *A C*; and, from theſe points of meet-
raw ſtraight lines acroſs the paper, paral-
the firſt right line *A B*, and they will be
rallels of the ſun's altitude, in whole de-
from ſun-riſe till ſun-ſet, on all the days
year.—— Theſe parallels of altitude muſt
wn out to the right line *B D*, which muſt
rallel to *A C*, and as far from it as is equal
intended circumference of the cylinder on
the paper is to be paſted, when the dial is
upon it.

ide the ſpace between the right lines *A C*
D (at top and bottom) into twelve equal

parts, for the twelve ſigns of the ecliptic; and, from mark to mark of theſe diviſions at top and bottom, draw right lines parallel to *A C* and *B D*; and place the characters of the 12 ſigns in theſe twelve ſpaces, at the bottom, as in the figure: beginning with ♑ or Capricorn, and ending with ♓ or Piſces. The ſpaces including the ſigns ſhould be divided by parallel lines into halves; and if the breadth will admit of it without confuſion, into quarters alſo.

At the top of the dial, make a ſcale of the months and days of the year, ſo as the days may ſtand over the ſun's place for each of them in the ſigns of the ecliptic. The ſun's place, for every day of the year, may be found by any common ephemeris: and here it will be beſt to make uſe of an ephemeries for the ſecond year after leap year; as the neareſt mean for the ſun's place on the days of the leap-year, and on thoſe of the firſt, ſecond, and third year after.

Compute the ſun's altitude for every hour (in the latitude of your place) when he is in the beginning, middle, and end of each ſign of the ecliptic; his altitude at the end of each ſign being the ſame as at the beginning of the next. And, in the upright parallel lines, at the beginning and middle of each ſign, make marks for theſe computed altitudes among the norizontal parallels of altitude, reckoning them downward, according to the order of the numeral figures ſet to them at the right hand, anſwering to the like diviſions of the quadrant at the left. And, through theſe marks, draw the curve hour-lines, and ſet the hours to them, as in the figure, reckoning the forenoon hours downward, and

the

of the ſhadow, will fall upon the time of the day, as it is before or after noon, among the curve hour-lines; and will ſhew the ſun's altitude at that time, amongſt the croſs parallels of his altitude, which go round the cylinder: and, at the ſame time, it will ſhew in what ſign of the ecliptic the ſun then is, and you may very nearly gueſs at the degree of the ſign, by eſtimation of the eye.

The ninth plate, on which this dial is drawn, may be cut out of the book, and paſted round a cylinder whoſe length is 6 inches and 6 tenths of an inch below the moveable top; and its diameter 2 inches and 24 hundred parts of an inch——Or, I ſuppoſe the copper-plate prints of it may be had at Mr. *Cadell's*, bookſeller in the Strand, London. But it will only do for London, and other places of the ſame latitude.

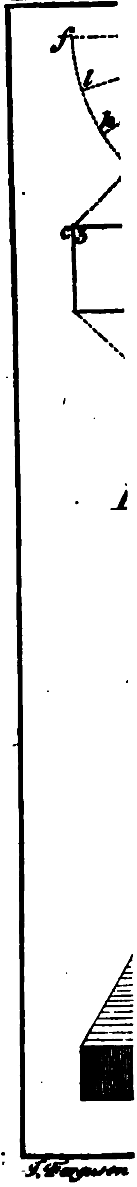
When a level table cannot be had, the dial may be hung by the ring *F* at the top. And when it is not uſed, the wire that ſerves for a ſtile may be drawn out, and put up within the cylinder; and the machine carried in the pocket.

To make three Sun-dials upon three different Planes, ſo as they may all ſhew the Time of the Day by one Gnomon.

On the flat board *A B C*, deſcribe a horizontal dial, according to any of the rules laid down in the Lecture on Dialling; and to it fix its gnomon *F G H*, the edge of the ſhadow from the ſide *F G* being that which ſhews the time of the day.

To this horizontal or flat board, join the upright board *E D C*, touching the edge *G H* of the gnomon. Then, making the top of the gnomon

PLATE



gnomon at *G* the center of the vertical ſouth dial, deſcribe a ſouth dial on the board *E D C*.

Laſtly, on a circular plate *I K* deſcribe an equinoctial dial, all the hours of which dial are equidiſtant from each other; and making a ſlit *c d* in that dial, from its edge to its center, in the XII o'clock line; put the ſaid dial perpendicularly on the gnomon *F G*, as far as the ſlit will admit of; and the triple dial will be finiſhed; the ſame gnomon ſerving all the three, and ſhewing the ſame time of the day on each of them.

An univerſal Dial on a plain Croſs.

This dial is repreſented by *Fig.* 1 of PLATE XI, and is moveable on a joint *C*, for elevating it to any given latitude, on the quadrant *C* o 90, as it ſtands upon the horizontal board *A*. The arms of the croſs ſtand at right angles to the middle part; and the top of it from *a* to *n*, is of equal length with either of the arms *n e* or *m k*.

Having ſet the middle line *t u* to the latitude of your place, on the quadrant, the board *A* level, and the point *N* northward by the needle; the plane of the croſs will be parallel to the plane of the equator; and the machine will be rectified.

Then, from III o'clock in the morning, till VI, the upper edge *k l* of the arm *i o* will caſt a ſhadow on the time of the day on the ſide of the arm *c m*: from VI till IX the lower edge *i* of the arm *i o* will caſt a ſhadow on the hours on the ſide *o q*. From IX in the morning to XII at noon, the edge *a b* of the top part *a n* will caſt a ſhadow on the hours on the arm *n e f*: from XII to III in the afternoon, the edge *c d* of the top part will caſt a ſhadow on the hours on the arm *k l m*: from III to VI in the evening, the edge *g h* will caſt a ſhadow on the hours on the part *p s*; and from VI till IX, the ſhadow of the edge *e f* will ſhew the time on the top part *a n*.

The breadth of each part, *a b*, *e f*, &c. muſt be ſo great as never to let the ſhadow fall quite without the part or arm on which the hours are marked, when the ſun is at his greateſt declination from the equator.

To determine the breadth of the ſides of the arms which contain the hours, ſo as to be in juſt proportion to their length; make an angle *A B C* (Fig. 2.) of $23\frac{1}{2}$ degrees, which is equal to the ſun's greateſt declination: and ſuppoſe the length of each arm, from the ſide of the long middle part, and alſo the length of the top-part above the arms, to be equal to *B d*.

Then, as the edges of the ſhadow from each of the arms, will be parallel to *B e*, making an angle of $23\frac{1}{2}$ degrees with the ſide *B d* of the arm when the ſun's declination is $23\frac{1}{2}$ degrees; 'tis plain, that if the length of the arm be *B d*, the leaſt breadth that it can have, to keep the edge *B e* of the ſhadow *B e g d* from going off the ſide of the arm *d e* before it comes to the end *e d* thereof, muſt be equal to *e d* or *d B*. But in order to keep the ſhadow within the quarter diviſions of the hours, when it comes near the end of the arm, the breadth thereof ſhould be ſtill greater, ſo as to be almoſt doubled, on account of the diſtance between the tips of the arms.

To place the hours right on the arms, take the following method.

Lay down the croſs *a c b d* (*Fig.* 3.) on a ſheet of paper; and with a black lead pencil, held cloſe to it, draw its ſhape and ſize on the paper. Then taking the length *a e* in your compaſſes, and ſetting one foot in the corner *a*, with the other foot deſcribe the quadrantal arc *e f*.—— Divide this arc into ſix equal parts, and through the diviſion marks draw right lines *a g*, *a h*, &c. continuing three of them to the arm *c e*, which are all that can fall upon it; and they will meet the arm in theſe points through which the lines that divide the hours from each other (as in *Fig.* 1.) are to be drawn right acroſs it.

Divide each arm, for the three hours it contains, in the ſame manner; and ſet the hours to their proper places (on the ſides of the arms) as they are marked in *Fig.* 3. Each of the hour ſpaces ſhould be divided into four equal parts, for the half hours and quarters, in the quadrant *e f* and right lines ſhould be drawn through theſe diviſion marks in the quadrant, to the arms of the croſs in order to determine the places thereon where the ſub-diviſions of the hours muſt be marked.

This is a very ſimple kind of univerſal dial; it is very eaſily made, and will have a pretty uncommon appearance in a garden.——I have ſeen a dial of this ſort, but never ſaw one of the kind that follows.

An univerſal Dial, ſhewing the Hours of the Day by a terreſtrial Globe, and by the Shadows of ſeveral Gnomons, at the ſame Time: together with all the Places of the Earth which are then enlightened by the Sun; and thoſe to which the Sun is then riſing, or on the Meridian, or Setting.

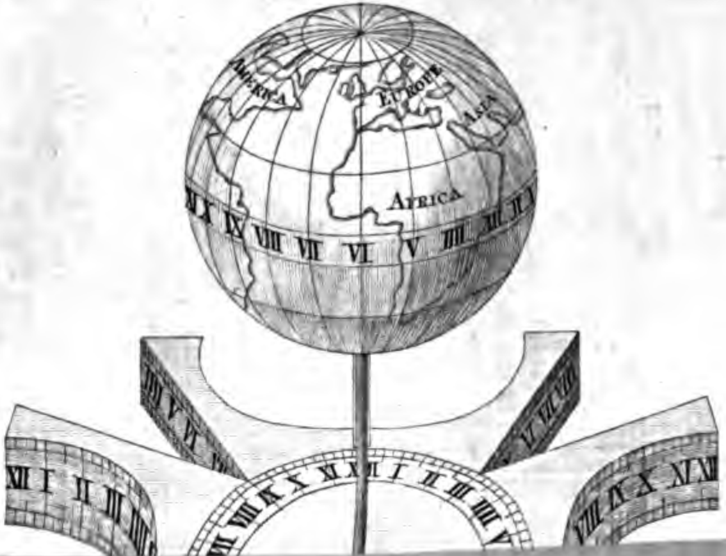
This dial (*See* PLATE XII.) is made of a thick ſquare piece of wood, or hollow metal. The ſides are cut into ſemicircular hollows, in which the hours are placed; the ſtile of each hollow coming out from the bottom thereof, as far as the ends of the hollows project. The corners are cut out into angles, in the inſides of which, the hours are alſo marked; and the edge of the end of each ſide of the angle ſerves as a ſtile for caſting a ſhadow on the hours marked on the other ſide.

In the middle of the uppermoſt ſide or plane, there is an equinoctial dial; in the center whereof, an upright wire is fixt, for caſting a ſhadow on the hours of that dial, and ſupporting a ſmall terreſtrial globe on its top.

The whole dial ſtands on a pillar, in the middle of a round horizontal board, in which there is a compaſs and magnetic needle, for placing the *meridian* ſtile toward the ſouth. The pillar has a joint with a quadrant upon it, divided into 90 degrees (ſuppoſed to be hid from ſight under the dial in the figure) for ſetting it to the latitude of any given place; the ſame way as already deſcribed in the dial on the croſs.

The equator of the globe is divided into 24 equal parts, and the hours are laid down upon it at

PLATE XII.


AMERICA
EUROPE
ASIA
AFRICA
IX VIII VII VI V IIII III II
XII I II III IIII
IIII V VI
VI VII VIII
VIII IX X XI XII
VIII IX X XI XII I II III IIII V

[illegible] parts. The time of the day may be [illegible] by theſe hours, when the ſun ſhines upon [illegible]obe.

[illegible] rectify and uſe this dial, ſet it on a level [illegible] or ſole of a window, where the ſun ſhines, [illegible]ng the meridian ſtile due ſouth, by means [illegible]he needle; which will be, when the needle [illegible] as far from the north fleur-de-lis toward [illegible]weſt, as it declines weſtward, at your place. [illegible]en bend the pillar in the joint, till the black [illegible]e on the pillar comes to the latitude of your [illegible]ace in the quadrant.

The machine being thus rectified, the plane of [illegible] dial-part will be parallel to the equator, the wire or axis that ſupports the globe will be parallel to the earth's axis, and the north pole of the globe will point toward the north pole of the heavens.

The ſame hour will then be ſhewn in ſeveral of the hollows, by the ends of the ſhadows of their reſpective ſtiles: The axis of the globe will caſt a ſhadow on the ſame hour of the day, in the equinoctial dial, in the center of which it is placed, from the 20th of March to the 23d of September; and, if the meridian of your place on the globe be ſet even with the meridian ſtile, all the parts of the globe that the ſun ſhines upon, will anſwer to thoſe places of the real earth which are then enlightened by the ſun. The places where the ſhade is juſt coming upon the globe, anſwer to all thoſe places of the earth to which the ſun is then ſetting; as the places where it is going off, and the light coming on, anſwer to all the places of the earth where the ſun is then riſing. And laſtly, if the hour of VI be marked on the equator in the meridian of your place (as it is marked on the meridian of London in the figure) the diviſion of the light and ſhade on the globe will ſhew the time of the day.

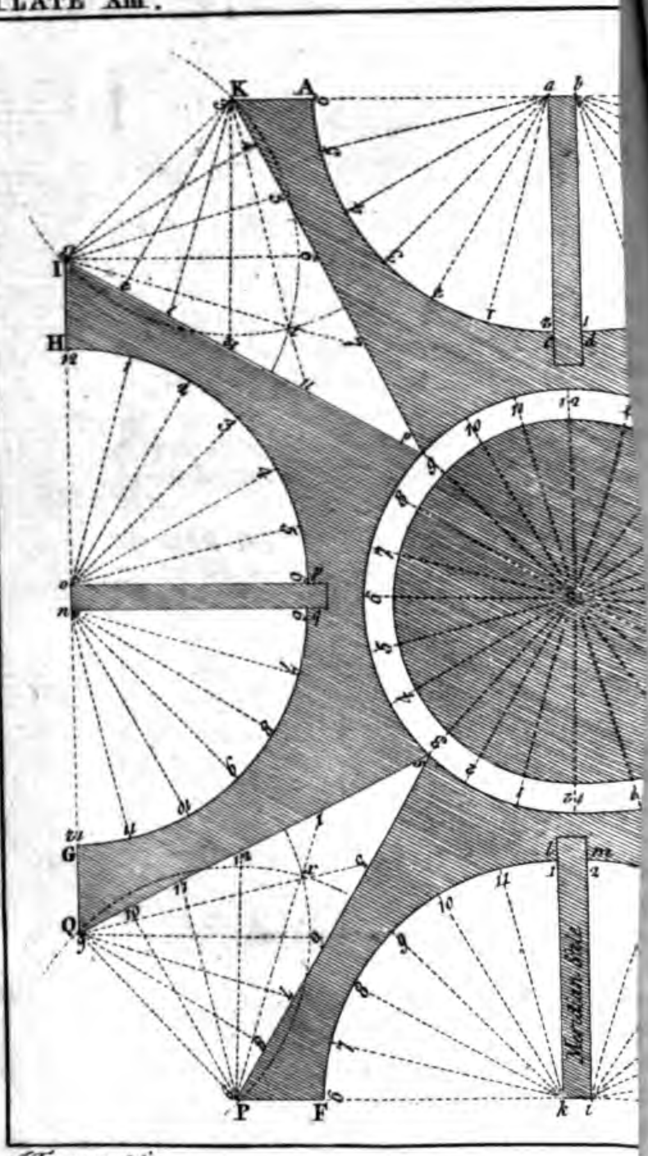
The northern ſtile of the dial (oppoſite to the ſouthern or meridian one) is hid from ſight in the figure, by the axis of the globe. The hours in the hollow to which that ſtile belongs, are alſo ſuppoſed to be hid by the oblique view of the figure: but they are the ſame as the hours in the front-hollow. Thoſe alſo in the right and left hand ſemicircular hollows are moſtly hid from ſight; and ſo alſo are all thoſe on the ſides next the eye of the four acute angles.

The conſtruction of this dial is as follows. *See* PLATE XIII.

On a thick ſquare piece of wood, or metal, draw the lines *a c* and *b d*, as far from each other as you intend for the thickneſs of the ſtile *a b c d* and in the ſame manner, draw the like thickneſs of the other three ſtiles, *e f g h*, *i k l m*, and *n o p q*, all ſtanding outright as from the center.

With any convenient opening of the compaſſes, as *a A* (ſo as to leave proper ſtrength of ſtuff when *K I* is equal to *a A*) ſet one foot in *a*, as a center, and with the other foot deſcribe the quadrantal arc *A c*. Then without altering the compaſſes, ſet one foot in *b* as a center, and with the other foot deſcribe the quadrant *d B*. All the other quadrants in the figure muſt be deſcribed in the ſame manner, and with

PLATE XIII.



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opening of the compaſſes, on their
e, *f*; *i*, *k*; and *n*, *o*: and each quadrant
into 6 equal parts, for ſo many hours,
he figure; each of which parts muſt be
ided into 4, for the half hours and quar-

equal diſtances from each corner, draw the
ines *Ip*, and *Kp*, *Lq*, and *Mq*, *Nr*, and
P s, and *Q s*; to form the four angular
ws *lpK*, *LqM*, *NrO*, and *PsQ*; mak-
he diſtances between the tips of theſe hol-
as *IK*, *LM*, *NO*, and *PQ*, each equal
e radius of the quadrants; and leaving ſuffi-
room within the angular points, *p*, *q*, *r*,
s, for the equinoctial circle in the middle.

To divide the inſides of theſe angles properly
the hour-ſpaces thereon, take the following
ethod.

Set one foot of the compaſſes in the point *I*,
a center; and open the other to *K*, and with
hat opening, deſcribe the arc *Kt*: then, with-
ut altering the compaſſes, ſet one foot in *K*,
nd with the other foot deſcribe the arc *It*.
ivide each of theſe arcs, from *I* and *K* to their
nterſection at *t*, into four equal parts; and
rom their centers *I* and *K*, through the points
f diviſion, draw the right lines *I* 3, *I* 4, *I* 5,
6, *I* 7; and *K* 2, *K* 1, *K* 12, *K* 11; and they
ill meet the ſides *Kp* and *Ip* of the angle *IpK*
here the hours thereon muſt be placed. And
eſe hour-ſpaces in the arcs muſt be ſubdivided
nto four equal parts, for the half hours and
uarters.——Do the like for the other three
gles, and draw the dotted lines, and ſet the
hours

hours in the inſides where thoſe lines meet them, as in the figure: and the like hour-lines will be parallel to each other in all the quadrants and in all the angles.

Mark points for all theſe hours, on the upper ſide and cut out all the angular hollows, and the quadrantal ones quite through the places where their four gnomons muſt ſtand; and lay down the hours on their inſides, as in PLATE XII, and then ſet in their four gnomons, which muſt be as broad as the dial is thick; and this breadth and thickneſs muſt be large enough to keep the ſhadows of the gnomons from ever falling quite out at the ſides of the hollows, even when the ſun's declination is at the greateſt.

Laſtly, draw the equinoctial dial in the middle, all the hours of which are equidiſtant from each other; and the dial will be finiſhed.

As the ſun goes round, the broad end of the ſhadow of the ſtile *a b c d* will ſhew the hours in the quadrant *A c*, from ſun riſe till VI in the morning; the ſhadow from the end *M* will ſhew the hours on the ſide *L q* from V to IX in the morning; the ſhadow of the ſtile *e f g h* in the quadrant *D g* (in the long days) will ſhew the hours from ſun-riſe till VI in the morning; and the ſhadow of the end *N* will ſhew the morning hours, on the ſide *O r*, from III to VII.

Juſt as the ſhadow of the northern ſtile *a b c d* goes off the quadrant *A c*, the ſhadow of the ſouthern ſtile *i k l m* begins to fall within the quadrant *F l*, at VI in the morning; and ſhews the time, in that quadrant, from VI till XII at noon; and from noon till VI in the evening in the quadrant *m E*. And the ſhadow of the end *O* ſhews the time from XI in the forenoon till III in the afternoon, on the ſide *r N*; as the ſhadow of the end *P* ſhews the time from IX in the morning till I o'clock in the afternoon, on the ſide *Q s*.

At noon, when the ſhadow of the eaſtern ſtile *e f g h* goes off the quadrant *h C* (in which it ſhewed the time from VI in the morning till noon, as it did in the quadrant *g D* from ſun-riſe till VI in the morning) the ſhadow of the weſtern ſtile *n o p q* begins to enter the quadrant *Hp*; and ſhews the hours thereon from XII at noon till VI in the evening; and after *that* till ſun-ſet, in the quadrant *q G*: and the end *Q* caſts a ſhadow on the ſide *P s* from V in the evening till IX at night, if the ſun be not ſet before that time.

The ſhadow of the end *I* ſhews the time on the ſide *K p* from III till VII in the afternoon; and the ſhadow of the ſtile *a b c d* ſhews the time from VI in the evening till the ſun ſets.

The ſhadow of the upright central wire, that ſupports the globe at top, ſhews the time of the day, in the middle or equinoctial dial, all the ſummer half year, when the ſun is on the north ſide of the equator.

In this ſupplement to my book of Lectures, all the machines that I have added to my apparatus, ſince that book was printed, are deſcribed, excepting two; one of which is a model of a mill for ſawing timber, and the other is a model of the great engine at London-bridge, for raiſing water. And my reaſons for leaving them out are as follow.

Firſt, I found it impoſſible to make ſuch a drawing of the ſaw-mill as could be underſtood; becauſe, in whatever view it be taken, a great many parts of it hid others from ſight. And, in order to ſhew it in my Lectures, I am obliged to turn it into all manner of poſitions.

Secondly, Becauſe any perſon who looks on *Fig.* 1. of PLATE XII in the book, and reads the account of it in the fifth Lecture therein, will be able to form a very good idea of the London-bridge engine, which has only two wheels and two trundles more than there are in Mr. *Alderſea*'s engine, from which the ſaid figure was taken.

FINIS.